# UNIT-I
# POWER SYSTEM

## HISTORY OF ELECTRIC POWER SYSTEMS

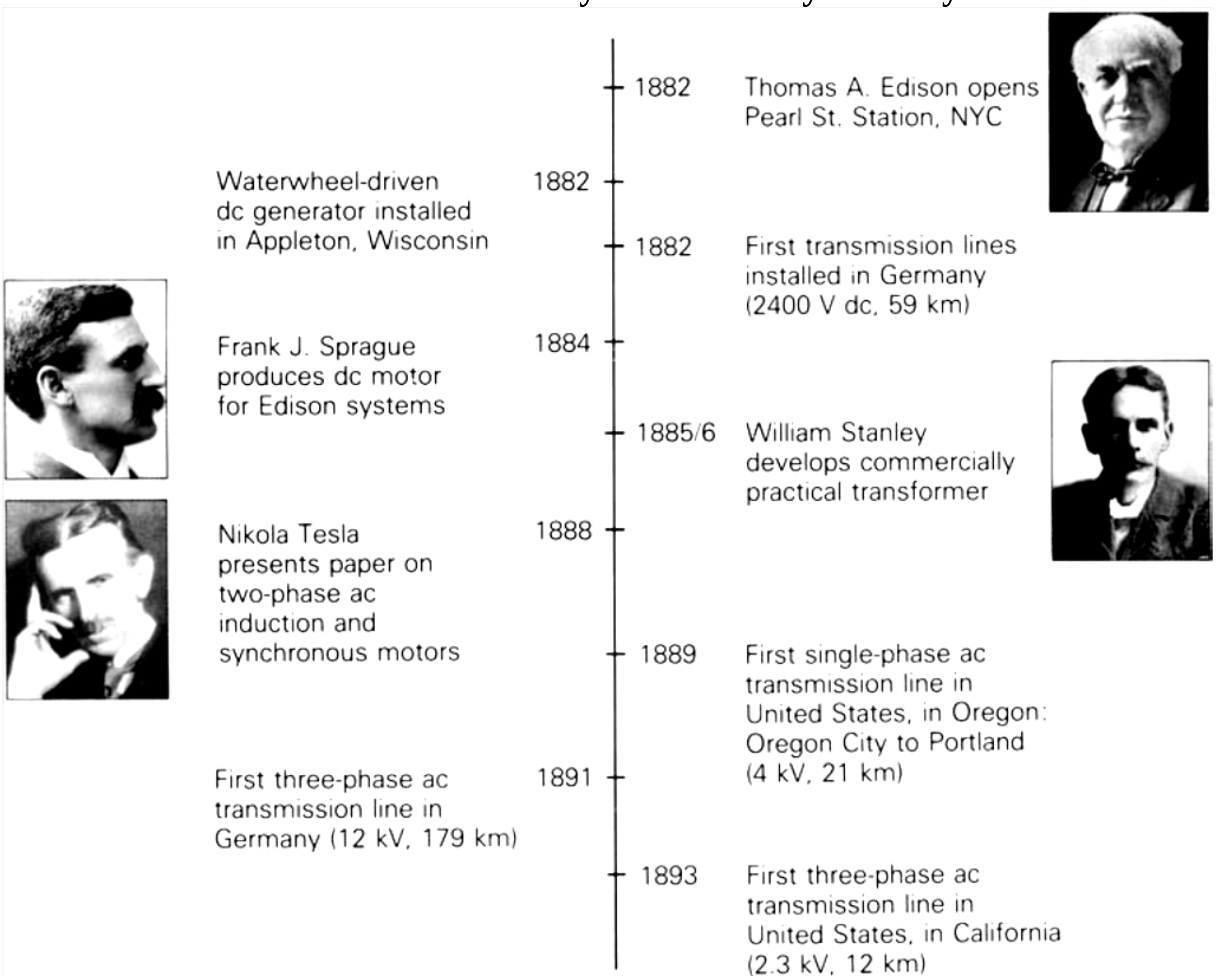
In 1878, Thomas A. Edison began work on the electric light and formulated the concept of a centrally located power station with distributed lighting serving a surrounding area. He perfected his light by October 1879, and the opening of his historic Pearl Street Station in New York City on September 4, 1882, marked the beginning of the electric utility industry. At Pearl Street, dc generators, then called dynamos, were driven by steam engines to supply an initial load of 30 kW for 110-V incandescent lighting to 59 customers in a one-square-mile (2.5-square-km) area. From this beginning in 1882 through 1972, the electric utility industry grew at a remarkable pace—a growth based on continuous reductions in the price of electricity due primarily to technological accomplishment and creative engineering.

The introduction of the practical dc motor by Sprague Electric, as well as the growth of incandescent lighting, promoted the expansion of Edison's dc systems. The development of three-wire 220-V dc systems allowed load to increase somewhat, but as transmission distances and loads continued to increase, voltage problems were encountered. These limitations of maximum distance and load were overcome in 1885 by William Stanley's development of a commercially practical transformer. Stanley installed an ac distribution system in Great Barrington, Massachusetts, to supply 150 lamps. With the transformer, the ability to transmit power at high voltage with corresponding lower current and lower line-voltage drops made ac more attractive than dc. The first single-phase ac line in the United States operated in 1889 in Oregon, between Oregon City and Portland—21 km at 4 kV.

The growth of ac systems was further encouraged in 1888 when Nikola Tesla presented a paper at a meeting of the American Institute of Electrical Engineers describing two-phase induction and synchronous motors, which made evident the advantages of poly phase versus single-phase systems. The first three phase line in Germany became operational in 1891, transmitting power 179 km at 12 kV. The first three-phase line in the United States (in California) became operational in 1893, transmitting power 12 km at 2.3 kV. The three-phase induction motor conceived by Tesla went on to become the workhorse of the industry.

In the same year that Edison's steam-driven generators were inaugurated, a waterwheel-driven generator was installed in Appleton, Wisconsin. Since then, most electric energy has been generated in steam-powered and in water powered (called hydro) turbine plants. Today, steam turbines account for more than 85% of U.S. electric energy generation, whereas hydro turbines account for about 6%. Gas turbines are used in some cases to meet peak loads. Also, the addition of wind turbines into the bulk power system is expected to grow considerably in the near future.

**Milestones of the early electric utility industry**




Steam plants are fueled primarily by coal, gas, oil, and uranium. Of these, coal is the most widely used fuel in the United States due to its abundance in the country. Although many of these coal-fueled power plants were converted to oil during the early 1970s, that trend has been reversed back to coal since the 1973–74 oil embargo, which caused an oil shortage and created a national desire to reduce dependency on foreign oil. In 2008, approximately 48% of electricity in the United States was generated from coal. In 1957, nuclear units with 90-MW steam-turbine capacity, fueled by uranium, were installed, and today nuclear units with 1312-MW steamturbine capacity are in service.

In 2008, approximately 20% of electricity in the United States was generated from uranium from 104 nuclear power plants. However, the growth of nuclear capacity in the United States has been halted by rising construction costs, licensing delays, and public opinion. Although there are no emissions associated with nuclear power generation, there are safety issues and environmental issues, such as the disposal of used nuclear fuel and the impact of heated cooling-tower water on aquatic habitats. Future technologies for nuclear power are concentrated on safety and environmental issues.

Starting in the 1990s, the choice of fuel for new power plants in the United States has been natural gas due to its availability and low cost as well as the higher efficiency, lower emissions, shorter construction-lead times, safety, and lack of controversy associated with power plants that use natural gas. Natural gas is used to generate electricity by the following processes:

(1) gas combustion turbines use natural gas directly to fire the turbine;

(2) steam turbines burn natural gas to create steam in a boiler, which is then run through the steam turbine;

(3) combined cycle units use a gas combustion turbine by burning natural gas, and the hot exhaust gases from the combustion turbine are used to boil water that operates a steam turbine; and

(4) fuel cells powered by natural gas generate electricity using electrochemical reactions by passing streams of natural gas and oxidants over electrodes that are separated by an electrolyte.

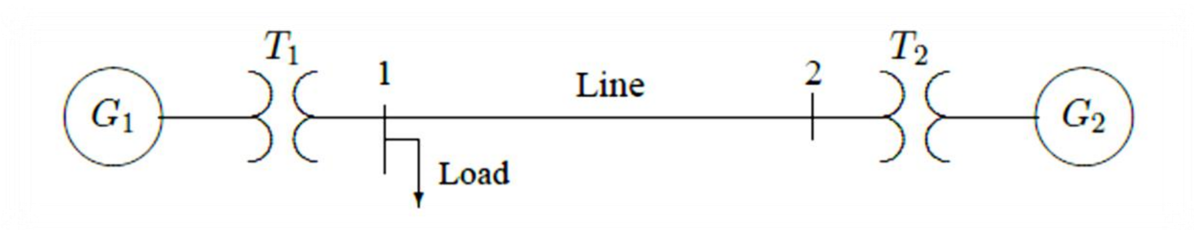
In 2008, approximately 21% of electricity in the United States was generated from natural gas. In 2008, in the United States, approximately 9% of electricity was generated by renewable sources and 1% by oil. Renewable sources include conventional hydroelectric (water power), geothermal, wood, wood waste, all municipal waste, landfill gas, other biomass, solar, and wind power. Renewable sources of energy cannot be ignored, but they are not expected to supply a large percentage of the world's future energy needs. On the other hand, nuclear fusion energy just may. Substantial research efforts have shown nuclear fusion energy to be a promising technology for producing safe, pollution-free, and economical electric energy later in the 21st century and beyond. The fuel consumed in a nuclear fusion reaction is deuterium, of which a virtually inexhaustible supply is present in seawater.

The early ac systems operated at various frequencies including 25, 50, 60, and 133 Hz. In 1891, it was proposed that 60 Hz be the standard frequency in the United States. In 1893, 25-Hz systems were introduced with the synchronous converter. However, these systems were used primarily for railroad electrification (and many are now retired) because they had the disadvantage of causing incandescent lights to flicker. In California, the Los Angeles Department of Power and Water operated at 50 Hz, but converted to 60 Hz when power from the Hoover Dam became operational in 1937. In 1949, Southern California Edison also converted from 50 to 60 Hz. Today, the two standard frequencies for generation, transmission, and distribution of electric power in the world are 60 Hz (in the United States, Canada, Japan, Brazil) and 50 Hz (in Europe, the former Soviet republics, South America except Brazil, and India). The advantage of 60-Hz systems is that generators, motors, and transformers in these systems are generally smaller than 50-Hz equipment with the same ratings. The advantage of 50-Hz systems is that transmission lines and transformers have smaller reactances at 50 Hz than at 60 Hz.

**PROBLEM.1**

Draw an impedance diagram for the electric power system shown in Figure showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

$G1$ : 90 MVA 20 kV $X$ = 9%
$T1$ : 80 MVA 20/200 kV $X$ = 16%
$T2$ : 80 MVA 200/20 kV $X$ = 20%
$G2$ : 90 MVA 18 kV $X$ = 9%
Line: 200 kV $X$ = 120 -
Load: 200 kV $S$ = 48 MW +$j$64 Mvar



The base voltage *VBG*1on the LV side of *T*1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of *T*2 at *VB*2 = 200 kV, and on its LV side at

$$V_{BG2} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base,

$$G: \qquad X = 0.09\left(\frac{100}{90}\right) = 0.10 \ \text{pu}$$

$$T_1: \qquad X = 0.16\left(\frac{100}{80}\right) = 0.20 \ \text{pu}$$

$$T_2: \qquad X = 0.20\left(\frac{100}{80}\right) = 0.25 \ \text{pu}$$

$$G_2: \qquad X = 0.09\left(\frac{100}{90}\right)\left(\frac{18}{20}\right)^2 = 0.081 \ \text{pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \ \Omega$$

The per unit line reactance is

$$Line: \qquad X = \left(\frac{120}{400}\right) = 0.30 \ \text{pu}$$

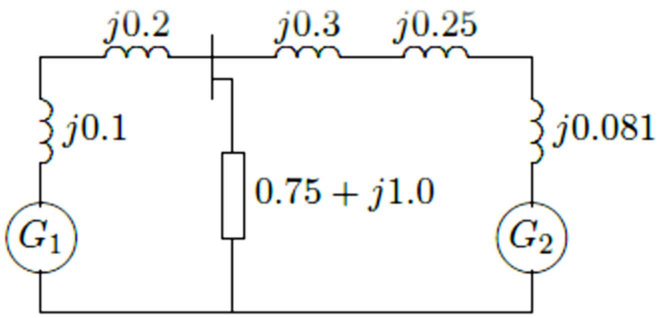
The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \ \Omega$$

The load impedance in per unit is

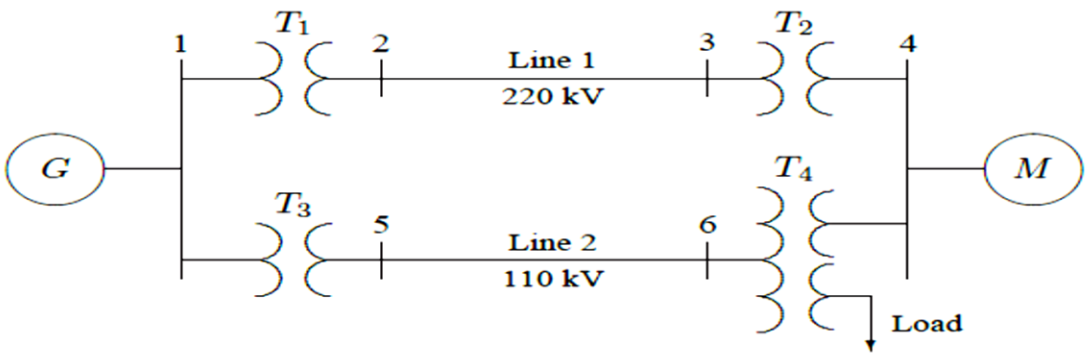
$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \ \text{pu}$$

The per unit equivalent circuit is shown in Figure




**PROBLEM.2**

The one-line diagram of a power system is shown in Figure.




The three-phase power and line-line ratings are given below.

*G*: 80 MVA 22 kV *X* = 24%

*T*1: 50 MVA 22/220 kV *X* = 10%

*T*2: 40 MVA 220/22 kV *X* = 6.0%

*T*3: 40 MVA 22/110 kV *X* = 6.4%

Line 1: 220 kV *X* = 121 -Ω

Line 2: 110 kV *X* = 42.35 -Ω

*M*: 68.85 MVA 20 kV *X* = 22.5%

Load: 10 Mvar 4 kV Δ-connected capacitors

The three-phase ratings of the three-phase transformer are

Primary: Y-connected 40MVA, 110 kV

Secondary: Y-connected 40 MVA, 22 kV

Tertiary: Δ-connected 15 MVA, 4 kV

The per phase measured reactances at the terminal of a winding with the second one short-circuited and the third open-circuited are

*ZPS* = 9.6% 40 MVA, 110 kV / 22 kV

*ZPT* = 7.2% 40 MVA, 110 kV / 4 kV
*ZST* = 12% 40 MVA, 22 kV / 4 kV
Obtain the T-circuit equivalent impedances of the three-winding transformer to the common MVA base. Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 22 kV as the voltage base for generator. The base voltage $V_{B1}$ on the LV side of $T1$ is 22 kV. Hence the base on its HV side is

$$V_{B2} = 22\left(\frac{220}{22}\right) = 220 \quad \text{kV}$$

This fixes the base on the HV side of $T2$ at $V_{B3}$ = 220 kV, and on its LV side at

$$V_{B4} = 220\left(\frac{22}{220}\right) = 22 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22\left(\frac{110}{22}\right) = 110 \quad \text{kV}$$

Voltage base for the tertiary side of $T4$ is

$$V_{BT} = 110\left(\frac{4}{110}\right) = 4 \quad \text{kV}$$

The per unit impedances on a 100 MVA base are:

$$G: \qquad X = 0.24\left(\frac{100}{80}\right) = 0.30 \ \text{pu}$$

$$T_1: \qquad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu}$$

$$T_2: \qquad X = 0.06\left(\frac{100}{40}\right) = 0.15 \ \text{pu}$$

$$T_3: \qquad X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}$$

The motor reactance is expressed on its nameplate rating of 68.85 MVA, and 20 kV. However, the base voltage at bus 4 for the motor is 22 kV, therefore

$$M: \qquad X = 0.225\left(\frac{100}{68.85}\right)\left(\frac{20}{22}\right)^2 = 0.27 \ \text{pu}$$

Impedance bases for lines 1 and 2 are

$$Z_{B2} = \frac{(220)^2}{100} = 484 \;\; \Omega$$
$$Z_{B5} = \frac{(110)^2}{100} = 121 \;\; \Omega$$

Line 1 and 2 per unit reactances are

$$Line_1: \qquad X = \left(\frac{121}{484}\right) = 0.25 \;\; \text{pu}$$
$$Line_2: \qquad X = \left(\frac{42.35}{121}\right) = 0.35 \;\; \text{pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(4)^2}{j10} = -j1.6 \;\; \Omega$$

The base impedance for the load is

$$Z_{BT} = \frac{(4)^2}{100} = 0.16 \;\; \Omega$$

Therefore, the load impedance in per unit is

$$Z_{L(pu)} = \frac{-j1.6}{0.16} = -j10 \;\; \text{pu}$$

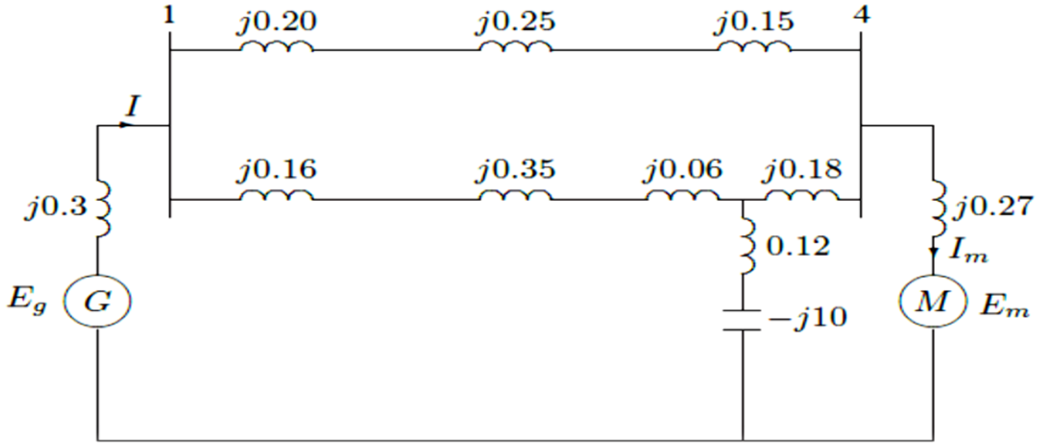
The three-winding impedances on a 100 MVA base are

$$Z_{PS} = 0.096\left(\frac{100}{40}\right) = 0.24 \;\; \text{pu}$$
$$Z_{PT} = 0.072\left(\frac{100}{40}\right) = 0.18 \;\; \text{pu}$$
$$Z_{ST} = 0.120\left(\frac{100}{40}\right) = 0.30 \;\; \text{pu}$$

The equivalent T circuit impedances are

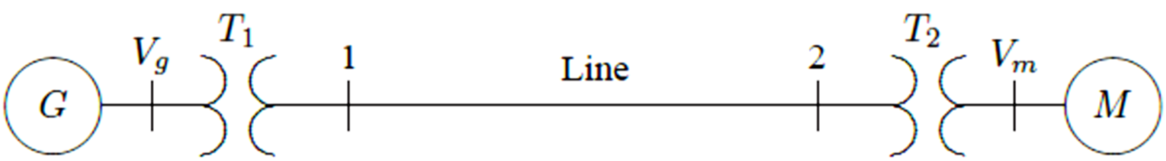
$$Z_P = \frac{1}{2}(j0.24 + j0.18 - j0.30) = j0.06 \text{ pu}$$
$$Z_S = \frac{1}{2}(j0.24 + j0.30 - j0.18) = j0.18 \text{ pu}$$
$$Z_T = \frac{1}{2}(j0.18 + j0.30 - j0.24) = j0.12 \text{ pu}$$

Per unit impedance diagram




**PROBLEM.3**

The three-phase power and line-line ratings of the electric power system shown in Figure are given below.




*G*1 : 60 MVA 20 kV *X* = 9%
*T*1 : 50 MVA 20/200 kV *X* = 10%
*T*2 : 50 MVA 200/20 kV *X* = 10%
*M* : 43.2 MVA 18 kV *X* = 8%
Line: 200 kV *Z* = 120 + *j*200 -

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

(a)

The base voltage *VBG*1on the LV side of *T*1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of *T*2 at *VB*2 = 200 kV, and on its LV side at

$$V_{Bm} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base,

$$\begin{aligned} G: &\quad X = 0.09\left(\frac{100}{60}\right) = 0.15 \ \text{pu} \\ T_1: &\quad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu} \\ T_2: &\quad X = 0.10\left(\frac{100}{50}\right) = 0.20 \ \text{pu} \\ M: &\quad X = 0.08\left(\frac{100}{43.2}\right)\left(\frac{18}{20}\right)^2 = 0.15 \ \text{pu} \end{aligned}$$

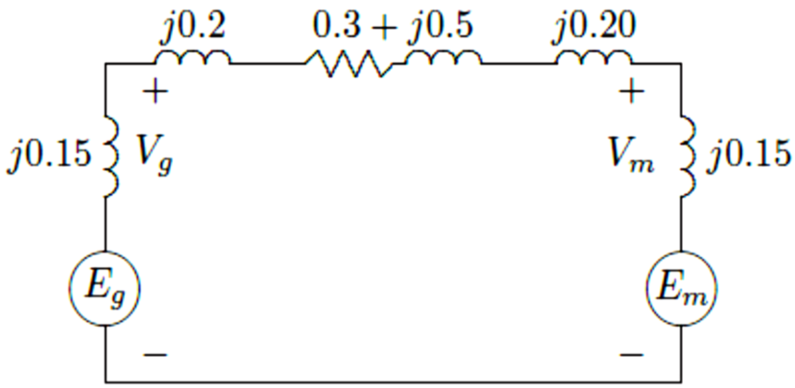
The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line impedance is

$$Line: \quad Z_{line} = \left(\frac{120 + j200}{400}\right) = 0.30 + j0.5 \ \text{pu}$$

The per unit equivalent circuit is shown in Figure.




(b) The motor complex power in per unit is

$$S_m = \frac{45\angle 36.87^\circ}{100} = 0.45\angle 36.87^\circ \ \text{ pu}$$

and the motor terminal voltage is

$$V_m = \frac{18\angle 0^\circ}{20} = 0.90\angle 0^\circ \ \text{ pu}$$

$$I = \frac{0.45\angle{-36.87^\circ}}{0.90\angle 0^\circ} = 0.5\angle{-36.87^\circ} \ \text{ pu}$$

$$V_g = 0.90\angle 0^\circ + (0.3 + j0.9)(0.5\angle{-36.87^\circ} = 1.31795\angle 11.82^\circ \ \text{ pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \ \text{ kV}$$

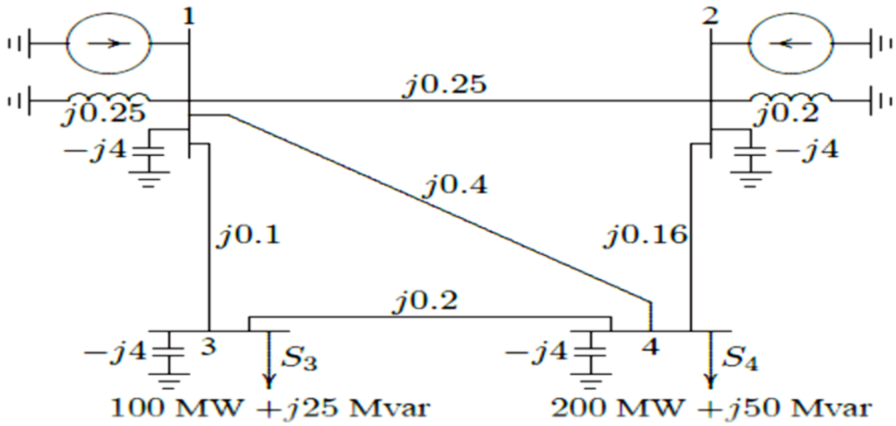
$$E_g = 0.90\angle 0^\circ + (0.3 + j1.05)(0.5\angle{-36.87^\circ} = 1.375\angle 13.88^\circ \ \text{ pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \ \text{ kV}$$

**PROBLEM.4**

A power system network is shown in Figure. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by π model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar. Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.



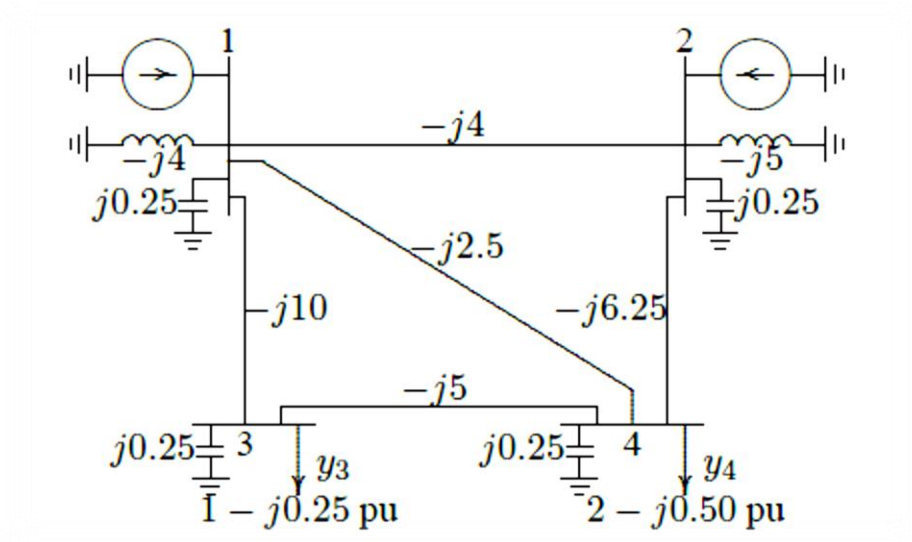

The load impedance in per unit is found from

$$Z = \frac{|V_{L-L}|^2}{S_L^*}\ \Omega \quad \& \quad Z_B = \frac{|V_B|^2}{S_B^*}\ \Omega \quad \text{or} \quad Z = \frac{|V_{pu}|^2}{S_{pu}^*}\ \text{pu}$$

$$Z_3 = \frac{(1.0)^2}{1 - j0.25} = 0.9412 + j0.2353\ \text{pu}$$

$$Z_4 = \frac{(1.0)^2}{2 - j0.5} = 0.4706 + j0.11765\ \text{pu}$$

Converting all impedances to admittances results in the admittance diagram shown in Figure




The self admittances are

$$Y_{11} = -j4 + j0.25 - j4 - j10 - j2.5 = -j20.25$$
$$Y_{22} = -j5 + j0.25 - j4 - j6.25 = -j15$$
$$Y_{33} = (1 - j0.25) + j0.25 - j10 - j5 = 1 - j15$$
$$Y_{44} = (2 - j0.5) + j0.25 - j2.5 - j6.25 - j5 = 2 - j14$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} -j20.25 & j4 & j10 & j2.5 \\ j4 & -j15 & 0 & j6.25 \\ j10 & 0 & 1 - j15 & j5 \\ j2.5 & j6.25 & j5 & 2 - j14 \end{bmatrix}$$

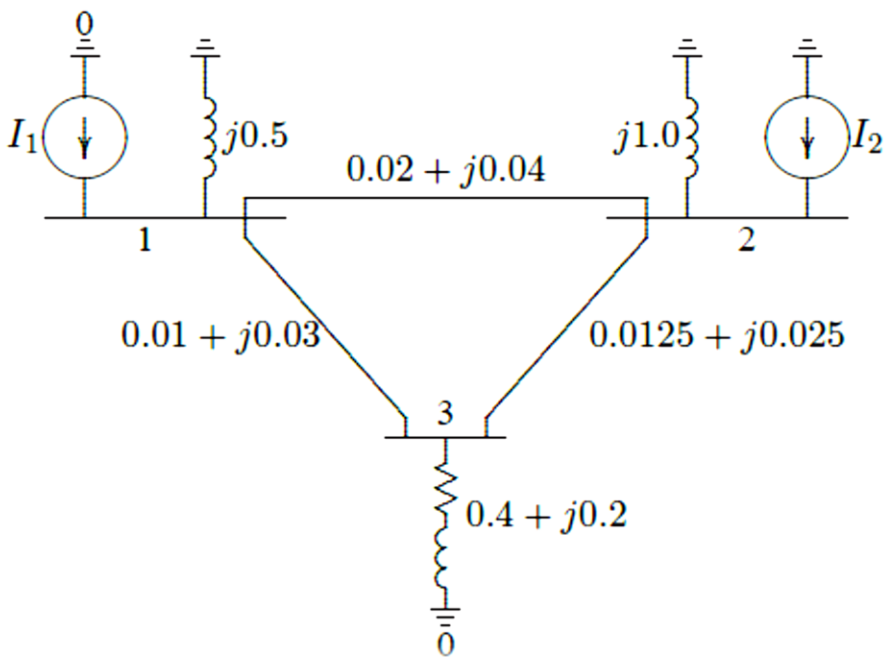
**PROBLEM.5**

A power system network is shown in Figure

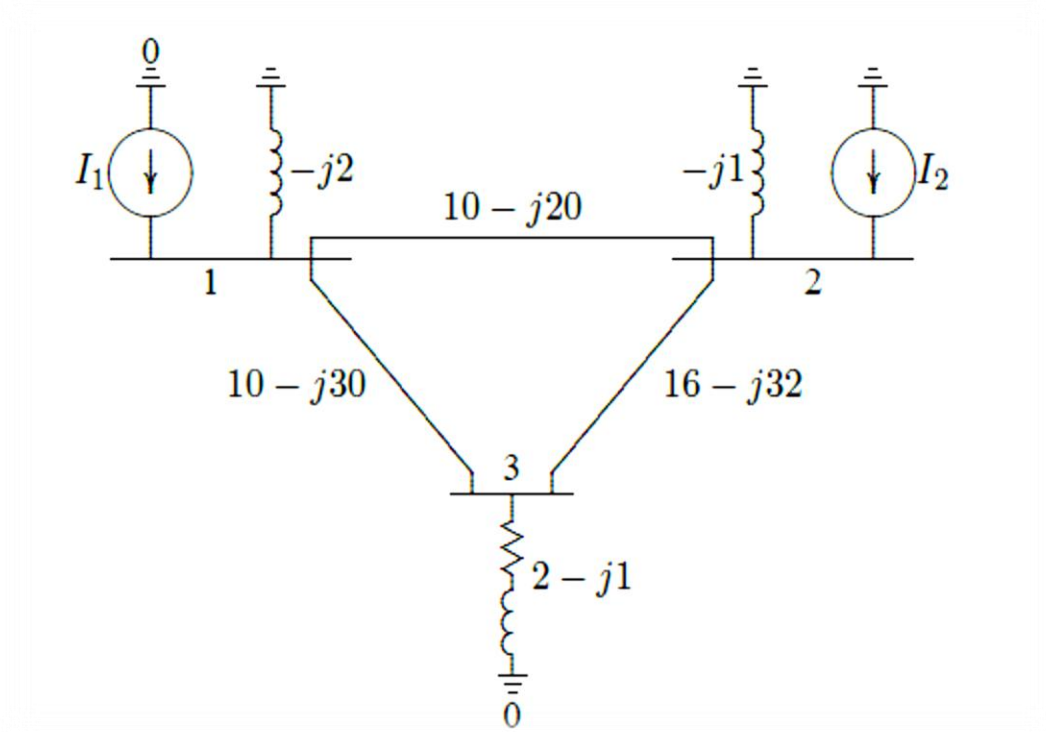
The values marked are impedances in per unit on a base of 100 MVA. The currents entering buses 1 and 2 are

$$I_1 = 1.38 - j2.72 \text{ pu}$$
$$I_2 = 0.69 - j1.36 \text{ pu}$$

Determine the bus admittance matrix by inspection.




Converting all impedances to admittances results in the admittance diagram shown in Figure



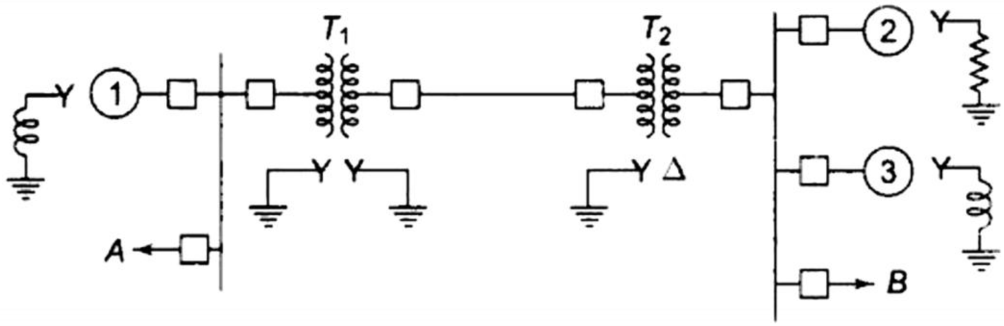
The self admittances are

$$Y_{11} = -j2 + (10 - j20) + (10 - j30) = 20 - j52$$
$$Y_{22} = -j1 + (10 - j20) + (16 - j32) = 26 - j53$$
$$Y_{33} = (2 - j1) + (10 - j30) + (16 - j32) = 28 - j63$$

Therefore, the bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 20 - j52 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j53 & -16 + j32 \\ -10 + j30 & -16 + j32 & 28 - j63 \end{bmatrix}$$

**PROBLEM:6**




Obtain the per unit impedance( reactance) diagram of the power system. Choose a common three-phase MVA base of 30 and a voltage base of 33 kV line-to-line on the transmission line. Then the voltage base in the circuit of generator 1 is 11 kV line-to-line and that in the circuits of generators 2 and 3 is 6.2 kV.

| | | | |
|---|---|---|---|
| Generator No. 1 | 30 MVA, | 10.5 kV, | $X'' = 1.6$ ohms |
| Generator No. 2 | 15 MVA, | 6.6 kV, | $X'' = 1.2$ ohms |
| Generator No. 3 | 25 MVA, | 6.6 kV, | $X'' = 0.56$ ohms |
| Transformer $T_1$ (3 phase) | 15 MVA, | 33/11 kV, | $X = 15.2$ ohms per phase on high tension side |
| Transformer $T_2$ | 15 MVA, (3 phase) | 33/6.2 kV, | $X = 16$ ohms per phase on high tension side |
| Transmission line | 20.5 ohms/phase | | |
| Load $A$ | 40 MW, | 11 kV, | 0.9 lagging power factor |
| Load $B$ | 40 MW, | 6.6 kV, | 0.85 lagging power factor |

Transmission line: $\dfrac{20.5\times 30}{(33)^2} = 0.564$

Transformer $T_1$: $\dfrac{15.2\times 30}{(33)^2} = 0.418$

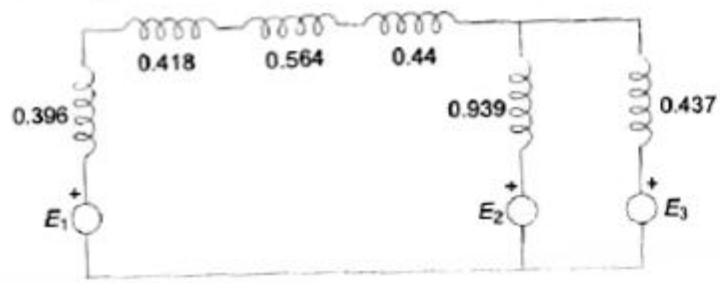
Transformer $T_2$: $\dfrac{16\times 30}{(33)^2} = 0.44$

Generator 1: $\dfrac{1.6\times 30}{(11)^2} = 0.396$

Generator 2: $\dfrac{1.2\times 30}{(6.2)^2} = 0.936$

Generator 3: $\dfrac{0.56\times 30}{(6.2)^2} = 0.437$

The reactance diagram of the system is shown in Fig. 4.8.




**PROBLEM:7**

The reactance data of generators and transformers are usually specified (or per cent) values, based on in pu equipment ratings rather than in actual ohmic values as given in Problem:6 assuming the following PU values of reactance's

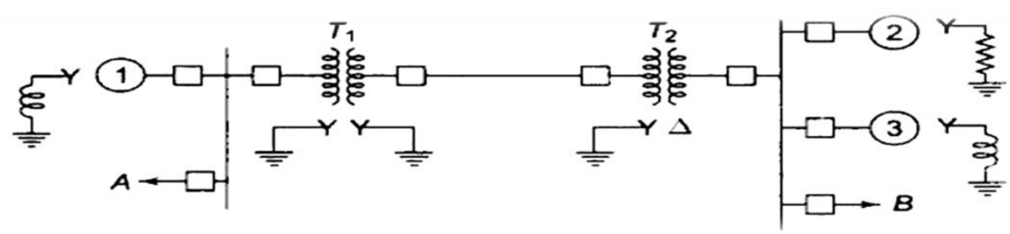
Transformer T1= 0.209 pu

Transformer T2= 0.220 pu

Generator G1 = 0.435 pu

Generator G2= 0.413 pu

Generator G 3= 0.3214 pu



| | | |
|---|---|---|
| Generator No. 1 | 30 MVA, | 10.5 kV, |
| Generator No. 2 | 15 MVA, | 6.6 kV, |
| Generator No. 3 | 25 MVA, | 6.6 kV, |
| Transformer $T_1$ (3 phase) | 15 MVA, | 33/11 kV, |
| Transformer $T_2$ | 15 MVA, (3 phase) | 33/6.2 kV, |

Transmission line 20.5 ohms/phase

| | | | |
|---|---|---|---|
| Load *A* | 40 MW, | 11 kV, | 0.9 lagging power factor |
| Load *B* | 40 MW, | 6.6 kV, | 0.85 lagging power factor |

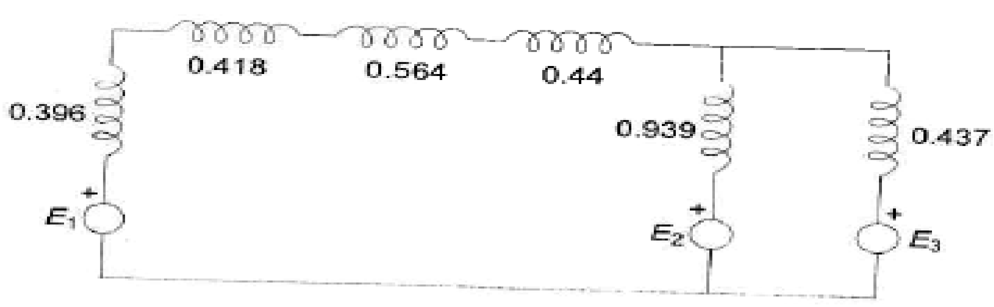
Transformer $T_1$: $$0.209 \times \frac{30}{15} = 0.418$$

Transformer $T_2$: $$0.22 \times \frac{30}{15} = 0.44$$

Generator 1: $$0.435 \times \frac{(10.5)^2}{(11)^2} = 0.396$$

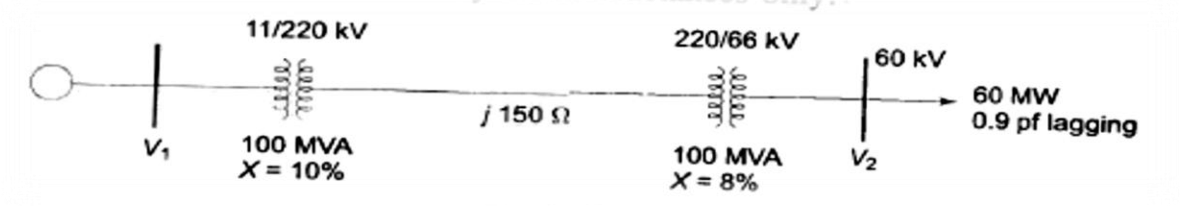
Generator 2: $$0.413 \times \frac{30}{15} \times \frac{(6.6)^2}{(6.2)^2} = 0.936$$

Generator 3: $$0.3214 \times \frac{30}{25} \times \frac{(6.6)^2}{(6.2)^2} = 0.437$$



**PROBLEM:8**

Figure shows the schematic diagram of a radial transmission system The ratings and reactances of the various components are shown therein. A load of 60 MW at 0.9-power factor lagging is tapped from the 66 kv substation which is to be maintained at 60 kv. calculate the terminal voltage of the synchronous machine. Represent the transmission line and the transformers by series reactances only.




Choose Base: 100 MVA 11 kV in generator circuit 220 kV transmission line
66 kV load bus
Reactance $T1$ = 0.1 pu
Reactance $T2$ = 0.08 pu

$$\text{Reactance transmission line} = \frac{150 \times 100}{(220)^2}$$

$$= 0.31 \text{ pu}$$

Load: $$\frac{60}{100} = 0.6 \text{ pu MW; } 0.9 \text{ pf lagging}$$

$$\text{Voltage } V_2 = \frac{60}{66} = 0.909 \angle 0^\circ$$

$$\text{Current } I_2 = \frac{0.6}{1 \times 0.9} \angle -25.8^\circ = 0.6667 \angle -25.8^\circ \text{ pu}$$

Generator terminal voltage

$$V_1 = V_2 + j\,(0.1 + 0.08 + 0.31) \times 0.6667 \angle -25.8^\circ$$

$$= 0.909 + 0.327 \angle 64.2^\circ$$

$$= 1.09 \angle 15.6^\circ$$

$$|V_1| \text{ (line)} = 1.09 \times 11 = \mathbf{12} \text{ kV}$$

**PROBLEM:9**

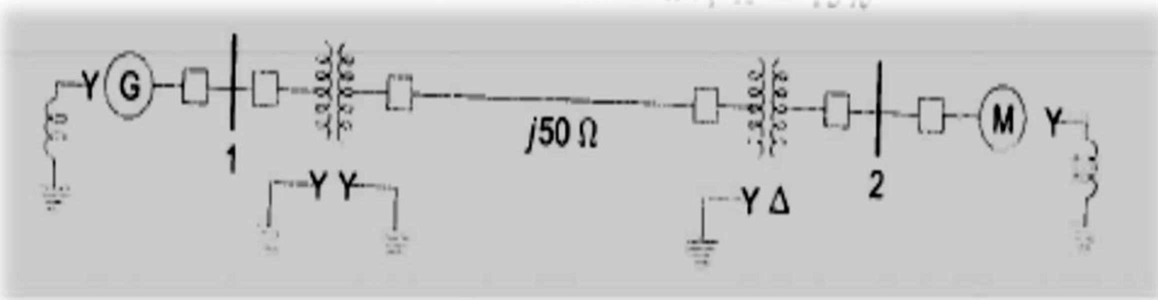
Draw the pu impedance diagram for the power system shown in fig. Neglect resistance and use a base of 100MVA, 220k v in 50Ω line. The ratings of the generator, motor and transformers are

Generator 40 MVA, 25 kV, X" = 20%

Motor 50 MVA, 11 kV, X" = 30%

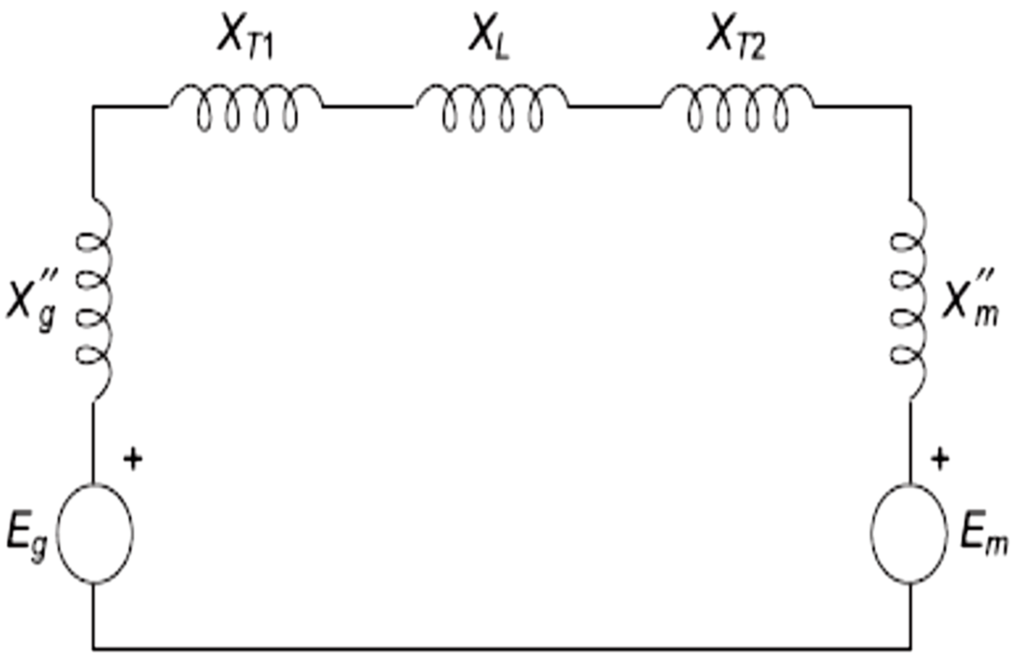
Y-Y transformer, 40 MVA, 33 Y-220 Y kV, X = 15%

Y-Δ transformer,30 MVA, 11Δ-220 Y kV, X = l5%




Base: 100 MVA

220 kV in line



$$220 \times \frac{33}{220} = 33 \text{ kV in generator}$$

$$220 \times \frac{11}{220} = 11 \text{ kV in motor}$$

Per unit reactances are:

$$X_g'' = 0.2 \left(\frac{100}{40}\right) \times \left(\frac{25}{33}\right)^2 = 0.287$$

$$X_m'' = 0.3 \times \left(\frac{100}{50}\right) = 0.6$$

$$X_{T1} = 0.15 \times \frac{100}{40} = 0.375$$

$$X_{T2} = 0.15 \times \left(\frac{100}{30}\right) = 0.5$$

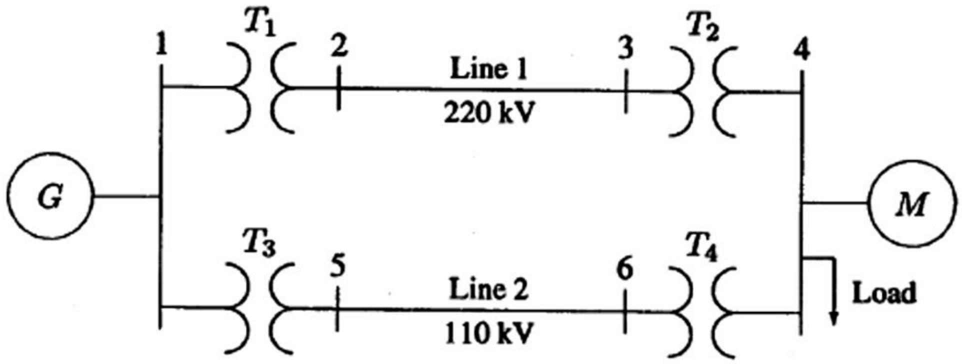
$$X_L = \frac{50 \times 100}{(220)^2} = 0.103$$

**PROBLEM:10**

The one line diagram of a three phase power system is shown in fig. Select a common base of 100MVA and 22Kv on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per unit. The manufacturer's data for each device is given as follows

| | | | |
|---|---|---|---|
| G | 90MVA | 22KV | X=18% |
| T1 | 50MVA | 22/220KV | X=10% |
| T2 | 40MVA | 220/11KV | X=6% |
| T3 | 40MVA | 22/110KV | X=6.4% |
| T4 | 40MVA | 110/11KV | X=8% |
| M | 66.5MVA | 10.45KV | X=18.5% |

The three phase load at bus 4 absorbs 57MVA, 0.6 power factor lagging at 10.45KV. Line 1 & 2 have reactance of 48.4 & 65.43Ω respectively.




First, the voltage bases must be determined for all sections of the network. The generator rated voltage is given as the base voltage at bus 1. This fixes the voltage bases for the remaining buses in accordance to the transformer turns ratios. The base voltage $V_{B1}$ on the LV side of $T_1$ is 22 kV. Hence the base on its HV side is

$$V_{B2} = 22(\frac{220}{22}) = 220 \quad \text{kV}$$

This fixes the base on the HV side of $T_2$ at $V_{B3} = 220$ kV, and on its LV side at

$$V_{B4} = 220(\frac{11}{220}) = 11 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22(\frac{110}{22}) = 110 \quad \text{kV}$$

$$G: \;\; X = 0.18\left(\frac{100}{90}\right) = 0.20 \quad \text{pu}$$

$$T_1: \;\; X = 0.10\left(\frac{100}{50}\right) = 0.20 \quad \text{pu}$$

$$T_2: \;\; X = 0.06\left(\frac{100}{40}\right) = 0.15 \quad \text{pu}$$

$$T_3: \;\; X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}$$

$$T_4: \;\; X = 0.08\left(\frac{100}{40}\right) = 0.2 \quad \text{pu}$$

$$M:\ X = 0.185\left(\frac{100}{66.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.25 \quad \text{pu}$$

$$Z_{B2} = \frac{(220)^2}{100} = 484 \quad \Omega$$
$$Z_{B5} = \frac{(110)^2}{100} = 121 \quad \Omega$$

Line 1 and 2 per-unit reactances are

$$\text{Line 1:}\quad X = \left(\frac{48.4}{484}\right) = 0.10 \quad \text{pu}$$
$$\text{Line 2:}\quad X = \left(\frac{65.43}{121}\right) = 0.54 \quad \text{pu}$$

The load apparent power at 0.6 power factor lagging is given by

$$S_{L(3\phi)} = 57\angle 53.13^\circ \quad \text{MVA}$$

Hence, the load impedance in ohms is

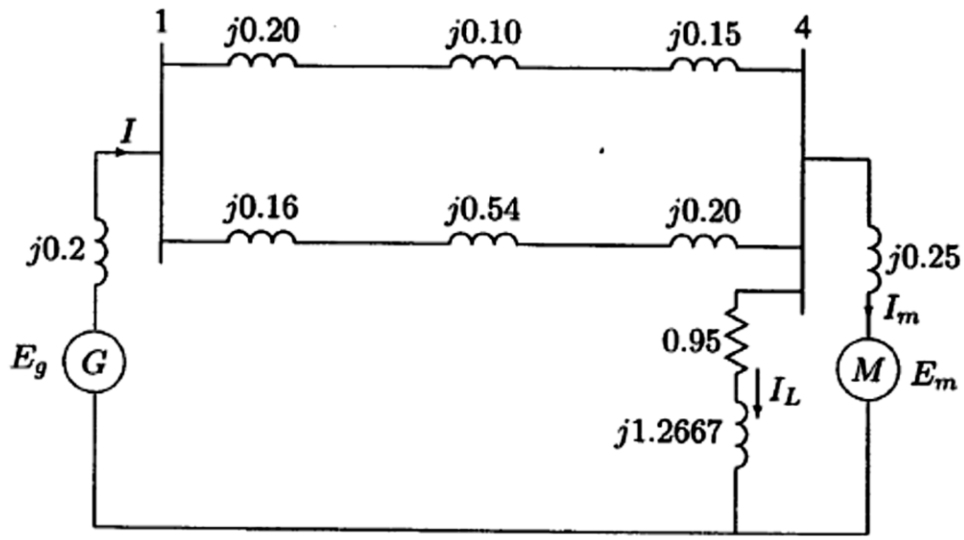
$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(10.45)^2}{57\angle -53.13^\circ} = 1.1495 + j1.53267 \quad \Omega$$

The base impedance for the load is

$$Z_{B4} = \frac{(11)^2}{100} = 1.21 \quad \Omega$$

Therefore, the load impedance in per-unit is

$$Z_{L(pu)} = \frac{1.1495 + j1.53267}{1.21} = 0.95 + j1.2667 \quad \text{pu}$$

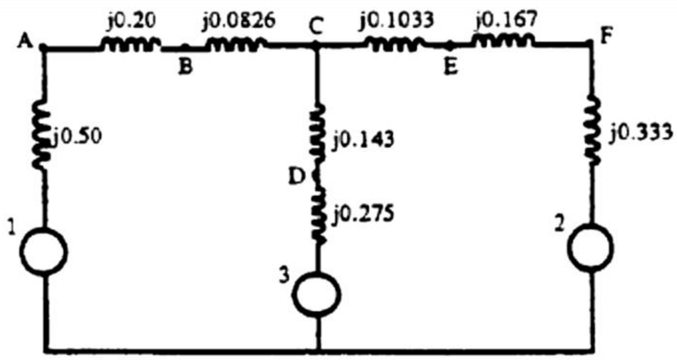



**PROBLEM:11**

The single line diagram of an unloaded power system is shown in figure. Reactances of the two sections of the transmission line are shown on the diagram. The generators and transformer are rated as follows:

| | | | |
|---|---|---|---|
| Generator1 | 20MVA | 13.8KV | $X''_d$=0.20pu |
| Generator2 | 30MVA | 18KV | $X''_d$=0.20pu |
| Generator3 | 30MVA | 20KV | $X''_d$=0.20pu |
| Transformer1 | 25MVA | 220Y/13.8ΔKV | X=10% |
| Transformer2 | 1ϕ units each rated 10MVA | 127/18KV | X=10% |
| Transformer3 | 35MVA | 220Y/22Y KV | X=10% |

Draw the impedance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the single line diagram. Choose a base of 50MVA, 13.8KV in the circuit of generator 1




$$\text{Gen 1: } X'' = 0.2 \times \frac{50}{20} = 0.50 \text{ per unit}$$

$$3\phi \text{ rating } T_2 = 220/18 \text{ kV, } 30 \text{ MVA}$$

$$\text{Base in trans. line: } 220 \text{ kV, } 50 \text{ MVA}$$

$$\text{Base for Gen 2} = 18 \text{ kV}$$

$$\text{Gen 2: } X'' = 0.2 \times \frac{50}{30} = 0.333 \text{ per unit}$$

$$\text{Base for Gen 3} = 22 \text{ kV}$$

$$\text{Gen 3: } X'' = 0.2\left(\frac{20}{22}\right)^2 \times \frac{50}{30} = 0.275 \text{ per unit}$$

$$\text{Transformer } T_1\text{: } X = .01 \times \frac{50}{25} = 0.20 \text{ per unit}$$

$$\text{Transformer } T_2\text{: } X = .01 \times \frac{50}{30} = 0.167 \text{ per unit}$$

$$\text{Transformer } T_3\text{: } X = .01 \times \frac{50}{35} = 0.143 \text{ per unit}$$

Transmission lines:

$$\text{Base } Z = \frac{220^2}{50} = 968\ \Omega$$

$$\frac{80}{968} = 0.0826 \text{ per unit} \qquad \frac{100}{968} = 0.1033 \text{ per unit}$$

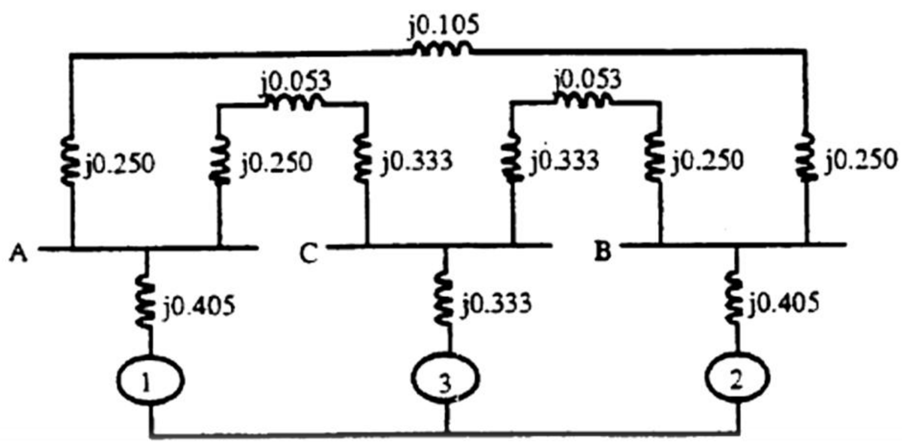
## PROBLEM:12

The ratings of the generators, motors and transformers are:

| | | | |
|---|---|---|---|
| Generator1 | 20MVA | 18KV | $X''_d$=20% |
| Generator2 | 20MVA | 18KV | $X''_d$=20% |

| | | | |
|---|---|---|---|
| Synchronous motor 3 | 30MVA | 13.8KV | X”$_d$=20% |
| 3ϕ Y-Y Transformers | 20MVA | 138Y/20Y KV | X=10% |
| 3ϕ Y-Δ Transformers | 15MVA | 138Y/13.8Δ KV | X=10% |

Draw the impedance diagram for the power system, Mark impedances in per unit. Neglect resistance and use a common base of 50MVA, 138KV in the 40Ω line.



Base voltages are:

| | |
|---|---|
| 40 Ω lines | 138 kV |
| 20 Ω lines | 138 kV |
| Gen. 1 & 2 | 20 kV |
| Motor 3 | 13.8 kV |

$$\text{Base impedance in lines} = \frac{138^2}{50} = 381\ \Omega$$

$$40\ \Omega \text{ line: } Z = \frac{40}{381} = 0.105 \text{ per unit}$$

$$20\ \Omega \text{ line: } Z = \frac{20}{381} = 0.053 \text{ per unit}$$

Transformers:

$$\text{Y-Y} = 0.1 \times \frac{50}{20} = 0.250 \text{ per unit}$$

$$\text{Y-}\Delta = 0.1 \times \frac{50}{15} = 0.333 \text{ per unit}$$

Gens. 1 & 2:

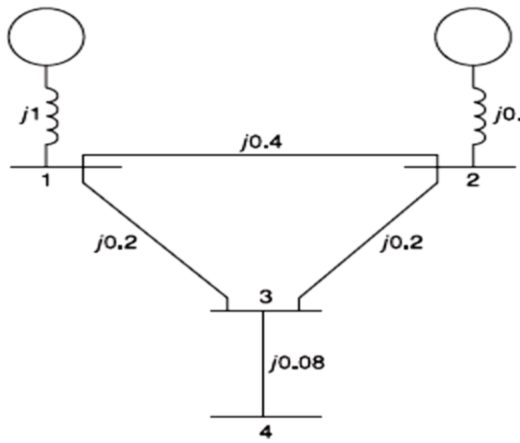
$$X'' = 0.20 \times \left(\frac{18}{20}\right)^2 \times \frac{50}{20} = 0.405 \text{ per unit}$$

Motor 3:

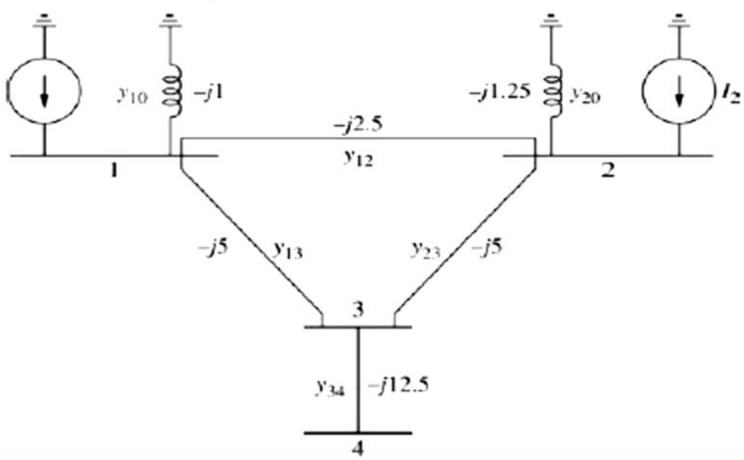
$$X'' = 0.20 \times \frac{50}{30} = 0.333 \text{ per unit}$$

**PROBLEM:13**

Given the impedance diagram of a simple system as shown in Figure, draw the admittance diagram for the system and develop the 4 x 4 bus admittance matrix Ybus by inspection.




The admittance diagram for the system is shown below:




$$\overline{Y}_{BUS} = \begin{bmatrix} \overline{Y}_{11} & \overline{Y}_{12} & \overline{Y}_{13} & \overline{Y}_{14} \\ \overline{Y}_{21} & \overline{Y}_{22} & \overline{Y}_{23} & \overline{Y}_{24} \\ \overline{Y}_{31} & \overline{Y}_{32} & \overline{Y}_{33} & \overline{Y}_{34} \\ \overline{Y}_{41} & \overline{Y}_{42} & \overline{Y}_{43} & \overline{Y}_{44} \end{bmatrix} = j \begin{bmatrix} -8.5 & 2.5 & 5.0 & 0 \\ 2.5 & -8.75 & 5.0 & 0 \\ 5.0 & 5.0 & -22.5 & 12.5 \\ 0 & 0 & 12.5 & -12.5 \end{bmatrix} S$$

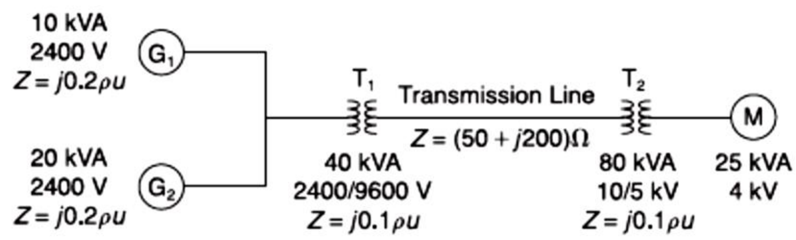
where $\overline{Y}_{11} = \overline{y}_{10} + \overline{y}_{12} + \overline{y}_{13}$; $\overline{Y}_{22} = \overline{y}_{20} + \overline{y}_{12} + \overline{y}_{23}$; $\overline{Y}_{23} = \overline{y}_{13} + \overline{y}_{23} + \overline{y}_{34}$

$\overline{Y}_{44} = y_{34}$; $\overline{Y}_{12} = \overline{Y}_{21} = -\overline{y}_{12}$; $\overline{Y}_{13} = \overline{Y}_{31} = -\overline{y}_{13}$; $\overline{Y}_{23} = \overline{Y}_{32} = -\overline{y}_{23}$

and $\overline{Y}_{34} = \overline{Y}_{43} = -\overline{y}_{34}$

**PROBLEM:14**

For the system shown in Figure, draw an impedance diagram in per unit, by choosing 100 kVA to be the base kVA and 2400 V as the base voltage for the generators.




$$G_1:\ \bar{Z} = j0.2\left(\frac{2400}{2400}\right)^2\left(\frac{100}{10}\right) = j2\,\text{pu}$$

$$G_2:\ \bar{Z} = j0.2\left(\frac{2400}{2400}\right)^2\left(\frac{100}{20}\right) = j1\,\text{pu}$$

$$T_1:\ \bar{Z} = j\,0.1\left(\frac{2400}{2400}\right)^2\left(\frac{100}{40}\right) = j0.25\,\text{pu}$$

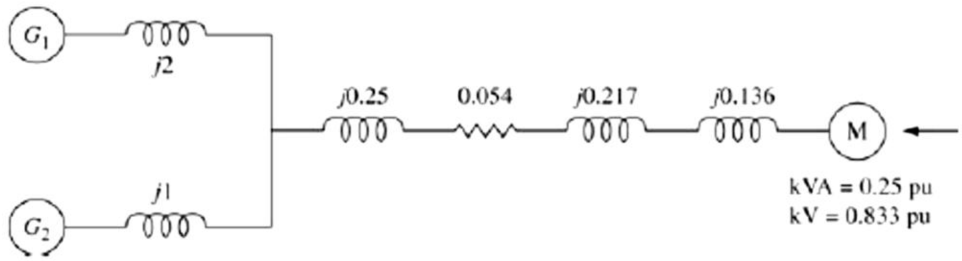
$$T_2:\ \bar{Z} = j0.1\left(\frac{10}{9.6}\right)^2\left(\frac{100}{80}\right) = j0.136\,\text{pu}$$

For the transmission-line zone, base impedance $= \dfrac{(9600)^2}{100\times 10^3}$

$$\therefore\ \bar{Z}_{LINE} = (50 + j200)\frac{100\times 10^3}{(9600)^2} = (0.054 + j0.217)\,\text{pu}$$

$$M;\ \text{kV}A = \frac{25}{100} = 0.25\,\text{pu};\ 4\,\text{kV} = \frac{4}{4.8} = 0.833\,\text{pu}$$

The impedance diagram for the system is shown below:




**PROBLEM:15**

Consider the single-line diagram of the power system shown in Figure Equipment ratings are:

Generator 1: 1000 MVA, 18 kV, X"=0.2 per unit

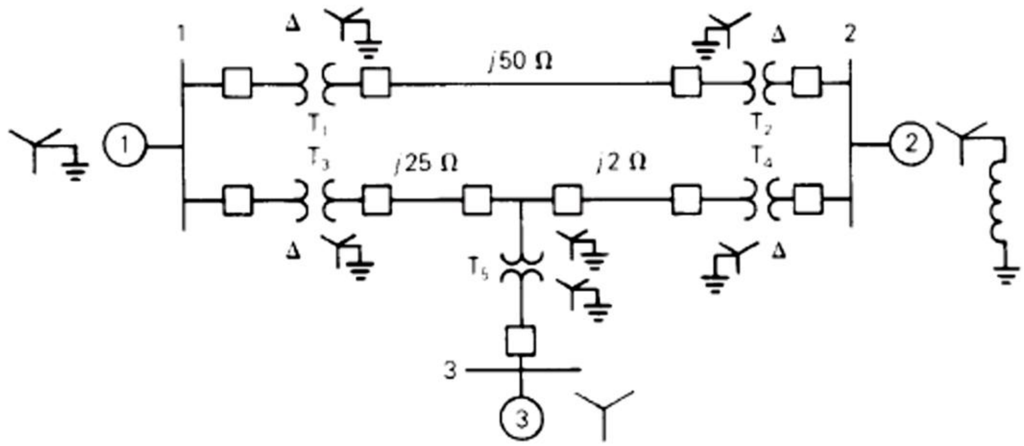
Generator 2: 1000 MVA, 18 kV, X"=0.2 per unit

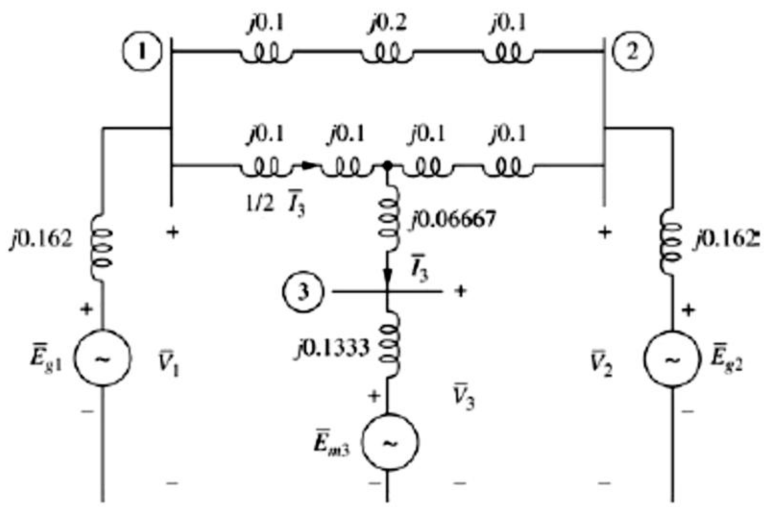
Synchronous motor 3: 1500 MVA, 20 kV, X"=0.2 per unit

Three-phase Δ–Y transformers T1, T2, T3, T4: 1000 MVA, 500 kV Y/20 kV Δ, X = 0.1

Three-phase Y–Y transformer T5: 1500 MVA, 500 kV Y/20 kV Y, X =0.1

Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the equivalent reactance diagram. Use a base of 100 MVA and 500 kV for the 50-ohm line. Determine the per-unit reactances.






$$S_{base} = 100 \text{ MVA}$$

$$V_{base\,H} = 500 \text{ kV in transmission-line Zones}$$

$$V_{base\,X} = 20 \text{ kV in motor/generator Zones}$$

$$X''_{g1} = X''_{g2} = 0.2\left(\frac{18}{20}\right)^2 = 0.162\,\text{pu}$$

$$X''_{m3} = 0.2\left(\frac{1000}{1500}\right) = 0.1333\,\text{pu}$$

$$X_{T1} = X_{T2} = X_{T3} = X_{T4} = 0.1\text{pu}$$

$$X_{T5} = 0.1\left(\frac{1000}{1500}\right) = 0.06667\,\text{pu}$$

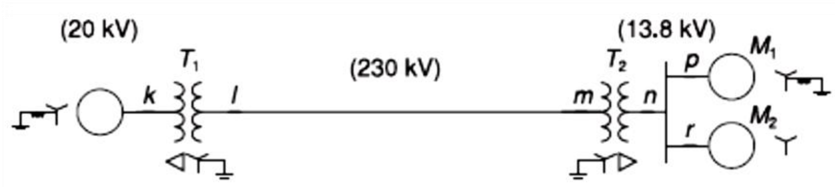
$$Z_{base\,H} = (500)^2/1000 = 250\,\Omega$$

$$X_{line}\,50 = 50/250 = 0.2\,\text{pu}$$

$$X_{line}\,25 = 25/250 = 0.1\,\text{pu}$$

## PROBLEM:16

Figure shows a one-line diagram of a system in which the three-phase generator is rated 300 MVA, 20 kV with a subtransient reactance of 0.2 per unit and with its neutral grounded through a 0.4-Ω reactor. The transmission line is 64 km long with a series reactance of 0.5 Ω/km. The three-phase transformer T1 is rated 350 MVA, 230/20 kV with a leakage reactance of 0.1 per unit. Transformer T2 is composed of three single-phase transformers, each rated 100 MVA, 127/13.2 kV with a leakage reactance of 0.1 per unit. Two 13.2-kV motors M1 and M2 with a subtransient reactance of 0.2 per unit for each motor represent the load. M1 has a rated input of 200 MVA with its neutral grounded through a 0.4-Ω current-limiting reactor. M2 has a rated input of 100 MVA with its neutral not connected to ground. Neglect phase shifts associated with the transformers. Choose the generator rating as base in the generator circuit and draw the positive-sequence reactance diagram showing all reactances in per unit.




Three-phase rating of transformer $T2$ is 3×100 = 300MVA and its line-to-line voltage ratio is √3 (127) :13.2 or 220 :13.2 kV. Choosing a common base of 300 MVA for the system, and selecting a base of 20 kV in the generator circuit, The voltage base in the transmission line is 230 kV and the voltage base in the motor circuit is 230 (13.2/220) = 13.8 kV transformer reactances converted to the proper base are given by

$$T_1 : X = 0.1 \times \frac{300}{350} = 0.0857; T_2 : 0.1\left(\frac{13.2}{13.8}\right)^2 = 0.0915$$

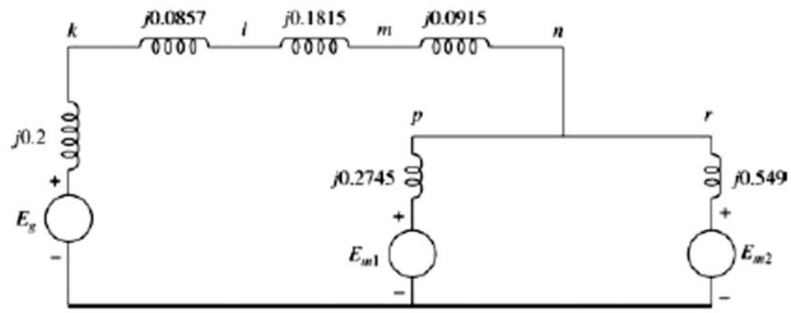
Base impedance for the transmission line is $(230)^2/300=176.3$ Ω

The reactance of the line in per unit is then $\frac{0.5\times 64}{176.3} = 0.1815$

Reactance $X''_\alpha$ of motor $M_1 : 0.2\left(\frac{300}{200}\right)\left(\frac{13.2}{13.8}\right)^2 = 0.2745$

Reactance $X''_\alpha$ of motor $M_2 : 0.2\left(\frac{300}{100}\right)\left(\frac{13.2}{13.8}\right)^2 = 0.549$

Neglecting transformer phase shifts, the positive-sequence reactance diagram is shown in figure below:




**PROBLEM:17**

Consider the single-line diagram of a power system shown in Figure with equipment ratings given below:

Generator G1: 50 MVA, 13.2 kV, x = 0.15 pu

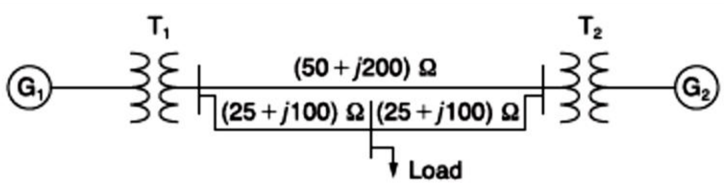
Generator G2: 20 MVA, 13.8 kV, x =0.15 pu

three-phase Δ–Y transformer T1: 80 MVA, 13.2 Δ/165 Y kV, X = 0.1 pu

three-phase Y–Δ transformer T2: 40 MVA, 165 Y/13.8 Δ kV, X = 0.1 pu

Load: 40 MVA, 0.8 p.f. lagging, operating at 150 kV

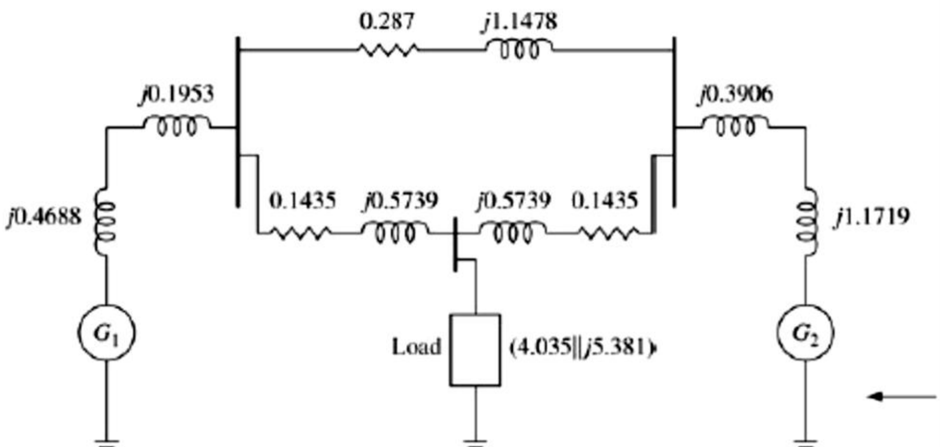
Choose a base of 100 MVA for the system and 132-kV base in the transmission-line circuit. Let the load be modeled as a parallel combination of resistance and inductance. Neglect transformer phase shifts. Draw a per-phase equivalent circuit of the system showing all impedances in per unit.



Base kV in transmission-line circuit = 132 kV

Base kV in the generator $G_1$ circuit = $132 \times \frac{13.2}{165} = 10.56\,\text{kV}$

Base kV in the generator $G_2$ circuit = $132 \times \frac{13.8}{165} = 11.04\,\text{kV}$




Impedance diagram of the system with pu values

On the common base of 100 MVA for the entire system,

$$G_1 : \bar{Z} = j0.15 \times \frac{100}{50} \times \left(\frac{13.2}{10.56}\right)^2 = j0.4688\,\text{pu}$$

$$G_2 : \bar{Z} = j0.15 \times \frac{100}{20} \times \left(\frac{13.8}{11.04}\right)^2 = j1.1719\,\text{pu}$$

$$T_1 : \bar{Z} = j0.1 \times \frac{100}{80} \times \left(\frac{13.2}{10.56}\right)^2 = j0.1953\,\text{pu}$$

$$T_2 : \bar{Z} = j0.1 \times \frac{100}{40} \times \left(\frac{13.8}{11.04}\right)^2 = j0.3906\,\text{pu}$$

Base impedance in transmission-line circuit is

$$\frac{(132)^2}{100} = 174.24\Omega$$

$$\bar{Z}_{TR.LINE\,1} = \frac{50 \times j200}{174.24} = 0.287 + j1.1478\,\text{pu}$$

$$\bar{Z}_{TR.LINE\,2} = \frac{25 \times j100}{174.24} = 0.1435 + j0.5739\,\text{pu}$$

$$LOAD: 40(0.8 + j0.6) = (32 + j24)\,\text{MVA}$$

$$R_{LOAD} = \frac{(150)^2}{32} = 703.1\Omega = \frac{703.1}{174.24}\,\text{pu} = 4.035\,\text{pu}$$

$$X_{LOAD} = \frac{(150)^2}{24} = 937.5\Omega = \frac{937.5}{174.24}\,\text{pu} = 5.381\,\text{pu}$$

$$\bar{Z}_{LOAD} = (R_{LOAD} \parallel jX_{LOAD})$$

# UNIT-II
# POWER FLOW ANALYSIS

## Introduction

A load flow (sometimes known as a power flow) is power system jargon for the steady-state solution of an electrical power network. It does not essentially differ from the solution of any other type of network except that certain constraints are peculiar to power systems and, in particular, the formulation is non-linear leading to the need for an iterative solution.

In previous chapters the manner in which the various components of a power system may be represented by equivalent circuits has been demonstrated. It should be stressed that the simplest representation of items of plant should always be used, consistent with the accuracy of the information available. There is no merit in using very complicated machine and line models when the load and other data are known only to a limited accuracy, for example, the long-line representation should only be used where absolutely necessary.

Similarly, synchronous machine models of more sophistication than those given in this text are needed only for very specialized purposes, for example in some stability studies. Usually, the size and complexity of the network itself provides more than sufficient intellectual stimulus without undue refinement of the components. Often, in high voltage networks, resistance may be neglected with little loss of accuracy and an immense saving in computation.

Load flow studies are performed to investigate the following features of a power system network:

1. Flow of MW and MVAr in the branches of the network.
2. Busbar (node) voltages.
3. Effect of rearranging circuits and incorporating new circuits on system loading.
4. Effect of temporary loss of generation and transmission circuits on system loading (mainly for security studies).
5. Effect of injecting in-phase and quadrature boost voltages on system loading.
6. Optimum system running conditions and load distribution.
7. Minimizing system losses.
8. Optimum rating and tap-range of transformers.
9. Improvements from change of conductor size and system voltage.

Planning studies will normally be performed for minimum-load conditions (examining the possibility of high voltages) and maximum-load conditions (investigating the possibility of low voltages and instability).

Having ascertained that a network behaves reasonably under these conditions, further load flows will be performed to optimize voltages, reactive power flows and real power losses. The design and operation of a power network to obtain optimum economy is of paramount importance and the furtherance of this ideal is achieved by the use of centralized automatic control of generating stations through system control centres.

These control systems often undertake repeated load flow calculations in close to real time. Although the same approach can be used to solve all load flow problems, for example the nodal voltage method, the object should be to use the quickest and most efficient method for the particular type of problem. Radial networks will require less sophisticated methods than closed loops. In very large networks the problem of organizing the data is almost as important as the method of solution, and the calculation must be carried out on a systematic basis and here the nodal voltage method is often the most convenient. Methods such as network reduction combined with the Thevenin or superposition theorems are at their best with smaller networks. In the nodal method, great numerical accuracy is required in the computation as the currents in the branches are derived from the voltage differences between the ends. These differences are small in well designed networks so the method is ideally suited for computation using digital computers and the per unit system.

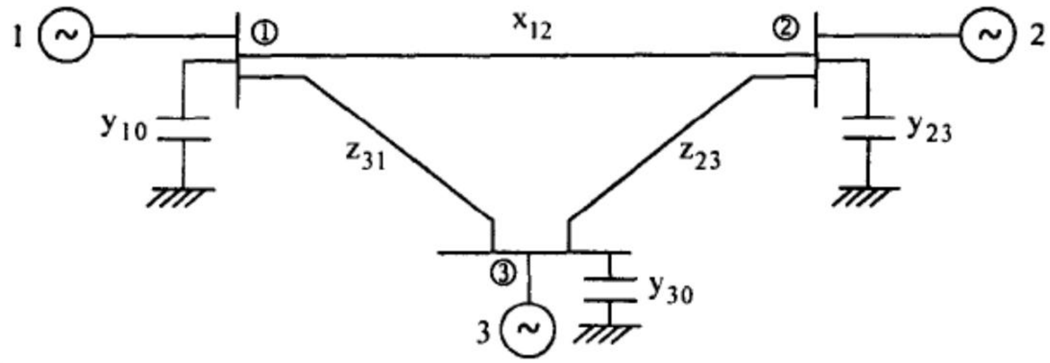
**Bus Classification**

| Bus | Specified variables | Computed variables |
|---|---|---|
| Slack - bus | Voltage magnitude and its phase angle | Real and reactive powers |
| Generator bus (PV - bus or voltage controlled bus) | Magnitudes of bus voltages and real powers (limit on reactive powers) | Voltage phase angle and reactive power. |
| Load bus | Real and reactive powers | Magnitude and phase angle of bus voltages |

**Modelling for Load Flow Studies**

Bus admittance formation

Consider the transmission system shown in Fig. Three bus transmission system The line impedances joining buses 1,2 and 3 are denoted by $z_{12}$, $Z_{22}$ and $z_{31}$ respectively. The corresponding line admittances are $Y_{12}$,$Y_{22}$ and $Y_{31}$ The total capacitive susceptances at the buses are represented by $Y_{IO}$,$Y_{20}$ and $Y_{30}$.




Applying Kirchoff's current law at each bus In matrix from
Where

$$I_1 = V_1\, y_{10} + (V_1 - V_2)\, y_{12} + (V_1 - V_3)\, y_{13}$$
$$I_2 = V_2\, y_{20} + (V_2 - V_1)\, y_{21} + (V_2 - V_3)\, y_{23}$$
$$I_3 = V_3\, y_{30} + (V_3 - V_1)\, y_{31} + (V_3 - V_2)\, y_{32}$$

In matrix from

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{10} + y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{20} + y_{12} + y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{30} + y_{13} + y_{23} \end{bmatrix} \times$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} , \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where

$$Y_{11} = y_{10} + y_{12} + y_{13}$$
$$Y_{22} = y_{20} + y_{12} + y_{23}$$
$$Y_{33} = y_{30} + y_{13} + y_{23}$$

are the self admittances forming the diagonal terms and

$$Y_{12} = Y_{21} = -y_{12}$$
$$Y_{13} = Y_{31} = -y_{13}$$
$$Y_{23} = Y_{32} = -y_{23}$$

are the mutual admittances forming the off-diagonal elements of the bus admittance matrix. For an n-bus system, the elements of the bus admittance matrix can be written down merely by inspection of the network as diagonal terms

$$Y_{ii} = y_{i0} + \sum_{\substack{k=1 \\ k \neq i}}^{n} y_{ik}$$

off and diagonal terms

$$Y_{ik} = -y_{ik}$$

If the network elements have mutual admittance (impedance), the above formulae will not apply. For a systematic formation of the y-bus, linear graph theory with singular transformations may be used.

***System Model for Load Flow Studies***

The variable and parameters associated with bus i and a neighboring bus k are represented in the usual notation as follows :

$$V_i = |V_i| \exp j\,\delta_i = V_i\,(\text{Cos}\,\delta_i + j\,\sin\,\delta_i)$$

**Bus admittance,**

$$Y_{ik} = |Y_{ik}| \exp j\,\theta_{ik} = |Y_{ik}|\,(\text{Cos}\,q_{ik} + j\,\sin\,\theta_{ik})$$

**Complex power,**

$$S_i = P_i + j\,Q_i = V_i\;I^*_I$$

**Using the indices G and L for generation and load,**

$$P_i = P_{Gi} - P_{Li} = \text{Re}\,[V_i\;I^*_i]$$
$$Q_i = Q_{Gi} - Q_{Li} = \text{Im}\,[V_i\;I^*_i]$$

**The bus current is given by**

$$I_{BUS} = Y_{BUS} \cdot V_{BUS}$$

$$I_i^* = \frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k$$

$$V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right]$$

Further,

$$P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

In the polar form

$$P_i + jQ_i = \sum_{k=1}^{n} |V_i \quad V_k \quad Y_{ik}| \exp j(\delta_i \quad - \delta_k \quad - \theta_{ik})$$

so that

$$P_i = \sum_{k=1}^{n} |V_i \quad V_k \quad Y_{ik}| \cos(\delta_i \quad - \delta_k \quad - \theta_{ik})$$

and

$$Q_i = \sum_{k=1}^{n} |V_i \quad V_k \quad Y_{ik}| \sin(\delta_i \quad - \delta_k \quad - \theta_{ik})$$

$i = 1, 2, \ldots.. n; \; i \neq$ slack bus

$$\Delta I_i = I_i - \sum_{k=1}^{n} Y_{ik} V_k$$

or using the voltage from

$$\Delta V_i = \frac{\Delta I_i}{Y_{ii}}$$

The convergence of the iterative methods depends on the diagonal dominance of the bus admittance matrix. The self-admittances of the buses, are usually large, relative to the mutual admittances and thus, usually convergence is obtained. Junctions of very high and low series impedances and large capacitances obtained in cable circuits long, EHV lines, series and shunt compensation are detrimental to convergence as these tend to weaken the diagonal dominance in the V-matrix. The choice of slack bus can affect convergence considerably.

In difficult cases, it is possible to obtain convergence by removing the least diagonally dominant row and column of Y. The salient features of the V-matrix iterative methods are that the elements in the summation terms are on the average only three even for well-developed power systems. The sparsity of the V-matrix and its symmetry reduces both the storage requirement and the computation time for iteration For a large, well conditioned system of n-buses, the number of iterations required are of the order of n and total computing time varies approximately as $n^2$

**Gauses - Seidel Iterative Method**

In this method, voltages at all buses except at the slack bus are assumed. The voltage at the slack bus is specified and remains fixed at that value. The (n-1) bus voltage relations.

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq 1}}^{n} Y_{ik} V_k\right]$$

i = 1, 2, ..... n; i ≠slack bus

are solved simultaneously for an improved solution. In order to accelerate'the convergence, all newly-computed values of bus voltages are substituted in eqn $V_1$ The bus voltage equation of the (m + 1 )$^{th}$ iteration may then be written as

$$V_i^{(m+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{(m)*}} - \sum_{\substack{k=1 \\ k\neq 1}}^{i-1} Y_{ik} V_k^{(m+1)} - \sum_{k=i-1}^{n} Y_{ik} V_k^{(m)}\right]$$

The method converges slowly because of the loose mathematical coupling between the buses. The rate of convergence of the process can be increased by using acceleration factors to the solution obtained after each iteration. A fixed acceleration factor α (1≤ α ≤ 2) is normally used for each voltage change,

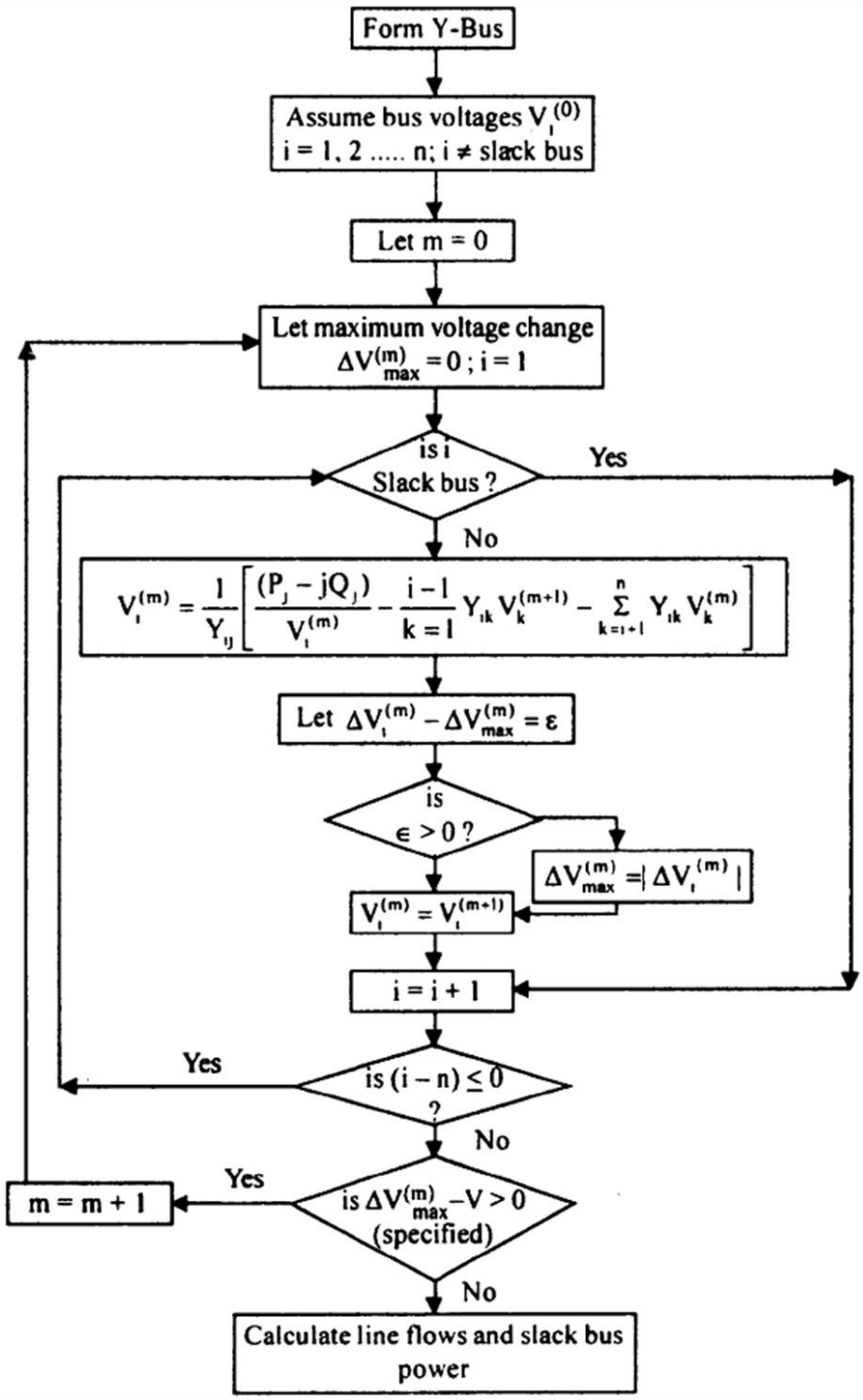
$$\Delta V_i = \alpha \frac{\Delta S_i^*}{V_i^* Y_{ii}}$$

The use of the acceleration factor amounts to a linear extrapolation of $V_1$ For a given system, it is quite often found that a near-optimal choice of α exists as suggested in literature over a range of operating conditions. Even though a complex value of α is suggested in literature, it is more convenient to operate with real values given by

$$\left|V_i^{(m)}\right| \angle \delta_i = |\alpha| \left|V_i^{(m)}\right| \angle \delta_i$$

Alternatively, different acceleration factors may be used for real and imaginary parts of the voltage.

While computing the reactive powers, the limits on the reactive source must be taken into consideration. If the calculated value of the reactive power is beyond limits. Then its value is fixed at the limit that is violated and it is no longer possible to hold the desired magnitude of the bus voltage, the bus is treated as a PQ bus or load bus.


Form Y-Bus
Assume bus voltages V_i^(0)
i = 1, 2 ..... n; i ≠ slack bus
Let m = 0
Let maximum voltage change
ΔV_max^(m) = 0 ; i = 1
is i
Slack bus ?
Yes
No
V_i^(m) = 1/Y_ii [ (P_i − jQ_i)/V_i^(m) − Σ_{k=1}^{i−1} Y_ik V_k^(m+1) − Σ_{k=i+1}^{n} Y_ik V_k^(m) ]
Let ΔV_i^(m) − ΔV_max^(m) = ε
is
ε > 0 ?
ΔV_max^(m) = | ΔV_i^(m) |
V_i^(m) = V_i^(m+1)
i = i + 1
Yes
is (i − n) ≤ 0
?
No
Yes
m = m + 1
is ΔV_max^(m) − V > 0
(specified)
No
Calculate line flows and slack bus
power

***Comparison of Various Methods for Power Flow Solution***

The requirements of a good power flow method are - high speed, low storage, and reliability for ill-conditioned problems. No single method meets all these requirements. It may be mentioned that for regular load flow studies NR-method in polar coordinates and for special applications fast decoupled load flow solution methods have proved to be most useful than other methods. NR-method is versatile, reliable and accurate. Fast decoupled load flow method is fast and needs the least storage. Convergence of iterative methods depends upon the dominance of the diagonal elements of the bus admittance matrix.

***Advantages of Gauss-Seidel method:***

1. The method is very simple in calculations and thus programing is easier.
2. The storage needed in the computer memory is relatively less.
3. In general, the method is applicable for smaller systems.

***Disadvantages of Gauss-Seidel Method:***

1. The number of iterations needed is generally high and is also dependent on the acceleration factor selected.
2. For large systems, use of Gauss-Seidel method is practically prohibitive.
3. The time for convergence also increases dramatically with increase of number of buses.

***Advantages of Newton-Raphsan Method,***

1. The method is more accurate, faster and reliable.
2. Requires less number of iteration for convergence. Infact, in three to four iterations good convergence is reached irrespective of the size of the system.
3. The number of iterations required is thus independent of the size of the system or the number of buses in the system.
4. The method is best suited for load flow solution to large size systems.
5. Decoupled and fast decoupled power flow solution can be obtained from Newton Raphsan Polar Coordinates method. Hence, it also can serve as a base for security and contingency studies.

***Disadvantages of Newton-Raphsan Method,***

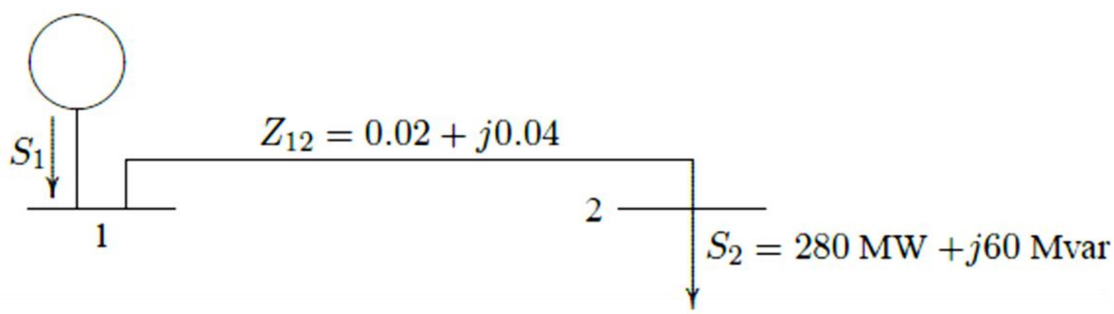
I. The memory needed is quite large for large size systems.
2. Calculations per iteration are also much larger than Gauss-Seidel method.
3. Since, it is a gradient method, the method is quite involved and hence, programming is also comparatively difficult and complicated.

**PROBLEM.1**

In the power system network shown in Figure, bus 1 is a slack bus with $V1 = 1.06 \angle 0^0$ per unit and bus 2 is a load bus with $S_2$ = 280 MW + $j$60 Mvar. The line impedance on a base of 100 MVA is $Z = 0.02 + j0.04$ per unit.

(a) Using Gauss-Seidel method, determine $V_2$ . Use an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and perform four iterations.

(b) If after several iterations voltage at bus 2 converges to $V_2 = 0.90 - j0.10$, determine $S_1$ and the real and reactive power loss in the line.




$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j29$$

The per unit load at bus 2 is

$$S_2 = -\frac{280 + j60}{100} = -2.8 - j0.60$$

Starting with an initial estimate of $V_2^{(0)}$= 1.0+$j$0.0, the voltage at bus 2 computed for three iterations are

$$V_2^{(1)} = \frac{\frac{-2.8+j0.60}{1.00000-j0.00000} + (10 - j20)(1)}{10 - j20} = 0.92000 - j0.10000$$

$$V_2^{(2)} = \frac{\frac{-2.8+j0.60}{0.92000+j0.10000} + (10 - j20)(1)}{10 - j20} = 0.90238 - j0.09808$$

$$V_2^{(3)} = \frac{\frac{-2.8+j0.60}{0.90238-j0.09808} + (10 - j20)(1)}{10 - j20} = 0.90050 - j0.10000$$

(b) Assuming voltage at bus 2 converges to $V_2 = 0.9 - j0.1$, the line flows are computed as follows

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1 + j0) - (0.9 - j0.10] = 3.0 - j1.0$$

$$I_{21} = -I_{12} = -3.0 + j1.0$$

$$S_{12} = V_1 I_{12}^* = (1.0 + j0.0)(3.0 + j1.0) = 3 + j1 \text{ pu}$$
$$= 300 \text{ MW} + j100 \text{ Mvar}$$
$$S_{21} = V_2 I_{21}^* = (0.9 - j0.1)(-3.0 - j1.0) = -2.8 - j0.6 \text{ pu}$$
$$= -280 \text{ MW} - j60 \text{ Mvar}$$

The line loss is

$$S_{L12} = S_{12} + S_{21} = (300 + j100) + (-280 - j60) = 20 \text{ MW} + j40 \text{ Mvar}$$

The slack bus real and reactive power are $P1$ = 300MW, and $Q1$ = 100 Mvar.

**PROBLEM.2**

Figure shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.06\angle\ 0^0$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

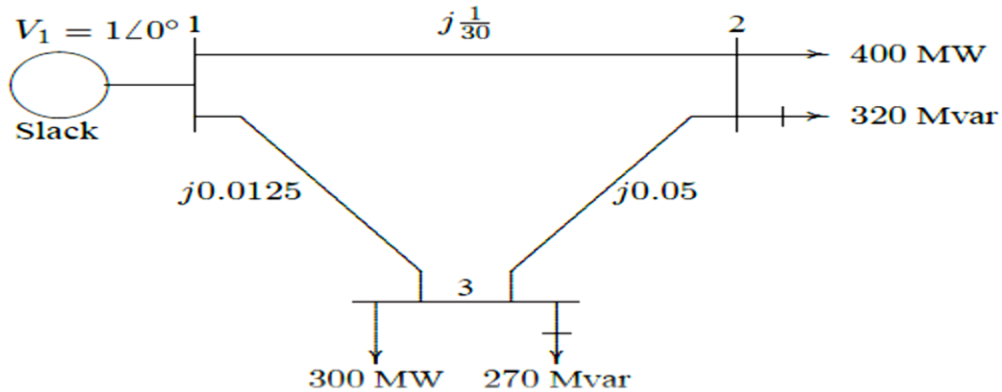
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0+j0$ and $V_3^{(0)}=1.0 + j0$, determine $V_2$ and $V_3$. Perform two iterations.

(b) If after several iterations the bus voltages converge to

$V_2 = 0.90 - j0.10$ pu

$V_3 = 0.95 - j0.05$ pu

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.




(a) Line impedances are converted to admittances

$$y_{12} = -j30$$
$$y_{13} = \frac{1}{j0.0125} = -j80$$
$$y_{23} = \frac{1}{j0.05} = -j20$$

At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(400 + j320)}{100} = -4.0 - j3.2 \text{ pu}$$
$$S_3^{sch} = -\frac{(300 + j270)}{100} = -3.0 - j2.7 \text{ pu}$$

For hand calculation,. Bus 1 is taken as reference bus (slack bus).
Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, $V_2$ and $V_3$ are computed as follows

$$V_2^{(1)} = \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$
$$= \frac{\frac{-4.0+j3.2}{1.0-j0} + (-j30)(1.0 + j0) + (-j20)(1.0 + j0)}{-j50}$$
$$= 0.936 - j0.08$$

$$V_3^{(1)} = \frac{\frac{S_3^{sch*}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{-3.0+j2.7}{1-j0} + (-j80)(1.0 + j0) + (-j20)(0.936 - j0.08)}{-j100}$$
$$= 0.9602 - j0.046$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-4.0+j3.2}{0.936+j0.08} + (-j30)(1.0 + j0) + (-j20)(0.9602 - j0.046)}{-j50}$$
$$= 0.9089 - j0.0974$$

and

$$V_3^{(2)} = \frac{\frac{-3.0+j2.7}{0.9602+j0.046} + (-j80)(1.0 + j0) + (-j20)(0.9089 - j0.0974)}{(-j100)}$$
$$= 0.9522 - j0.0493$$

The process is continued and a solution is converged with an accuracy of 5£10¡5 per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9020 - j0.0993 \qquad V_3^{(3)} = 0.9505 - j0.0498$$
$$V_2^{(4)} = 0.9004 - j0.0998 \qquad V_3^{(4)} = 0.9501 - j0.0500$$
$$V_2^{(5)} = 0.9001 - j0.1000 \qquad V_3^{(5)} = 0.9500 - j0.0500$$
$$V_2^{(6)} = 0.9000 - j0.1000 \qquad V_3^{(6)} = 0.9500 - j0.0500$$
$$V_2^{(7)} = 0.9000 - j0.1000 \qquad V_3^{(7)} = 0.9500 - j0.0500$$

The final solution is

$$V_2 = 0.90 - j0.10 = 0.905554\angle{-6.34^\circ} \quad \text{pu}$$
$$V_3 = 0.95 - j0.05 = 0.9513\angle{-3.0128^\circ} \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.0[1.0(-j30 - j80) - (-j30)(0.9 - j0.1) - \\ &\quad (-j80)(0.95 - j0.05)] \\ &= 7.0 - j7.0 \end{aligned}$$

or the slack bus real and reactive powers are $P1$ = 7.0 pu = 700MWand $Q1$ = 7.0 pu = 700 Mvar.

To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (-j30)[(1.0 + j0) - (0.90 - j0.10)] = 3.0 - j3.0$$
$$I_{21} = -I_{12} = -3.0 + j3.0$$
$$I_{13} = y_{13}(V_1 - V_3) = (-j80)[(1.0 + j0) - (0.95 - j.05)] = 4.0 - j4.0$$
$$I_{31} = -I_{13} = -4.0 + j4.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (-j20)[(0.90 - j0.10) - (0.95 - j.05)] = -1.0 + j1.0$$
$$I_{32} = -I_{23} = 1.0 - j1.0$$

The line flows are

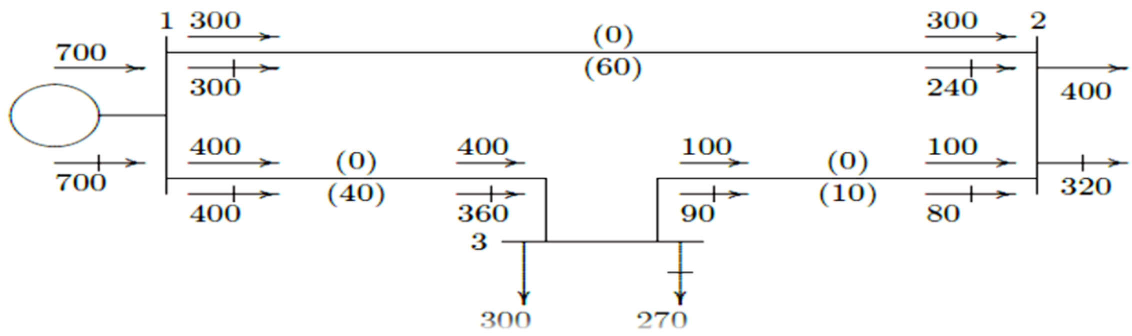
$$S_{12} = V_1 I_{12}^* = (1.0 + j0.0)(3.0 + j3) = 3.0 + j3.0 \text{ pu}$$
$$= 300 \text{ MW} + j300 \text{ Mvar}$$

$$S_{21} = V_2 I_{21}^* = (0.90 - j0.10)(-3 - j3) = -3.0 - j2.4 \text{ pu}$$
$$= -300 \text{ MW} - j240 \text{ Mvar}$$
$$S_{13} = V_1 I_{13}^* = (1.0 + j0.0)(4.0 + j4.0) = 4.0 + j4.0 \text{ pu}$$
$$= 400 \text{ MW} + j400 \text{ Mvar}$$
$$S_{31} = V_3 I_{31}^* = (0.95 - j0.05)(-4.0 - j4.0) = -4.0 - j3.6 \text{ pu}$$
$$= -400 \text{ MW} - j360 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.90 - j0.10)(-1.0 - j1.0) = -1.0 - j0.80 \text{ pu}$$
$$= -100 \text{ MW} - j80 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (0.95 - j0.05)(1 + j1) = 1.0 + j0.9 \text{ pu}$$
$$= 100 \text{ MW} + j90 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 0.0 \text{ MW} + j60 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 0.0 \text{ MW} + j40 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.0 \text{ MW} + j10 \text{ Mvar}$$

The power flow diagram is shown in Figure, where real power direction is indicated by → and the reactive power direction is indicated by ⊢→ The values within parentheses are the real and reactive losses in the line.



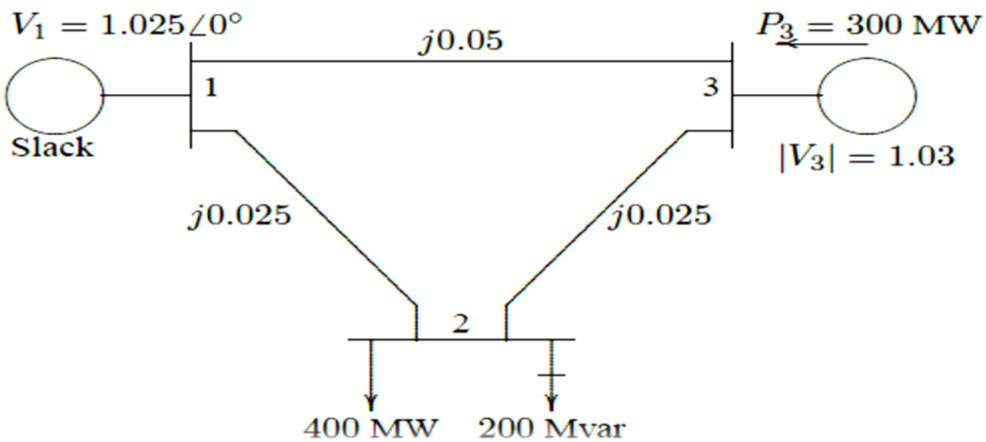

**PROBLEM.3**

Figure shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1$ = 1.0256∟ $0^0$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)}$= 1.0+*j*0 and $V_3^{(0)}$ =1.03 + *j*0 and keeping │*V*3│ = 1:03 pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.

(b) If after several iterations the bus voltages converge to

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.




(a) Line impedances converted to admittances are $y_{12}$ = -*j*40, $y_{13}$ = -*j*20 and $y_{23}$ = -*j*40. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \text{ pu}$$

$$P_3^{sch} = \frac{300}{100} = 3.0 \text{ pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)}$= 1.0 + *j*0.0 and $V_3^{(0)}$= 1.03 + *j*0.0, $V_2$ and $V_3$ are computed

$$V_2^{(1)} = \frac{\frac{S_2^{sch*}}{V_2^{(0)*}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12}+y_{23}}$$
$$= \frac{\frac{-4.0+j2.0}{1.0-j0} + (-j40)(1.025+j0) + (-j40)(1.03+j0)}{(-j80)}$$
$$= 1.0025 - j0.05$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed

$$Q_3^{(1)} = -\Im\{V_3^{(0)^*}[V_3^{(0)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\}$$
$$= -\Im\{(1.03-j0)[(1.03+j0)(-j60) - (-j20)(1.025+j0)$$
$$-(-j40)(1.0025-j0.05)]\}$$
$$= 1.236$$

The value of $Q_3^{(1)}$ is used as $Q_3^{sch}$ for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{S_3^{sch}{}^*}{V_3^{(0)}{}^*} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13}+y_{23}}$$
$$= \frac{\frac{3.0-j1.236}{1.03-j0} + (-j20)(1.025+j0) + (-j40)(1.0025-j0.05)}{(-j60)}$$
$$= 1.0300 + j0.0152$$

Since $|V3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained,i.e, $f_3^{(1)} = 0{:}0152$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.03)^2 - (0.0152)^2} = 1.0299$$

$$V_3^{(1)} = 1.0299 + j0.0152$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{S_2^{sch*}}{V_2^{(1)*}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-4.0+j2.0}{1.0025+j.05} + (-j40)(1.025) + (-j40)(1.0299 + j0.0152)}{(-j80)}$$

$$= 1.0001 - j0.0409$$

$$Q_3^{(2)} = -\Im\{V_3^{(1)*}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$

$$= -\Im\{(1.0299 - j0.0152)[(1.0299 + j0.0152)(-j60)$$

$$-(-j20)(1.025 + j0) - (-j40)(1.0001 - j0.0409)]\}$$

$$= 1.3671$$

$$V_{c3}^{(2)} = \frac{\frac{S_3^{sch*}}{V_3^{(1)*}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{3.0-j1.3671}{1.0299-j0.0152} + (-j20)(1.025) + (-j40)(1.0001 - j.0409)}{(-j60)}$$

$$= 1.0298 + j0.0216$$

Since $|V3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained,i.e, $f_3^{(2)} = 0.0216$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.03)^2 - (0.0216)^2} = 1.0298$$

$$V_3^{(2)} = 1.0298 + j0.0216$$

The process is continued and a solution is converged with an accuracy of $5*10^{-5}$ pu to

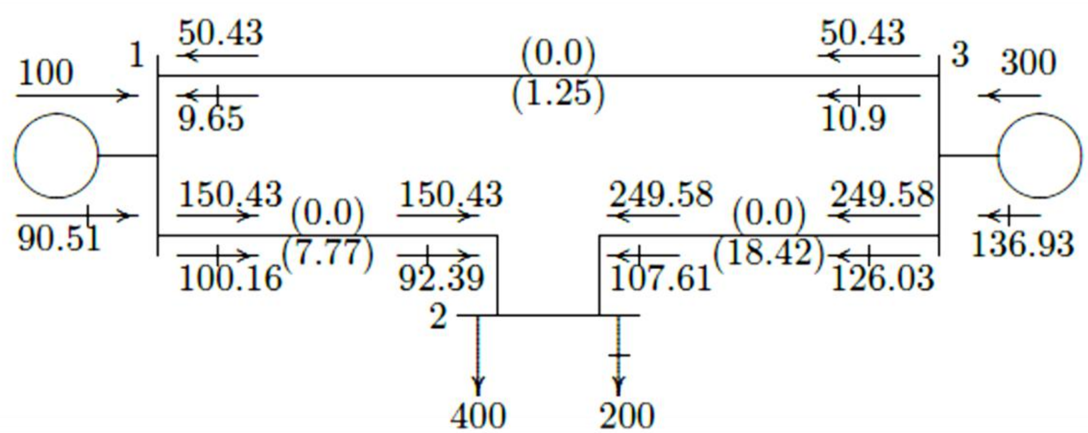
$$V_2 = 1.001243\angle{-2.1^\circ} = 1.000571 - j.0366898 \ \text{pu}$$
$$S_3 = 3.0 + j1.3694 \ \text{pu} = 300 \ \text{MW} + j136.94 \ \text{Mvar}$$
$$V_3 = 1.03\angle 1.36851^\circ \ \text{pu} = 1.029706 + j0.0246$$

(b) Line flows and line losses are computed as in Problem.2, and the results expressed in MW and Mvar are

$$S_{12} = 150.43 + j100.16 \quad S_{21} = -150.43 - j92.39 \quad S_{L12} = 0.0 + j7.77$$
$$S_{13} = -50.43 - j9.65 \quad S_{31} = 50.43 + j10.90 \quad S_{L13} = 0.0 + j1.25$$
$$S_{23} = -249.58 - j107.61 \quad S_{32} = 249.58 + j126.03 \quad S_{L23} = 0.0 + j18.42$$

The slack bus real and reactive powers are

$$S_1 = S_{12} + S_{13} = (150.43 + j100.16) + (-50.43 - j9.65)$$
$$= 100 \ \text{MW} + j90.51 \ \text{Mvar}$$



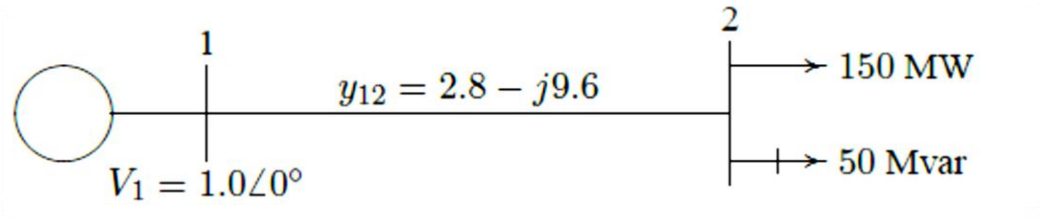

Power flow diagram of Problem.3(powers in MW and Mvar).

**PROBLEM.4**

In the two-bus system shown in Figure 58, bus 1 is a slack bus with $V_1 = 1.06\angle 0^0$ pu. A load of 150MWand 50 Mvar is taken from bus 2. The line admittance is $y_{12} = 10\angle{-73.74^0}$ pu on a base of 100 MVA. The expression for real and reactive power at bus 2 is given by

$$P_2 = 10|V_2||V_1|\cos(106.26^\circ - \delta_2 + \delta_1) + 10|V_2|^2\cos(-73.74^\circ)$$
$$Q_2 = -10|V_2||V_1|\sin(106.26^\circ - \delta_2 + \delta_1) - 10|V_2|^2\sin(-73.74^\circ)$$

Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0^0$. Perform two iterations. Partial derivatives of $P_2$, and $Q_2$ with respect to $|V2|$, and $\delta_2$ are




$$\frac{\partial P_2}{\partial \delta_2} = 10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1)$$
$$\frac{\partial P_2}{\partial |V_2|} = 10|V_1|\cos(106.26° - \delta_2 + \delta_1) + 20|V_2|\cos(-73.74°)$$
$$\frac{\partial Q_2}{\partial \delta_2} = 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -10|V_1|\sin(106.26° - \delta_2 + \delta_1) - 20|V_2|\sin(-73.74°)$$

The load expressed in per unit is

$$S_2^{sch} = -\frac{(150 + j50)}{100} = -1.5 - j0.5 \text{ pu}$$

The slack bus voltage is $V1 = 1.06\angle 0$ pu. Starting with an initial estimate of $|V_2^{(0)}|$ $j = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed

$$\begin{aligned}\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} &= -1.5 - [10\cos(106.26°) + 10\cos(-73.74°)] \\ &= -1.5 \text{ pu} \\ \Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} &= -0.5 - [-10\sin(106.26°) - 10\sin(-73.74°)] \\ &= -0.5 \text{ pu}\end{aligned}$$

The elements of the Jacobian matrix at the initial estimate are

$$\begin{aligned}J_1^{(0)} &= 10(1)(1)\sin(106.26°) = 9.6 \\ J_2^{(0)} &= 10(1)\cos(106.26°) + 20(1)\cos(-73.74°) = 2.8 \\ J_3^{(0)} &= 10(1)(1)\cos(106.26°) = -2.8 \\ J_4^{(0)} &= -10(1)\sin(106.26°) - 20(1)\sin(-73.74°) = 9.6\end{aligned}$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 9.6 & 2.8 \\ -2.8 & 9.6 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\Delta\delta_2^{(0)} = -0.13 \qquad \delta_2^{(1)} = 0 + (-0.13) = -0.13 \text{ radian}$$
$$\Delta|V_2^{(0)}| = -0.09 \qquad |V_2^{(1)}| = 1 + (-0.09) = 0.91 \text{ pu}$$

For the second iteration, we have

$$\Delta P_2^{(1)} = P_2^{sch} - P_2^{(1)} = -1.5 - (-1.3403) = -0.1597 \text{ pu}$$
$$\Delta Q_2^{(1)} = Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3822) = -0.1178 \text{ pu}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

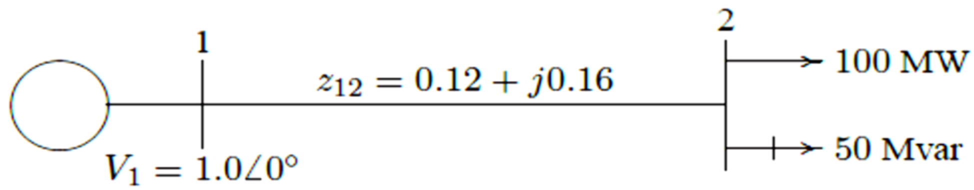
$$\begin{bmatrix} -0.1597 \\ -0.1178 \end{bmatrix} = \begin{bmatrix} 8.332 & 1.0751 \\ -3.659 & 8.3160 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\Delta\delta_2^{(1)} = -0.0164 \qquad \delta_2^{(2)} = -0.13 + (-0.0164) = -0.1464 \text{ radian}$$
$$\Delta|V_2^{(1)}| = -0.0214 \qquad |V_2^{(2)}| = 0.91 + (-0.0214) = 0.8886 \text{ pu}$$

**PROBLEM.5**

In the two-bus system shown in Figure, bus 1 is a slack bus with $V_1 = 1.06 \angle 0^0$ pu. A load of 100MWand 50 Mvar is taken from bus 2. The line impedanceis $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2^{(0)}| = 1.0$ pu and $\delta_2^{(0)} = 0^0$. Perform two iterations.



The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

The bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13^\circ & 5\angle 126.87^\circ \\ 5\angle 126.87^\circ & 5\angle -53.13^\circ \end{bmatrix}$$

Substituting for admittances, the expression for real and reactive power at bus 2 Becomes

$$P_2 = 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 5|V_2|^2\cos(-53.13^\circ)$$
$$Q_2 = -5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 5|V_2|^2\sin(-53.13^\circ)$$

Partial derivatives of $P_2$, and $Q_2$ with respect to $|V_2|$, and are $\delta_2$

$$\frac{\partial P_2}{\partial \delta_2} = 5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1)$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 10|V_2|\cos(-53.13^\circ)$$
$$\frac{\partial Q_2}{\partial \delta_2} = 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 10|V_2|\sin(-73.74^\circ)$$

The load expressed in per units is

$$S_2^{sch} = -\frac{(100 + j50)}{100} = -1.0 - j0.5 \text{ pu}$$

The slack bus voltage is $V_1 = 1.06\angle 0^0$pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -1.0 - [5\cos(126.87^\circ) + 5\cos(-53.13^\circ)]$$
$$= -1.0 \text{ pu}$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -0.5 - [-5\sin(126.87^\circ) - 5\sin(-53.13^\circ)]$$
$$= -0.5 \text{ pu}$$

The elements of the Jacobian matrix at the initial estimate are

$$J_1^{(0)} = 5(1)(1)\sin(126.87^\circ) = 4$$
$$J_2^{(0)} = 5(1)\cos(126.87^\circ) + 10(1)\cos(-53.13^\circ) = 3$$
$$J_3^{(0)} = 5(1)(1)\cos(126.87^\circ) = -3$$
$$J_4^{(0)} = -5(1)\sin(126.87^\circ) - 10(1)\sin(-53.13^\circ) = 4$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\Delta\delta_2^{(0)} = -0.10 \qquad \delta_2^{(1)} = 0 + (-0.10) = -0.10 \text{ radian}$$
$$\Delta|V_2^{(0)}| = -0.2 \qquad |V_2^{(1)}| = 1 + (-0.2) = 0.8 \text{ pu}$$

For the second iteration, we have

$$\Delta P_2^{(1)} = P_2^{sch} - P_2^{(1)} = -1.0 - (-0.7875) = -0.2125 \text{ pu}$$
$$\Delta Q_2^{(1)} = Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3844) = -0.1156 \text{ pu}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

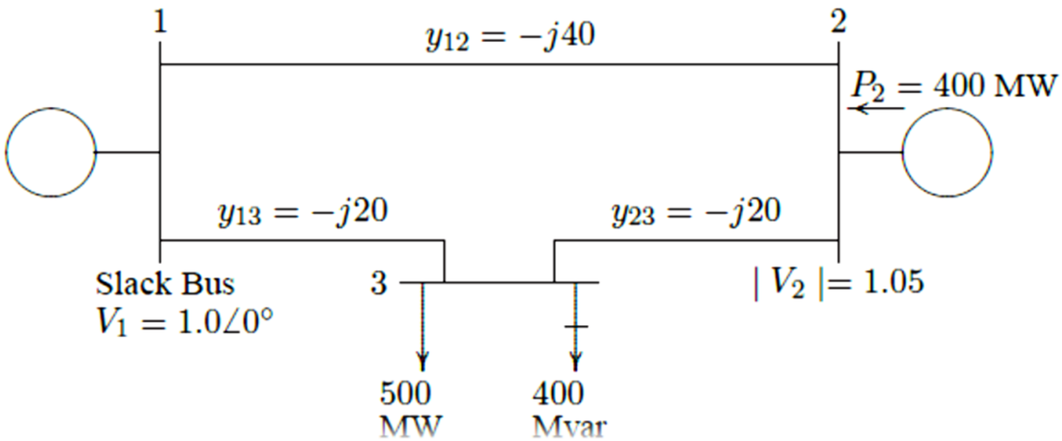
$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\Delta\delta_2^{(1)} = -0.0350 \qquad \delta_2^{(2)} = -0.1 + (-0.0350) = -0.135 \text{ radian}$$
$$\Delta|V_2^{(1)}| = -0.0773 \qquad |V_2^{(2)}| = 0.8 + (-0.0773) = 0.7227 \text{ pu}$$

**Problem.6**

Figure shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.06\angle 0^0$ per unit. Voltage magnitude at bus 2 is fixed at 1:05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.




(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90° - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90° - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90° - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90° - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90° - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90° - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.05 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V2| = 1.05$ pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.

By inspection, the bus admittance matrix in polar form is

$$Y_{bus} = \begin{bmatrix} 60\angle -\frac{\pi}{2} & 40\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 40\angle \frac{\pi}{2} & 60\angle -\frac{\pi}{2} & 20\angle \frac{\pi}{2} \\ 20\angle \frac{\pi}{2} & 20\angle \frac{\pi}{2} & 40\angle -\frac{\pi}{2} \end{bmatrix}$$

(a) The power flow equation with voltages and admittances expressed in polar form Is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Substituting the elements of the bus admittance matrix in the above equations for $P_2$, $P_3$, and $Q_3$ will result in the given equations.

(b) Elements of the Jacobian matrix are obtained by taking partial derivatives of the given equations with respect to $\delta_2$, $\delta_3$ and $|V_3|$.

$$\frac{\partial P_2}{\partial \delta_2} = 40|V_2||V_1|\sin(\frac{\pi}{2} - \delta_2 + \delta_1) + 20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -20|V_2||V_3|\sin(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = 20|V_2|\cos(\frac{\pi}{2} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = 20|V_3||V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 20|V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20|V_3||V_1|\cos(\frac{\pi}{2} - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(\frac{\pi}{2} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -20|V_1|\sin(\frac{\pi}{2} - \delta_3 + \delta_1) - 20|V_2|\sin(\frac{\pi}{2} - \delta_3 + \delta_2) + 80|V_3|$$

The load and generation expressed in per units are

$$P_2^{sch} = \frac{400}{100} = 4.0 \text{ pu}$$
$$S_3^{sch} = -\frac{(500 + j400)}{100} = -5.0 - j4.0 \text{ pu}$$

The slack bus voltage is $V_1 = 1.06\angle 0^0$ pu, and the bus 2 voltage magnitude is $|V_2|$ =1.05 pu. Starting with an initial estimate of $|V_3^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)}$ =0.0, the power residuals are

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 4.0 - (0) = 4.0$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = -5.0 - (0) = -5.0$$
$$\Delta Q_3^{(0)} = Q_3^{sch} - Q_3^{(0)} = -4.0 - (-1.0) = -3.0$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_3^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= 0.0275 & \delta_2^{(1)} &= 0 + 0.0275 = 0.0275 \text{ radian} = 1.5782^\circ \\ \Delta\delta_3^{(0)} &= -0.1078 & \delta_3^{(1)} &= 0 + (-0.1078) = -0.1078 \text{ radian} = -6.1790^\circ \\ \Delta|V_3^{(0)}| &= -0.0769 & |V_3^{(1)}| &= 1 + (-0.0769) = 0.9231 \text{ pu} \end{aligned}$$

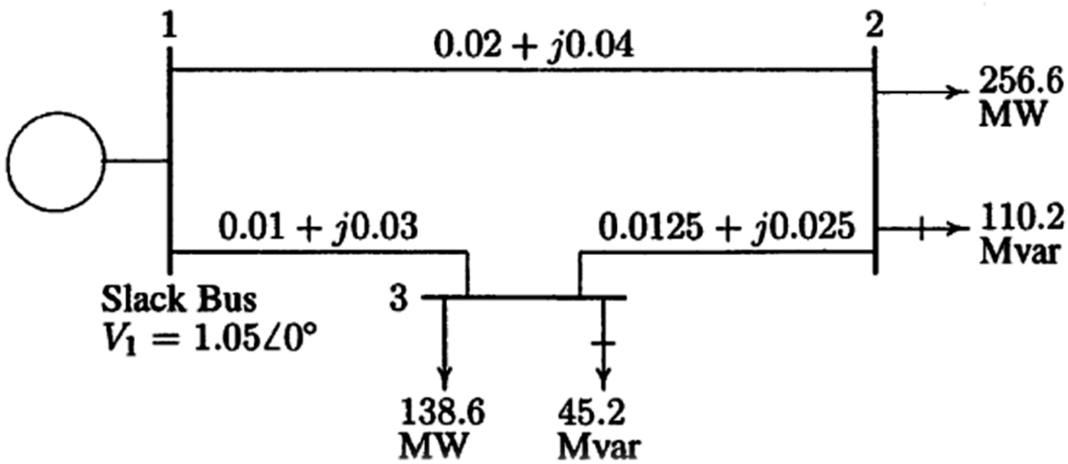
For the second iteration, we have

$$\begin{bmatrix} 0.2269 \\ -0.3965 \\ -0.5213 \end{bmatrix} = \begin{bmatrix} 61.1913 & -19.2072 & 2.8345 \\ -19.2072 & 37.5615 & -4.9871 \\ 2.6164 & -4.6035 & 33.1545 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_3^{(1)}| \end{bmatrix}$$

$$\begin{aligned} \Delta\delta_2^{(1)} &= 0.0006 & \delta_2^{(2)} &= 0.0275 + 0.0006 = 0.0281 \text{ radian} = 1.61^\circ \\ \Delta\delta_3^{(1)} &= -0.0126 & \delta_3^{(2)} &= -0.1078 + (-0.0126) = -0.1204 \text{ radian} = -6.898^\circ \\ \Delta|V_3^{(1)}| &= -0.0175 & |V_3^{(2)}| &= 0.9231 + (-0.0175) = 0.9056 \text{ pu} \end{aligned}$$

**PROBLEM:7**

The one line diagram of a simple three bus power system with generator at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05per unit. The scheduled loads at buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100-MVA base and the line charging susceptances are neglected.




(a) Using the Gauss-seidel method, Determine the phasor values of the voltage at the load buses 2 and 3 accurate to four decimal places.

(b) Find the slack bus real and reactive power.

(c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly

$y_{13}$=10-j30

$y_{23}$=16-j32

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-2.566+j1.102}{1.0-j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0 + j0)}{(26 - j52)}$$

$$= 0.9825 - j0.0310$$

and

$$V_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-1.386+j0.452}{1-j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9825 - j0.0310)}{(26 - j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0011 - j0.0353)}{(26 - j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011+j0.0353} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9816 - j0.052)}{(26 - j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035°} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624°} \quad \text{pu}$$

(b)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

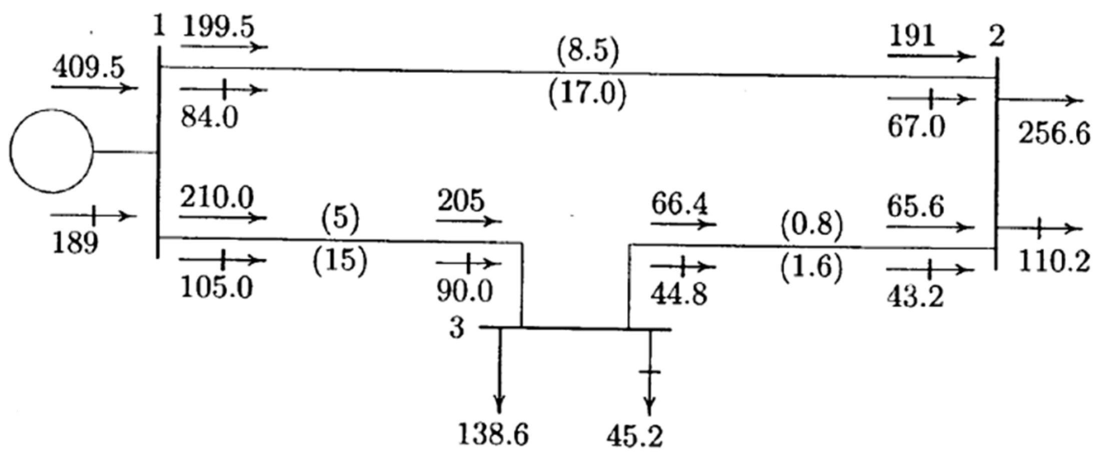
$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \quad \text{pu} \\ &= 199.5 \text{ MW} + j84.0 \text{ Mvar} \end{aligned}$$

$$S_{21} = V_2 I_{21}^* = (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \text{ pu}$$
$$= -191.0 \text{ MW} - j67.0 \text{ Mvar}$$
$$S_{13} = V_1 I_{13}^* = (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \text{ pu}$$
$$= 210.0 \text{ MW} + j105.0 \text{ Mvar}$$

$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu}$$
$$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu}$$
$$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu}$$
$$= 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

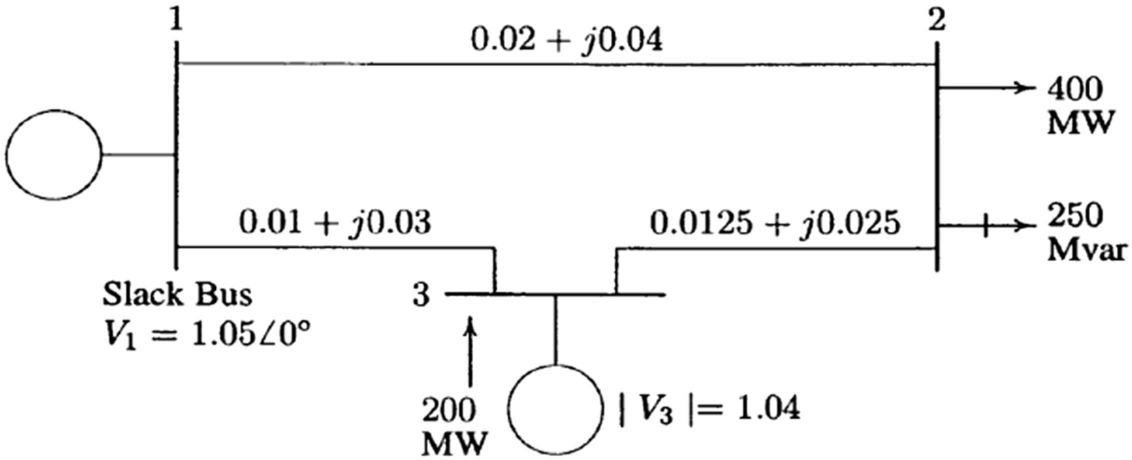
and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$



**PROBLEM:8**

One line diagram of a simple three bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05pu. Voltage magnitude at bus 3 is fixed at 1.04pu with a real power generation of 200MW. A load consisting of 400MW and 250Mvar is taken from bus 2. Line impedances are marked in per unit on a common100 MVA base and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-seidel method including line flows and line losses.




Line impedances converted to admittances are $y_{12} = 10-j20$, $y_{13} = 10-j30$ and $y_{23} = 16-j32$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400+j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

$$V_2^{(1)} = \frac{\frac{P_2^{sch}-jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12}+y_{23}}$$

$$= \frac{\frac{-4.0+j2.5}{1.0-j0} + (10-j20)(1.05+j0) + (16-j32)(1.04+j0)}{(26-j52)}$$

$$= 0.97462 - j0.042307$$

$$Q_3^{(1)} = -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\}$$
$$= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - (16 - j32)(0.97462 - j0.042307)]\}$$
$$= 1.16$$

The value of $Q_3^{(1)}$ is used as $Q_3^{sch}$ for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$
$$= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$
$$= 1.03783 - j0.005170$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$
$$= \frac{\frac{-4.0 + j2.5}{.97462 + j.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 + j0.005170)}{(26 - j52)}$$
$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$
$$= -\Im\{(1.039987 + j0.005170)[(1.039987 - j0.005170)(26 - j62) - (10 - j30)(1.05 + j0) - (16 - j32)(0.971057 - j0.043432)]\}$$
$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.38796}{1.039987 + j0.00517} + (10 - j30)(1.05) + (16 - j32)(.971057 - j.043432)}{(26 - j62)}$$

$$= 1.03908 - j0.00730$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ pu in seven iterations as given below.

$V_2^{(3)} = 0.97073 - j0.04479$ $Q_3^{(3)} = 1.42904$ $V_3^{(3)} = 1.03996 - j0.00833$

$V_2^{(4)} = 0.97065 - j0.04533$ $Q_3^{(4)} = 1.44833$ $V_3^{(4)} = 1.03996 - j0.00873$

$V_2^{(5)} = 0.97062 - j0.04555$ $Q_3^{(5)} = 1.45621$ $V_3^{(5)} = 1.03996 - j0.00893$

$V_2^{(6)} = 0.97061 - j0.04565$ $Q_3^{(6)} = 1.45947$ $V_3^{(6)} = 1.03996 - j0.00900$

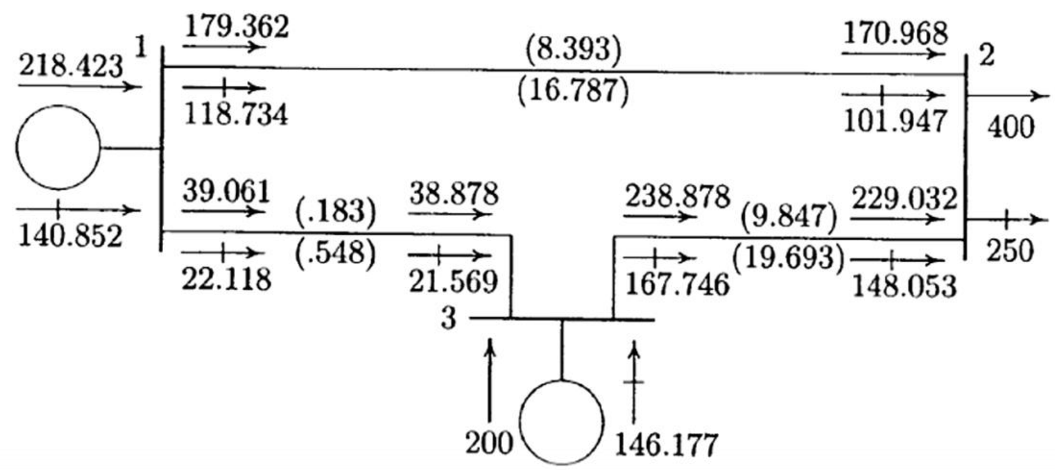
$V_2^{(7)} = 0.97061 - j0.04569$ $Q_3^{(7)} = 1.46082$ $V_3^{(7)} = 1.03996 - j0.00903$

The final solution is

$$V_2 = 0.97168\angle{-2.6948^\circ} \quad \text{pu}$$

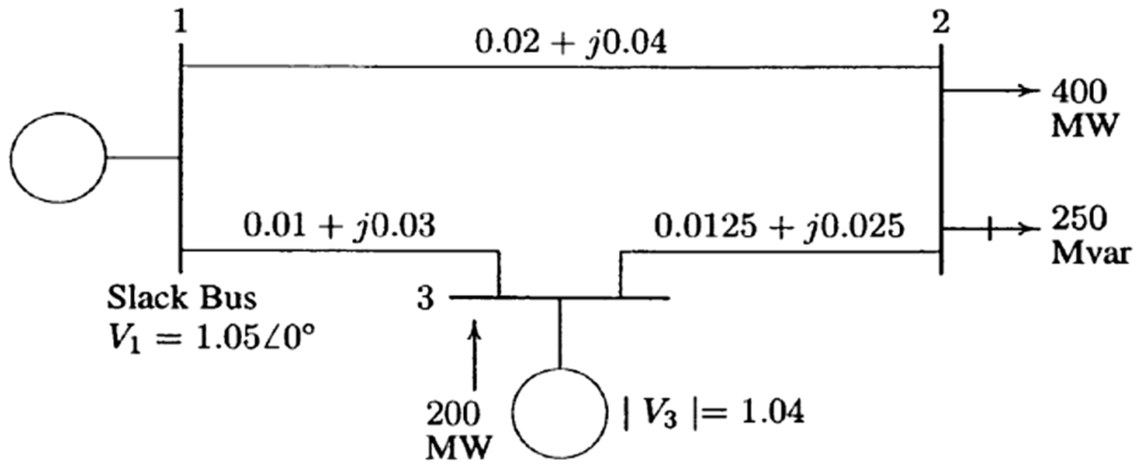
$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$
$$V_3 = 1.04\angle{-.498^\circ} \quad \text{pu}$$
$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$

$$S_{12} = 179.36 + j118.734 \quad S_{21} = -170.97 - j101.947 \quad S_{L12} = 8.39 + j16.79$$
$$S_{13} = 39.06 + j22.118 \quad S_{31} = -38.88 - j21.569 \quad S_{L13} = 0.18 + j0.548$$
$$S_{23} = -229.03 - j148.05 \quad S_{32} = 238.88 + j167.746 \quad S_{L23} = 9.85 + j19.69$$




## PROBLEM:9

One line diagram of a simple three bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05pu. Voltage magnitude at bus 3 is fixed at 1.04pu with a real power generation of 200MW. A load consisting of 400MW and 250Mvar is taken from bus 2. Line impedances are marked in per unit on a common100 MVA base and the line charging susceptances are neglected. Obtain the power flow solution by the Newton - Raphson method



Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$, and $y_{23} = 16 - j32$. This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in radian yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33}$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to $\delta_2$, $\delta_3$ and $|V_2|$.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\Delta\delta_2^{(0)} = -0.045263 \qquad \delta_2^{(1)} = 0 + (-0.045263) = -0.045263$$

$$\Delta\delta_3^{(0)} = -0.007718 \qquad \delta_3^{(1)} = 0 + (-0.007718) = -0.007718$$

$$\Delta|V_2^{(0)}| = -0.026548 \qquad |V_2^{(1)}| = 1 + (-0.026548) = 0.97345$$

Voltage phase angles are in radians. For the second iteration, we have

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

and

$$\Delta\delta_2^{(1)} = -0.001795 \qquad \delta_2^{(2)} = -0.045263 + (-0.001795) = -0.04706$$
$$\Delta\delta_3^{(1)} = -0.000985 \qquad \delta_3^{(2)} = -0.007718 + (-0.000985) = -0.00870$$
$$\Delta|V_2^{(1)}| = -0.001767 \qquad |V_2^{(2)}| = 0.973451 + (-0.001767) = 0.971684$$

For the third iteration, we have

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.147447 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_2^{(2)}| \end{bmatrix}$$

and

$$\Delta\delta_2^{(2)} = -0.000038 \qquad \delta_2^{(3)} = -0.047058 + (-0.0000038) = -0.04706$$
$$\Delta\delta_3^{(2)} = -0.0000024 \qquad \delta_3^{(3)} = -0.008703 + (-0.0000024) = 0.008705$$
$$\Delta|V_2^{(2)}| = -0.0000044 \qquad |V_2^{(3)}| = 0.971684 + (-0.0000044) = 0.97168$$

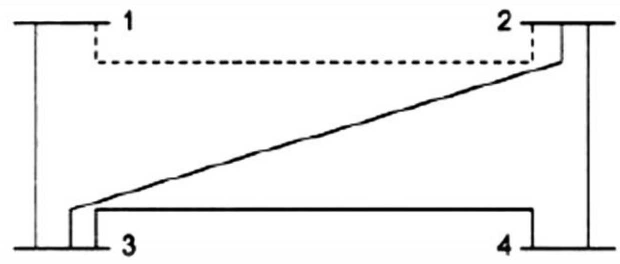
$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33}$$
$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3| |Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3)$$
$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3| |Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3)$$

Upon substitution, we have

$$Q_3 = 1.4617 \text{ pu}$$
$$P_1 = 2.1842 \text{ pu}$$
$$Q_1 = 1.4085 \text{ pu}$$

**PROBLEM:10**

For the sample system of Fig the generators are connected at all the four buses, while loads are at buses 2 and 3. Values of real and reactive powers are listed in table. All buses other than the slack are pq type. Assuming a flat voltage start, find the voltages and bus angles at the three buses at the end of the first GS iteration.




| Line data | R.pu | X.pu |
|---|---|---|
| 1-2 | 0.05 | 0.15 |
| 1-3 | 0.10 | 0.30 |
| 2-3 | 0.15 | 0.45 |
| 2-4 | 0.10 | 0.30 |
| 3-4 | 0.05 | 0.15 |

| Bus data | P.pu | Q.pu | V.pu | Remarks |
|---|---|---|---|---|
| 1 | - | - | $1.04\angle 0^0$ | Slack bus |
| 2 | 0.5 | -0.2 | - | PQ bus |
| 3 | -1.0 | 0.5 | - | PQ bus |
| 4 | 0.3 | -0.1 | - | PQ bus |

$$Y_{BUS}=\begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

$$V^1{}_2 = \frac{1}{Y_{22}}\left\{\frac{P_2 - jQ_2}{(V_2^0)*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right\}$$

$$= \frac{1}{Y_{22}}\left\{\frac{0.5 + j0.2}{1 - j0} - 1.04(-2 + j6) - (-0.666 + j2) - (-1 + j3)\right\}$$

$$= \frac{4.246 - j11.04}{3.666 - j11} = 1.019 + j0.046 \text{ pu}$$

$$V_3^1 = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_3^1 - Y_{34}V_4^0\right\}$$

$$= \frac{1}{Y_{33}}\left\{\frac{-1 - j0.5}{1 - j0} - 1.04\,(-1 + j3)\right.$$

$$\left. - (-0.666 + j2)\,(1.019 + j0.046) - (-2 + j6)\right\}$$

$$= \frac{2.81 - j11.627}{3.666 - j11} = 1.028 - j0.087 \text{ pu}$$

$$V_4^1 = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right\}$$

$$= \frac{1}{Y_{44}}\left\{\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)\,(1.019 + j0.046)\right.$$

$$\left. - (-2 + j6)\,(1.028 - j0.087)\right\}$$

$$= \frac{2.991 - j9.253}{3 - j9} = 1.025 - j0.0093 \text{ pu}$$

**PROBLEM:11**

In problem10 let bus 2 be a PV bus now with l$V_2$l = 1.04 pu. Once again assuming a flat voltage start, find $Q_2$, $\delta_2$, $V_3$, $V_4$ at the end of the first GS iteration. Given: 0.2 < Q, < l.

$$Q_2^1 = -\,\text{Im}\,\{(V_2^0)^* Y_{21} V_1 + (V_2^0)^*[Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0]\}$$

$$= -\,\text{Im}\,\{1.04\,(-2 + j6)\,1.04 + 1.04\,[(3.666 - j11)\,1.04$$

$$+ (-0.666 + j2) + (-1 + j3)]\}$$

$$= -\,\text{Im}\,\{-0.0693 - j0.2079\} = 0.2079 \text{ pu}$$

$$\therefore Q_2^1 = 0.2079 \text{ pu}$$

$$\delta_2^1 = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666 - j11}\left[\frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)(1.04 + j0) - (-0.666 + j2)(1 + j0) - (-1 + j3)(1 + j0)\right]\right\}$$

$$= \angle\left(\frac{4.2267 - j11.439}{3.666 - j11}\right) = \angle(1.0512 + j0.0339)$$

or $\delta_2^1 = 1.84658° = 0.032$ rad

$$\therefore V_2^1 = 1.04(\cos\delta_2^1 + j\sin\delta_2^1)$$

$$= 1.04(0.99948 + j0.0322)$$

$$= 1.03946 + j0.03351$$

$$V_3^1 = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{(1 - j0)} - (-1 + j3)\,1.04 - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right\}$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) - (-2 + j6)(1.0317 - j0.08937)\right]$$

$$= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031$$

**PROBLEM:12**

Consider the three-bus system of Fig. Each of the three lines has a series Impedance of 0.02 + j0.08pu and a total shunt admittance of j0.02 pu. The Specified quantities at the buses are tabulated below:

Controllable reactive power sources available at bus 3 with the constraint

$0 \leq Q_{G3} \leq 1.5$ Pu

| Bus | $P_D$ | $Q_D$ | $P_G$ | $Q_G$ | Voltage |
|---|---|---|---|---|---|
| 1 | 2 | 1 | - | - | $V_1$=1.04+j0 (Slack bus) |
| 2 | 0 | 0 | 0.5 | 1 | Unspecified (PQ bus) |
| 3 | 1.5 | 0.6 | 0.0 | $Q_{G3}$=? | $V_3$=1.04 (PV bus) |

Find the load flow solution using the NR method

*Solution* Using the nominal-π model for transmission lines, $Y_{BUS}$ for the given system is obtained as follows:

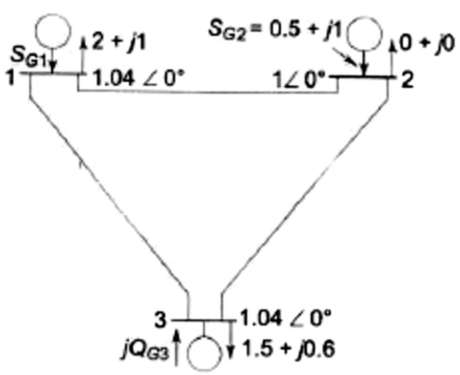
For each line

$$y_{series} = \frac{1}{0.02 + j0.08} = 2.941 - j11.764 = 12.13 \angle -75.96°$$

Each off-diagonal term = − 2.941 + j11.764

Each self term = 2[(2.941 − j11.764) + j0.01]

= 5.882 − j23.528 = 24.23 ∠−75.95°



$$\therefore \quad Y_{BUS} = \begin{bmatrix} 24.23\angle-75.95° & 12.13\angle104.04° & 12.13\angle104.04° \\ 12.13\angle104.04° & 24.23\angle-75.95° & 12.13\angle104.04° \\ 12.13\angle104.04° & 12.13\angle104.04° & 24.23\angle-75.95° \end{bmatrix}$$

$$P_2 = |V_2|\,|V_1|\,|Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2 |Y_{22}| \cos \theta_{22} + |V_2|\,|V_3|\,|Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2)$$

$$P_3 = |V_3|\,|V_1|\,|Y_{31}| \cos(\theta_{31} + \delta_1 - \delta_3) + |V_3|\,|V_2|\,|Y_{32}| \times \cos(\theta_{32} + \delta_2 - \delta_3) + |V_3|^2 |Y_{33}| \cos \theta_{33}$$

$$Q_2 = -|V_2|\,|V_1|\,|Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) - |V_2|^2 |Y_{22}| \times \sin \theta_{22} - |V_2|\,|V_3|\,|Y_{23}| \sin(\theta_{23} + \delta_2 - \delta_3)$$

$$P_2^0 = -0.23 \text{ pu}$$

$$P_3^0 = 0.12 \text{ pu}$$

$$Q_2^0 = -0.96 \text{ pu}$$

$$\Delta P_2^0 = P_2 \text{ (specified)} - P_2^0 \text{ (calculated)}$$

$$= 0.5 - (-0.23) = 0.73$$

$$\Delta P_3^0 = -1.5 - (0.12) = -1.62$$

$$\Delta Q_2^0 = 1 - (-0.96) = 1.96$$

The changes in variables at the end of the first iteration are obtained as follows:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\Delta P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

Jacobian elements can be evaluated by differentiating the expressions given above for $P_2$,$P_3$,$Q_2$ with respect to $\delta_2$, $\delta_3$ and $|V_2|$ and substituting the given and assumed values at the start of iteration. The changes in variables are obtained as

$$\begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \\ |V_2|^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ |V_2|^0 \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 1.089 \end{bmatrix}$$

We can now calculate

$$Q_3^1 = 0.4677$$

$$Q_{G3}^1 = Q_3^1 + Q_{D3} = 0.4677 + 0.6 = 1.0677$$

which is within limits.

If' the same problem is solved using a digital computer, the solution Converges in three iterations. The final results are given below:

$$V_2 = 1.081 \angle -0.024 \text{ rad}$$

$$V_3 = 1.04 \angle -0.0655 \text{ rad}$$

$$Q_{G3} = -0.15 + 0.6 = 0.45 \text{ (within limits)}$$

$$S_1 = 1.031 + j(-0.791)$$

$$S_2 = 0.5 + j1.00$$

$$S_3 = -1.5 - j0.15$$

Transmission loss = 0.031 pu

***Line flows***

The following matrix shows the real part of line flows

$$\begin{bmatrix} 0.0 & 0.191312E00 & 0.839861E00 \\ -0.184229E00 & 0.0 & 0.684697E00 \\ -0.826213E00 & -0.673847E00 & 0.0 \end{bmatrix}$$

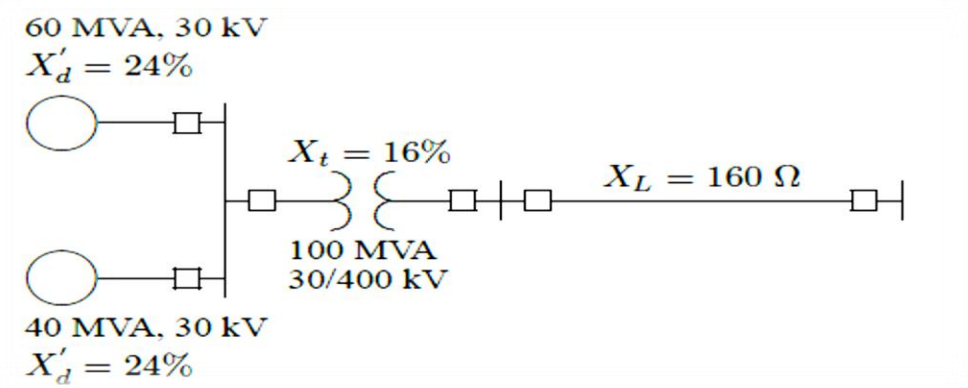
The following matrix shows the imaginary part of line flows

$$\begin{bmatrix} 0.0 & -0.599464E00 & -0.191782E00 \\ 0.605274E00 & 0.0 & 0.396045E00 \\ 0.224742E00 & -0.375165E00 & 0.0 \end{bmatrix}$$

# UNIT-III
# SYMMETRICAL FAULT ANALYSIS

## PROBLEM.1

The system shown in Figure is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160\Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the shortcircuit current and the short-circuit MVA.




The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600\ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375\ \ \text{A}$$

The reactances on a common 100 MVA base are

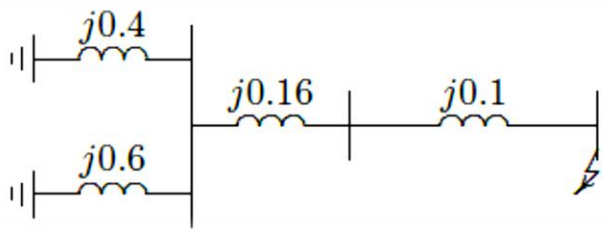
$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \text{ pu}$$

$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \text{ pu}$$

$$X_t = \frac{100}{100}(0.16) = 0.16 \text{ pu}$$

$$X_{line} = \frac{160}{1600} = 0.1 \text{ pu}$$

The impedance diagram is as shown in Figure




Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$I_f = \frac{1}{j0.5} = 2\angle{-90^\circ} \text{ pu}$$
$$= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \text{ A}$$

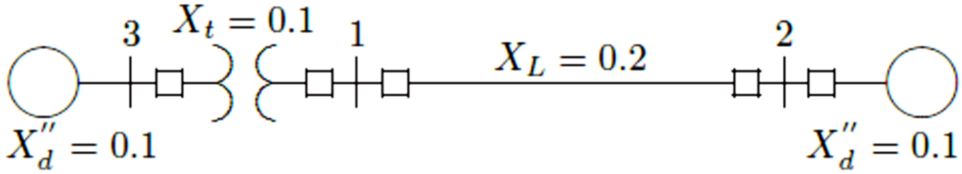
The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

**PROBLEM.2**

The one-line diagram of a simple power system is shown in Figure. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.

(a) Using Th´evenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.



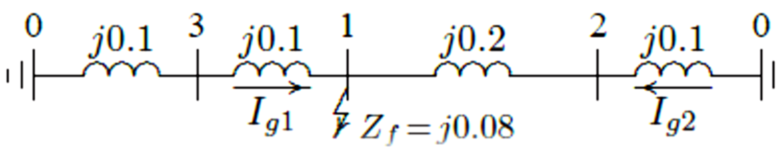

(a) Impedance to the point of fault is

$$X = j\frac{(0.2)(0.3)}{0.2 + 0.3} = j0.12 \text{ pu}$$

The fault current is

$$I_f = \frac{1}{j0.12 + j0.08} = 5\angle{-90^\circ} \text{ pu}$$

The impedance diagram




(b)

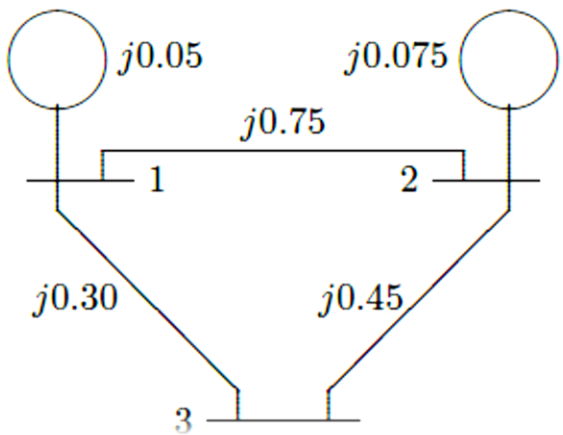
$$V_1 = (j0.08)(-j5) = 0.4 \text{ pu}$$
$$I_{g1} = \frac{j0.3}{j0.5}(5)\angle{-90^\circ} = 3\angle{-90^\circ} \text{ pu}$$
$$I_{g2} = \frac{j0.2}{j0.5}(5)\angle{-90^\circ} = 2\angle{-90^\circ} \text{ pu}$$
$$V_2 = 0.4 + (j0.2)(-j2) = 0.8 \text{ pu}$$
$$V_3 = 0.4 + (j0.1)(-j3) = 0.7 \text{ pu}$$

**PROBLEM.3**

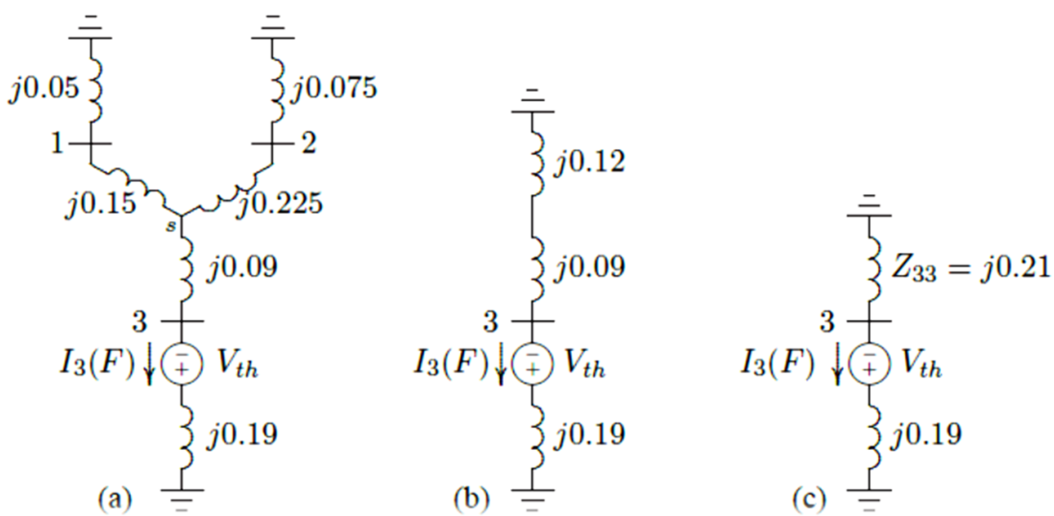
The one-line diagram of a simple three-bus power system is shown in Figure Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.

(a) Using Th´evenin's theorem obtain the impedance to the point of fault and the fault current in per unit.

(b) Determine the bus voltages and line currents during fault.




Converting the∆ formed by buses 123 to an equivalent Y




Reduction of Thevenin's equivalent network.

$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15 \qquad Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$

$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thevenin's impedance is

$$Z_{33} = \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09$$
$$= j0.12 + j0.09 = j0.21$$

From Figure(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

With reference to Figure(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure(a), we get

$$\Delta V_1 = 0 - (j0.05)(-j1.5) \quad = -0.075 \;\; \text{pu}$$
$$\Delta V_2 = 0 - (j0.075)(-j1) \quad = -0.075 \;\; \text{pu}$$
$$\Delta V_3 = (j0.19)(-j2.5) - 1.0 = -0.525 \;\; \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

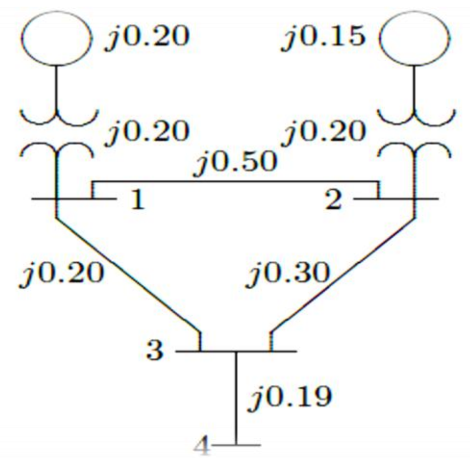
$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \;\; \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \;\; \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \;\; \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu}$$

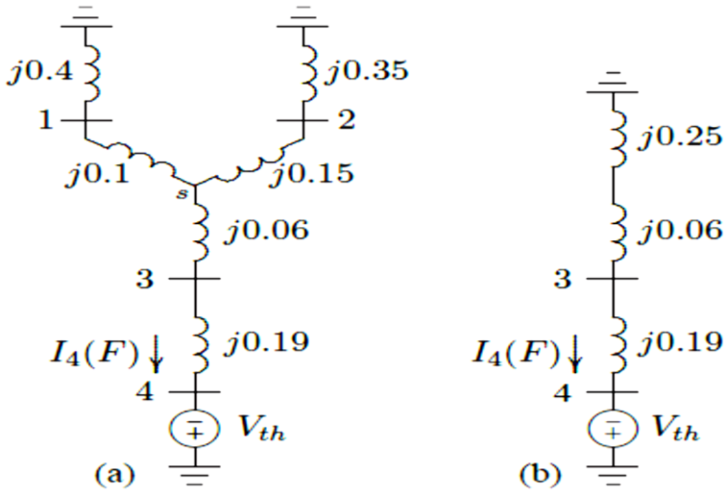
**PROBLEM.4**

The one-line diagram of a simple four-bus power system is shown in Figure Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A bolted three-phase fault occurs at bus 4.




(a) Using Thevenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.
(c) Repeat (a) and (b) for a fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

(a) Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure (a), we have




Reduction of Thevenin's equivalent network.

$$Z_{1s} = \frac{(j0.2)(j0.5)}{j1.0} = j0.10 \qquad Z_{2s} = \frac{(j0.5)(j0.3)}{j1.0} = j0.15$$
$$Z_{3s} = \frac{(j0.2)(j0.3)}{j1.0} = j0.06$$

Combining the parallel branches, Thevenin's impedance is

$$Z_{33} = \frac{(j0.5)(j0.5)}{j0.5 + j0.5} + j0.06 + j0.19 = j0.5$$

The fault current is

$$I_4(F) = \frac{V_4(F)}{Z_{44}} = \frac{1.0}{j0.5} = -j2.0 \quad \text{pu}$$

With reference to Figure(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$
$$I_{G2} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$

(b) For the bus voltage changes from Figure(a), we get

$$\Delta V_1 = 0 - (j0.4)(-j1.0) \quad = -0.4 \text{ pu}$$
$$\Delta V_2 = 0 - (j0.35)(-j1) \quad = -0.35 \text{ pu}$$
$$\Delta V_3 = 1 - (j0.19)(-j2) \quad = -0.62 \text{ pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.4 = 0.60 \text{ pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.35 = 0.65 \text{ pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.62 = 0.38 \text{ pu}$$
$$V_4(F) = 0 \text{ pu}$$

The short circuit-currents in the lines are

$$I_{21}(F) = \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.6}{j0.5} = 0.1\angle{-90^\circ} \text{ pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = 1.1\angle{-90^\circ} \text{ pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = 0.9\angle{-90^\circ} \text{ pu}$$
$$I_{34}(F) = \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = 2.0\angle{-90^\circ} \text{ pu}$$

(c) (a) Combining parallel branches between buses 1 and 2 results in the circuit shown in Figure (a). Combining the parallel branches, Thevenin's impedance is

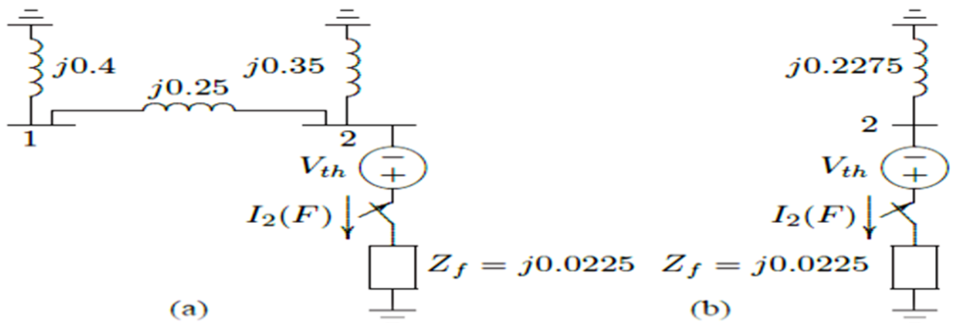
$$Z_{22} = \frac{(j0.65)(j0.35)}{j0.65 + j0.35} = j0.2275$$

The fault current is

$$I_2(F) = \frac{V_4(F)}{Z_{44} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4.0 \text{ pu}$$

With reference to Figure (a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.35}{j0.65 + j0.35} I_2(F) = -j1.4 \quad \text{pu}$$
$$I_{G2} = \frac{j0.65}{j0.65 + j0.35} I_2(F) = -j2.6 \quad \text{pu}$$




Reduction of Thevenin's equivalent network.

(c) (b) For the bus voltage changes from Figure (a), we get

$$\Delta V_1 = 0 - (j0.4)(-j1.4) \quad = -0.56 \ \text{pu}$$
$$\Delta V_2 = 0 - (j0.35)(-j2.6) \quad = -0.91 \ \text{pu}$$
$$\Delta V_3 = -j0.56 - (j0.2)(-\frac{j1.4}{2}) \quad = -0.70 \ \text{pu}$$
$$\Delta V_4 = \Delta V_3 = -0.70 \ \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

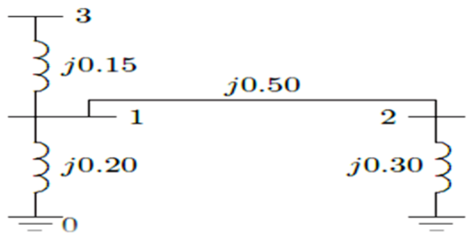
$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.56 = 0.44 \ \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.91 = 0.09 \ \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.70 = 0.30 \ \text{pu}$$
$$V_4(F) = V_4(0) + \Delta V_4 = 1.0 - 0.70 = 0.30 \ \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = 0.7\angle{-90^\circ} \text{ pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = 0.7\angle{-90^\circ} \text{ pu}$$
$$I_{32}(F) = \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = 0.7\angle{-90^\circ} \text{ pu}$$
$$I_{34}(F) = \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.3}{j0.19} = 0 \text{ pu}$$

**PROBLEM.5**

Using the method of building algorithm find the bus impedance matrix for the network shown in Figure




Add branch 1, $z_{10} = j0.2$ between node $q$ = 1 and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.3$ between node $q$ = 2 and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.15$ between the new node $q$ = 3 and the existing node $p$ = 1. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.5$ between node $q$ = 2 and node $p$ = 1.

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.3 & 0 & j0.3 \\ j0.2 & 0 & j0.35 & -j0.2 \\ -j0.2 & j0.3 & -j0.2 & Z_{44} \end{bmatrix}$$

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.5 + j0.2 + j0.3 - 2(j0) = j1.0$$

and

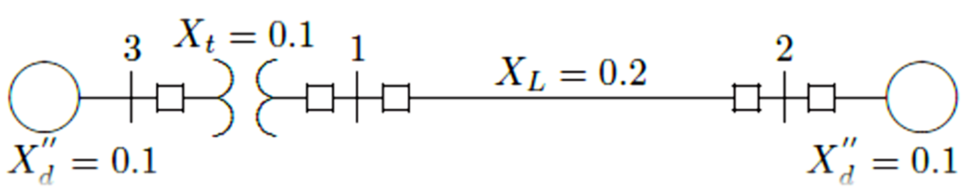
$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j1.0} \begin{bmatrix} -j0.2 \\ j0.3 \\ -j0.2 \end{bmatrix} \begin{bmatrix} -j0.2 & j0.3 & -j0.2 \end{bmatrix} = \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix}$$

the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix} = \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix}$$

## PROBLEM.6

Obtain the bus impedance matrix for the network




Add branch 1, $z_{20} = j0.1$ between node $q = 2$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{22} = z_{20} = j0.10$$

Next, add branch 2, $z_{30} = j0.1$ between node $q = 3$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{22} & 0 \\ 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.1$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{33} + z_{13} & 0 & Z_{33} \\ 0 & Z_{22} & 0 \\ Z_{33} & 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 4, $z_{12} = j0.2$ between node $q = 2$ and node $p = 1$.

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.2 & 0 & j0.1 & -j0.2 \\ 0 & j0.1 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & -j0.1 \\ -j0.2 & j0.1 & -j0.1 & Z_{44} \end{bmatrix}$$

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.2 + j0.2 + j0.1 - 2(j0) = j0.5$$

and

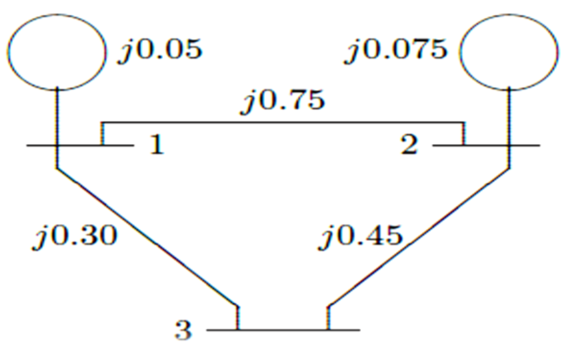
$$\frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{44}} = \frac{1}{j0.5}\begin{bmatrix} -j0.2 \\ j0.1 \\ -j0.1 \end{bmatrix}\begin{bmatrix} -j0.2 & j0.1 & -j0.1 \end{bmatrix} = \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix}$$

the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix} = \begin{bmatrix} j0.12 & j0.04 & j0.06 \\ j0.04 & j0.08 & j0.02 \\ j0.06 & j0.02 & j0.08 \end{bmatrix}$$

**PROBLEM.7**

Obtain the bus impedance matrix for the network




Add branch 1, $z_{10} = j0.05$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.05$$

Next, add branch 2, $z_{20} = j0.075$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 \\ 0 & j0.075 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.3$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2,

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.75$ between node $q = 2$ and node $p = 1$.

$$\begin{aligned}\mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.05 & 0 & j0.05 & -j0.05 \\ 0 & j0.075 & 0 & j0.075 \\ j0.05 & 0 & j0.35 & -j0.05 \\ -j0.05 & j0.075 & -j0.05 & Z_{44} \end{bmatrix}\end{aligned}$$

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.75 + j0.05 + j0.075 - 2(j0) = j0.875$$

and

$$\begin{aligned}\frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{44}} &= \frac{1}{j0.875}\begin{bmatrix} -j0.05 \\ j0.075 \\ -j0.05 \end{bmatrix}\begin{bmatrix} -j0.05 & j0.075 & -j0.05 \end{bmatrix} \\ &= \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}\end{aligned}$$

the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}$$

$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix}$$

Add link 5, $z_{23} = j0.45$ between node $q = 3$ and node $p = 2$.

$$\mathbf{Z}_{bus}^{(5)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} - Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23} - Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33} - Z_{32} \\ Z_{31} - Z_{21} & Z_{32} - Z_{22} & Z_{33} - Z_{23} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 & j0.042857 \\ j0.004286 & j0.068571 & j0.004286 & -j0.064286 \\ j0.047143 & j0.004286 & j0.347142 & j0.342857 \\ j0.042857 & -j0.064286 & j0.342857 & Z_{44} \end{bmatrix}$$

$$Z_{44} = z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.45 + j0.068571 + j0.347142 - 2(j0.004286) = j0.85714$$

and

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j0.85714} \begin{bmatrix} j0.042857 \\ -j0.064286 \\ j0.342857 \end{bmatrix} \begin{bmatrix} j0.042857 & -j0.064286 & j0.342857 \end{bmatrix}$$

$$= \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137143 \end{bmatrix}$$

the new bus impedance matrix is

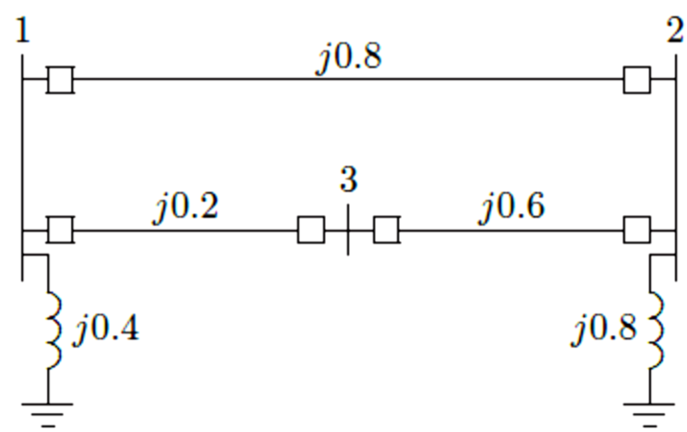
$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} -$$

$$\begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137142 \end{bmatrix} = \begin{bmatrix} j0.0450 & j0.00750 & j0.030 \\ j0.0075 & j0.06375 & j0.030 \\ j0.0300 & j0.03000 & j0.210 \end{bmatrix}$$

**PROBLEM.8**

The bus impedance matrix for the network shown in Figure is given by

$$Z_{bus} = j\begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$




There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

The line between buses 1 and 2 with impedance $Z_{12}$ = $j$0.8 is removed. The removal of this line is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{12}$ = -$j$0.8 between node $q$ = 2 and node $p$ = 1.

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$\mathbf{Z}_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 & -j0.100 \\ j0.200 & j0.400 & j0.25000 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & -j0.02500 & Z_{44} \end{bmatrix}$$

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = -j0.8 + j0.3 + j0.4 - 2(j0.2) = -j0.5$$

and

$$\frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{44}} = \frac{1}{-j0.5}\begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix}\begin{bmatrix} -j0.10 & j0.20 & -j0.025 \end{bmatrix}$$

$$= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix}$$

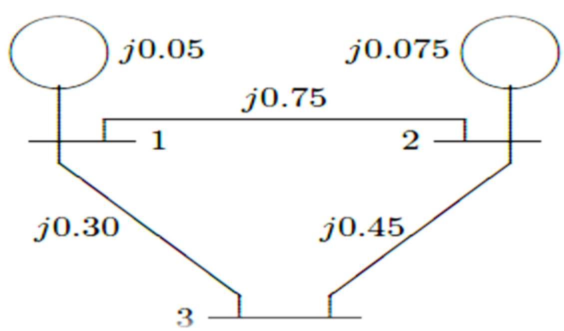
the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 \\ j0.200 & j0.400 & j0.25000 \\ j0.275 & j0.250 & j0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.00500 \\ j0.040 & -j0.080 & j0.01000 \\ -j0.005 & j0.010 & -j0.00125 \end{bmatrix}$$

$$= \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix}$$

**PROBLEM.9**

The per unit bus impedance matrix for the power system is given by



$$Z_{bus} = j\begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.

for a fault at bus 3 with fault impedance $Z_f = j0.19$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \quad \text{pu}$$

bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.03)(-j2.5) = 0.925 \ \text{pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.21)(-j2.5) = 0.475 \ \text{pu}$$

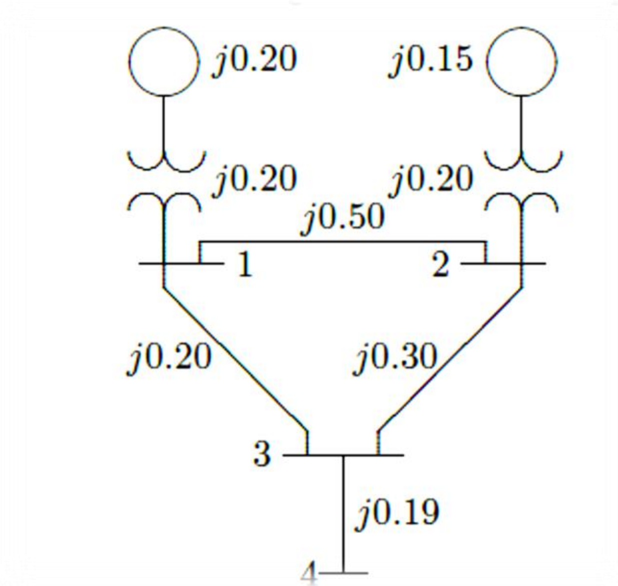
the short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \ \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \ \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \ \text{pu}$$

**PROBLEM.10**

The per unit bus impedance matrix for the power system is given by

$$Z_{bus} = j\begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$




(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
(b) Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.
(a) for a solid fault at bus 4 the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44}} = \frac{1.0}{j0.5} = -j2 \ \text{pu}$$

bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.200)(-j2) = 0.60 \ \text{pu} \\ V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.175)(-j2) = 0.65 \ \text{pu} \\ V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.310)(-j2) = 0.38 \ \text{pu} \\ V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.500)(-j2) = 0 \ \text{pu} \end{aligned}$$

the short circuit currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.60}{j0.5} = -j0.1 \ \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1 \ \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9 \ \text{pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = -j2 \ \text{pu} \end{aligned}$$

(b)for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit,
the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4 \quad \text{pu}$$

bus voltages during the fault are

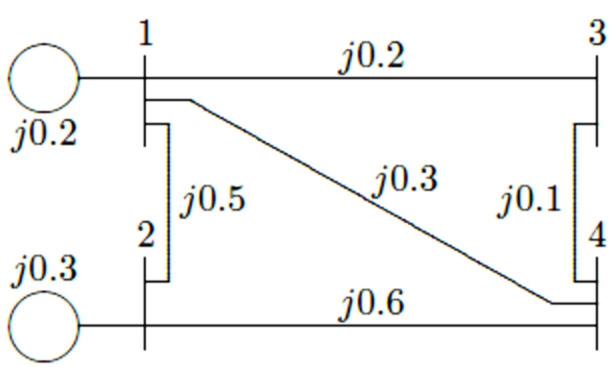
$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{12}I_2(F) = 1.0 - (j0.140)(-j4) = 0.44 \ \text{pu} \\
V_2(F) &= V_2(0) - Z_{22}I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09 \ \text{pu} \\
V_3(F) &= V_3(0) - Z_{32}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu} \\
V_4(F) &= V_4(0) - Z_{42}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu}
\end{aligned}$$

the short circuit currents in the lines are

$$\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7 \ \text{pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7 \ \text{pu} \\
I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = -j0.7 \ \text{pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \ \text{pu}
\end{aligned}$$

**PROBLEM.11**

The per unit bus impedance matrix for the power system shown in Figure
is given by



$$Z_{bus} = j \begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus4 through a fault impedance of $Z_f = j0.0025$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault.

for a fault at bus 4 with fault impedance $Zf = j0{:}0225$ per unit, the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44} + Z_f} = \frac{1.0}{j0.2475 + j0.0025} = -j4 \quad \text{pu}$$

bus voltages during the fault are

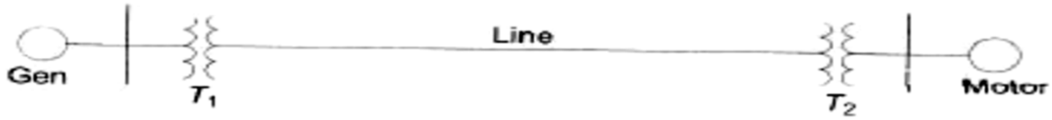
$$\begin{aligned} V_1(F) &= V_1(0) - Z_{14} I_4(F) = 1.0 - (j0.135)(-j4) = 0.46 \ \text{pu} \\ V_2(F) &= V_2(0) - Z_{24} I_4(F) = 1.0 - (j0.0975)(-j4) = 0.61 \ \text{pu} \\ V_3(F) &= V_3(0) - Z_{34} I_4(F) = 1.0 - (j0.210)(-j4) = 0.16 \ \text{pu} \\ V_4(F) &= V_4(0) - Z_{44} I_4(F) = 1.0 - (j0.2475)(-j4) = 0.01 \ \text{pu} \end{aligned}$$

the short circuit currents in the lines are

$$\begin{aligned} I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.46 - 0.16}{j0.2} = -j1.5 \ \text{pu} \\ I_{14}(F) &= \frac{V_1(F) - V_4(F)}{z_{14}} = \frac{0.46 - 0.01}{j0.3} = -j1.5 \ \text{pu} \\ I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.61 - 0.46}{j0.5} = -j0.3 \ \text{pu} \\ I_{24}(F) &= \frac{V_2(F) - V_4(F)}{z_{24}} = \frac{0.61 - 0.01}{j0.6} = -j1.0 \ \text{pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.16 - 0.01}{j0.10} = -j1.5 \ \text{pu} \end{aligned}$$

**PROBLEM:12**

For the system in Fig the ratings of the various components are:
Generator:25 MVA, 12.4 kV, l0% sub transient reactance
Motor:20 MVA, 3.8 kV, l5% sub transient reactance
Transformer T1: 25 MVA, 11/33 kv, 8% reactance
Transformer T2: 20 MVA, 33/3.3 kV, 10% reactance
Line: 20 ohms reactance




The system is loaded so that the motor is drawing 15 Mw at 0.9 loading power factor, the motor terminal voltage being 3.1 kv. Find the sub transient current in generator and motor for a fault at generator bus. Choose generator voltage base as 11Kv,the line voltage base is 33 kv and motor voltage base is 3.3 kv.

Base MVA = 25; Voltage base in gen circuit = 11 kV

voltage base in line circuit = 33 kV

voltage base in motor circuit = 3.3 kV

Calculation of pu reactances

$$\text{Gen} = 0.1 \times (12.4/11)^2 = 0.127$$

$$\text{Motor} = 0.15 \times (25/20) \times (3.8/3.3)^2 = 0.249$$

$$\text{Line} = 20 \times 25/(33)^2 = 0.459;$$

$$\text{Transformer } T_1 = 0.08$$

$$\text{Transformer } T_2 = 0.1 \times 25/20 = 0.125;$$

$$\text{Motor Load: } \frac{15}{25} = 0.6 \text{ MW (Pu) pf 0.9 leading or } \angle 25.8^\circ$$

$$\text{Terminal voltage} = 3.1/3.3 = 0.939 \text{ pu}$$

$$\text{Motor current} = 0.6/(0.939 \times 0.9) = 0.71 \ \angle 25.8^\circ \text{ pu}$$

Under conditions of steady load:

Voltage at generator terminals

$$= 0.939 \angle 0° + 0.71 \angle 25.8° (0.08 + 0.459 + 0.125) \angle 90°$$

$$= 0.734 + j\, 0.424 = 0.847 \angle 30°$$

Thévenin equivalent voltage as seen from $P$: $V° = 0.847 \angle 30°$

Current caused by fault in gen circuit (towards $P$) $= \dfrac{0.847 \angle 30°}{j\,0.127} = 6.67$ $\angle -60°$

$$(I_B \text{ (Gen)} = 25/(\sqrt{3} \times 11) = 1.312 \text{ kA};$$

$$I_B \text{ (Motor)} = 25/(\sqrt{3} \times 3.3) = 4.374 \text{ kA})$$

Current caused by fault in motor circuit (towards $P$) $= \dfrac{0.847 \angle 30°}{j\,0.913}$

$$= 0.93 \angle -60°$$

Motor current during fault $= -0.71 \angle 25.8° + 0.93 \angle -60°$

$$= -0.174 - j\, 1.114 \text{ pu} = \mathbf{4.93} \text{ kA}$$

**PROBLEM:13**

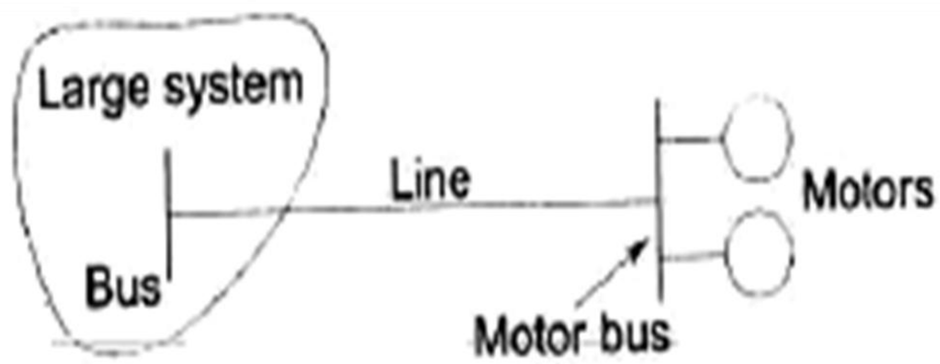
Two synchronous motors are connected to the bus of a large system through a short transmission line as shown in Fig. The ratings of various components are:

Motors (each): 1 MVA, 440 V,0.1 pu transient reactance

Line: 0.05 ohm reactance

Large system: Short circuit MVA at its bus at 440 V is 8.

when the motors are operating at 440 V, calculate the short circuit current (symmetrical) fed into a three-phase fault at motor bus.




Base: 1 MVA, 0.44 kV; Line reactance $= \dfrac{0.05 \times 1}{(0.44)^2} = 0.258$ pu

Reactance of large system = 1/8 = 0.125 pu

Operating voltage at motor bus before fault $= \dfrac{0.4}{0.44} = 0.909$ pu

Short circuit current fed to fault at motor bus $= 0.909 \left( \dfrac{1}{0.125 + 0.258} + 2 \times \dfrac{1}{0.1} \right) = 20.55$ pu

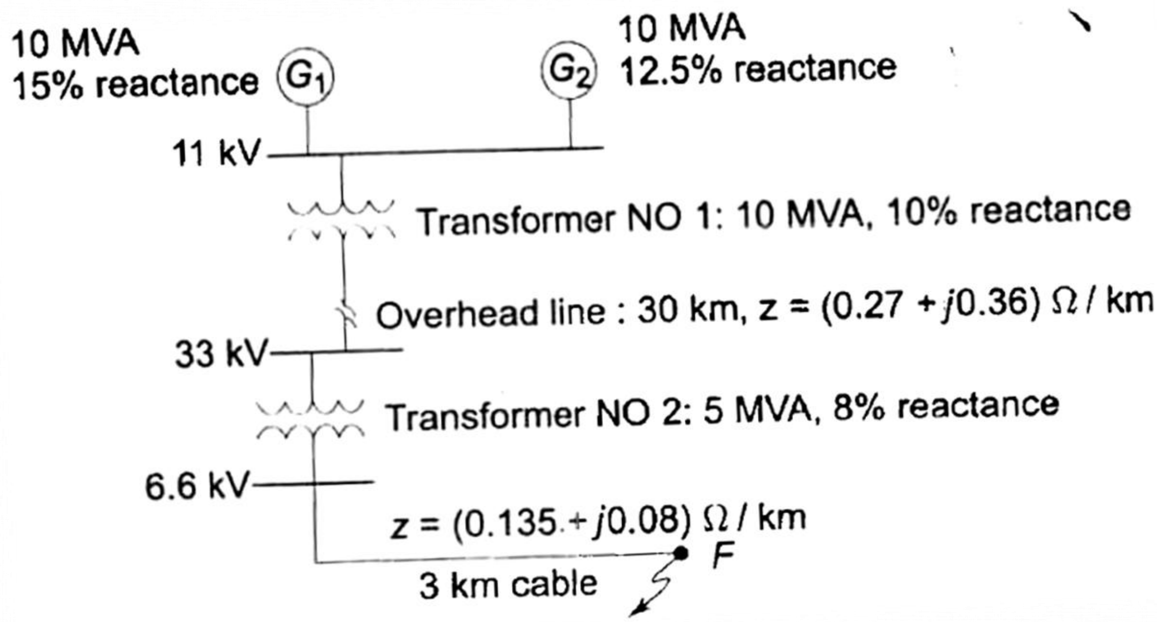
Base current $= 1/(\sqrt{3} \times 0.44) = 1.312$ kA

$\therefore$ Short circuit current = **26.96** kA

**PROBLEM:14**

For the radial network shown in Fig a three-phase fault occurs at F. Determine the fault current and the line voltage at ll kv bus under fault conditions

Select a system base of 100 MVA.Voltage bases are: 11kV-in generators, 33 kV for overhead line and 6.6 kV for cable.




$$\text{Reactance of } G_1 = j\frac{0.15\times100}{10} = j1.5 \text{ pu}$$

$$\text{Reactance of } G_2 = j\frac{0.125\times100}{10} = j1.25 \text{ pu}$$

$$\text{Reactance of } T_1 = j\frac{0.1\times100}{10} = j1.0 \text{ pu}$$

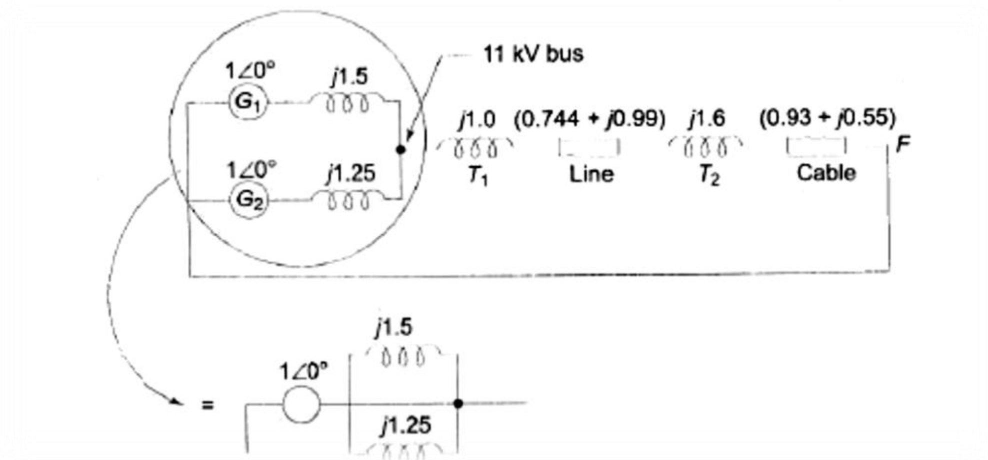
$$\text{Reactance of } T_2 = j\frac{0.08\times100}{5} = j1.6 \text{ pu}$$

$$\text{Overhead line impedance} = \frac{Z\,(\text{in ohms})\times \text{MVA}_{\text{Base}}}{(\text{kV}_{\text{Base}})^2}$$

$$= \frac{30\times(0.27+j0.36)\times100}{(33)^2}$$

$$= (0.744 + j0.99) \text{ pu}$$

$$\text{Cable impedance} = \frac{3(0.135+j0.08)\times100}{(6.6)^2} = (0.93 + j0.55) \text{ pu}$$




$$\text{Total impedance} = (j1.5 \parallel j1.25) + (j1.0) + (0.744 + j0.99) + (j1.6) + (0.93 + j0.55)$$

$$= 1.674 + j4.82 = 5.1 \angle 70.8^\circ \text{ pu}$$

$$I_{SC} = \frac{1\angle 0}{5.1\angle 70.8^\circ} = 0.196 \angle -70.8^\circ \text{ pu}$$

$$I_{Base} = \frac{100 \times 10^3}{\sqrt{3} \times 6.6} = 8{,}750 \text{ A}$$

$$\therefore \quad I_{SC} = 0.196 \times 8{,}750 = 1{,}715 \text{ A}$$

Total impedance between F and11 kV bus

$$= (0.93 + j055) + (j1.6) + (0.744 + j0.99) + (j1.0)$$

$$= 1.674 + j4.14 = 4.43 \angle 76.8^\circ \text{ pu}$$

$$\text{Voltage at 11 kV bus} = 4.43 \angle 67.8^\circ \times 0.196 \angle -70.8^\circ$$

$$= 0.88 \angle -3^\circ \text{ pu} = 0.88 \times 11 = 9.68 \text{ kV}$$

**PROBLEM:15**

A 25 MVA, 11 kV generator with $X''_d$ = 20% is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig. Each motor has $X''_d$ = 25% and $X'_d$ = 30% on a base of 5 MVA, 6.6 kV. The three-phase rating of the step-up transformer is 25 MVA, 11/66 kV with a leakage reactance of l0o/o and that of the step-down transformer is 25 MVA, 66/6.6k V with a leakage reactance of l0%.The bus voltage at the motors is 6.6 kV when a three-phase fault occurs at the point F. For the specified fault, calculate

(a) the sub transient current in the fault,

(b) the sub transient current in the breaker .8,
(c) the momentary current in breaker B, and
(d) the current to be interrupted by breaker B in five cycles.
Given: Reactance of the transmission line = l5%oo n a base of 25 MVA, 66kV. Assume that the system is operating on no load when the fault" occurs.
Choose a system base of 25 MVA.
For a generator voltage base of 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.


(a) For each motor

$$X''_{dm} = j0.25 \times (25/5) = j1.25\text{pu}$$

Line, transformers and generator reactances are already given on proper base values. The circuit model of the system for fault calculations is given in Fig. The system being initially on no load, the generator and motor induced emfs are identical.

$$I_{SC} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j4.22 \text{ pu}$$

$$\text{Base current in 6.6 kV circuit} = \frac{25 \times 1{,}000}{\sqrt{3} \times 6.6} = 2{,}187 \text{ A}$$

$$\therefore \quad I_{SC} = 4.22 \times 2{,}187 = 9{,}229 \text{ A}$$

(b) From Fig. 9.9c, current through circuit breaker $B$ is

$$I_{SC}(\text{B}) = 2 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j3.42$$

$$= 3.42 \times 2{,}187 = 7{,}479.5 \text{ A}$$

(c) For finding momentary current through the breaker, we must add the DC off-set current to the symmetrical sub transient current obtained in part (b). Rather than calculating the DC off-set current, allowance is made for it on an empirical basis momentary current through breaker B = 1.6 x 7479.5

=11967A

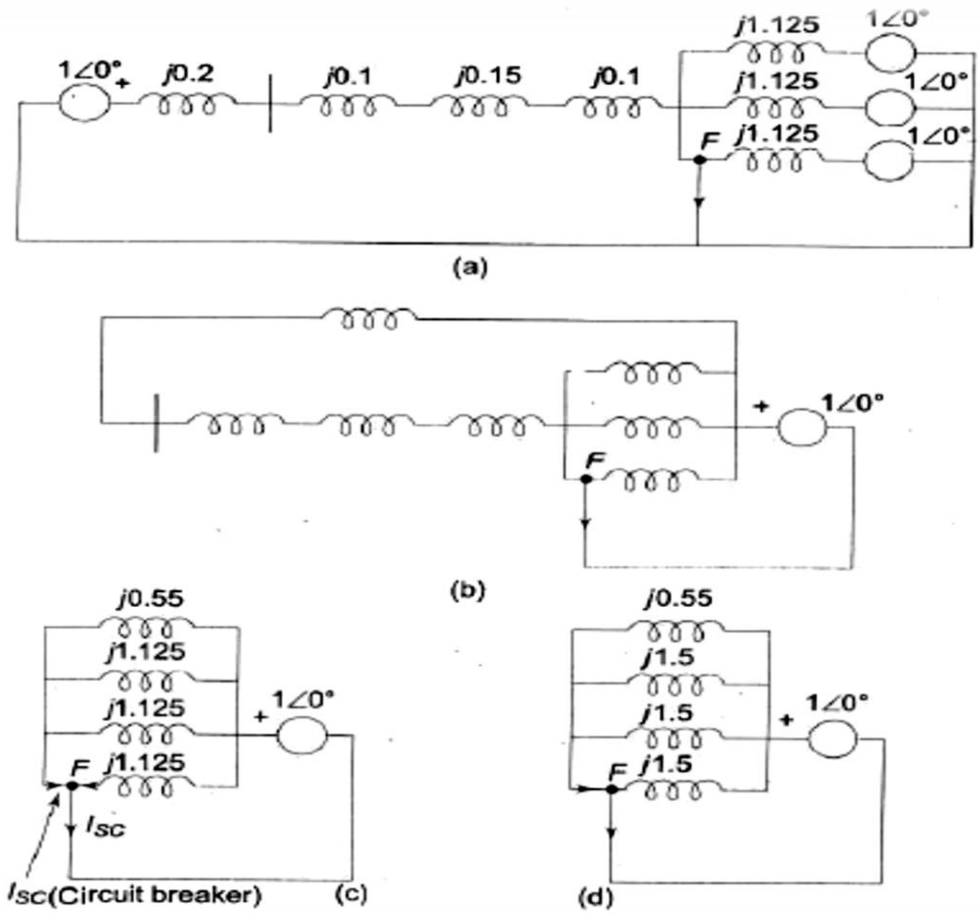
(d) To compute the current to be interrupted by the breaker, motor sub transient reactance ($X''_d$ = j0.25) is now replaced by transient reactance ($X'_d$ = j0.3O).

$$X_d'\text{ (motor)} = j0.3 \times \frac{25}{5} = j1.5 \text{ pu}$$

Current (symmetrical) to be interrupted by the breaker

$$= 2 \times \frac{1}{j1.5} + \frac{1}{j0.55} = 3.1515 \text{ pu}$$

Allowance is made for the DC off-set value by multiplying with a factor of 1.1
Therefore, the current to be interrupted is 1.1 x 3.1515 x 2.187 = 7.581 A




## PROBLEM:16

A synchronous generator and a synchronous motor each rated 25 MVA, 11Kv having l5% sub transient reactance are connected through transformers and a line as shown in Fig. The transformers are rated 25 MVA. 11/66kV and 66/11kV with leakage reactance of 10% each. The line has a reactance of 10% on a base of 25 MVA, 66 kv. The motor is drawing 15 Mw at 0.9 power factor leading and a terminal voltage of 10.6 kV when a symmetrical three-phase fault occurs at the motor terminals. Find the sub transient current in the generator, motor and fault.

Prefault voltage $V^o = \frac{10.6}{11} = 0.9636 \angle 0° \text{ pu}$

Load = 15 MW, 0.8 pf leading

$= \frac{15}{25} = 0.6$ pu, 0.8 pf leading

Prefault current $I^o = \frac{0.6}{0.9636 \times 0.8} \angle 36.9° = 0.7783 \angle 36.9° \text{ pu}$

Voltage behind subtransient reactance (generator)

$$E''_g = 0.9636 \angle 0° + j0.45 \times 0.7783 \angle 36.9°$$
$$= 0.7536 + j0.28 \text{ pu}$$

Voltage behind subtransient reactance (motor)

$$E''_m = 0.9636 \angle 0° - j0.15 \times 0.7783 \angle 36.9°$$
$$= 1.0336 - j0.0933 \text{ pu}$$

$$I''_g = \frac{0.7536 + j0.2800}{j0.45} = 0.6226 - j1.6746 \text{ pu}$$

$$I''_m = \frac{1.0336 - j0.0933}{j0.15} = -0.6226 - j6.8906 \text{ pu}$$

Current in fault

$$I^f = I''_g + I''_m = -j8.5653 \text{ pu}$$

Base current (gen/motor) $= \frac{25 \times 10^3}{\sqrt{3} \times 11} = 1{,}312.2$ A

Now

$$I''_g = 1{,}312.0\,(0.6226 - j1.6746) = (816.4 - j2{,}197.4) \text{ A}$$
$$I''_m = 1{,}312.2\,(-0.6226 - j6.8906) = (-816.2 - j9{,}041.8) \text{ A}$$
$$I^f = -j11{,}239 \text{ A}$$

**PROBLEM:17**

Three 6.6 kv generators A,B and C, each of 10% leakage reactance and MVA Ratings 40,50 and 25, respectively are interconnected electrically, as shown in fig by a tie bar through current limiting reactors, each of 12% reactance based upon the rating of the machine to which it is connected A. three-phase feeder is supplied from the bus bar of generator A at a line voltage of 6.6 kV. The feeder has a resistance of 0.06Ω/phase and an inductive reactance of 0.12Ω/phase. Estimate the maximum MVA that can be fed into a symmetrical short circuit at the far end of the feeder.

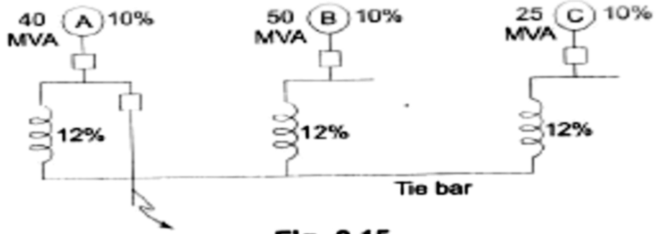



**Fig. 9.15**

*Solution* Choose as base 50 MVA, 6.6 kV.

Feeder impedance

$$= \frac{(0.06 + j0.12) \times 50}{(6.6)^2} = (0.069 + j0.138) \text{ pu}$$

$$\text{Gen A reactance} = \frac{0.1 \times 50}{40} = 0.125 \text{ pu}$$

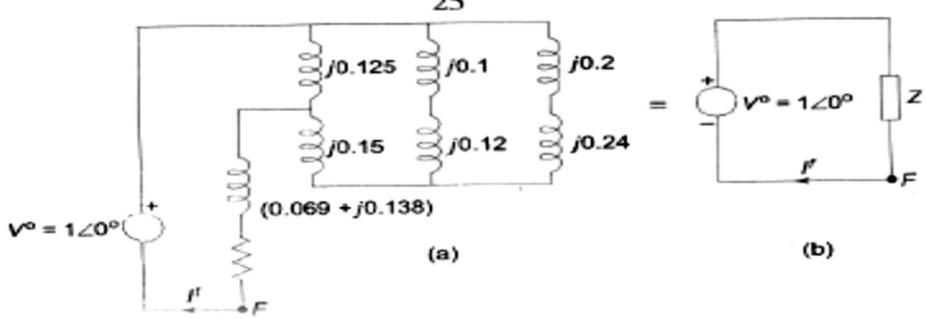
$$\text{Gen B reactance} = 0.1 \text{ pu}$$

$$\text{Gen C reactance} = 0.1 \times \frac{50}{25} = 0.2 \text{ pu}$$

$$\text{Reactor A reactance} = \frac{0.12 \times 50}{40} = 0.15 \text{ pu}$$

$$\text{Reactor B reactance} = 0.12 \text{ pu}$$

$$\text{Reactor C reactance} = \frac{0.12 \times 50}{25} = 0.24 \text{ pu}$$




Assume no load prefault conditions, i.e. prefault currents are zero. Postfault currents can then be calculated by the circuit model of Fig. 9.16a corresponding to Fig. 9.13d. The circuit is easily reduced to that of Fig. 9.16b, where

$$Z = (0.069 + j0.138) + j0.125 \parallel (j0.15 + j0.22 \parallel j0.44)$$

$$= 0.069 + j0.226 = 0.236\angle 73°$$

$$\text{SC MVA} = V^o I^f = V^o\left(\frac{V^o}{Z}\right) = \frac{1}{Z} \text{ pu (since } V^o = 1 \text{ pu)}$$

$$= \frac{1}{Z} \times (\text{MVA})_{\text{Base}}$$

$$= \frac{50}{0.236} = 212 \text{ MVA}$$

**PROBLEM:18**

A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are:

Generator: 10 MVA, 6.6 kV; $X''_d$= 0.1 pu, $X'_d$= 0.20pu and $X_d$= 0.80 Pu

Transformer:10MVA,6.9/33kV,reactance0.08pu

The system is operating no load at a line voltage of 30 kv, when a three phase fault occurs on the line just beyond the circuit breaker Find

(a) the initial symmetrical rms current in the breaker

(b) the maximum possible DC off-set current in the breaker,

(c) the momentary current rating of the breaker

(d) the current to be interrupted by the breaker and the interrupting kVA and

(e) the sustained short circuit current in the breaker

Bus: 10 MVA, 6.6 kV (Gen), 6.6/31.56 kV (transformer)

Base current = $10/(\sqrt{3} \times 31.56) = 0.183$ kA

Gen reactances: $x''_d = 0.1,\ x'_d = 0.2,\ x_d = 0.8$ pu

Transformer reactance: $0.08 \times (6.9/6.6)^2 = 0.0874$ pu

No load voltage before fault = 30/31.56 = 0.95 pu

(a) Initial symmetrical rms current = $\dfrac{0.95}{0.1 + 0.0874} = 5.069$ pu

= **0.9277** kA

(b) Max. possible dc off-set current = $\sqrt{2} \times 0.9277$ = **1.312** kA

(c) Momentary current (rms) rating of the breaker = 1.6 × 0.9277

= **1.4843** kA

(d) Current to be interrupted by the breaker (5 cycle) = 1.1 × 0.9277

= **1.0205** kA; Interrupting MVA = $\sqrt{3} \times 30 \times 1.0205$ = **53.03** MVA

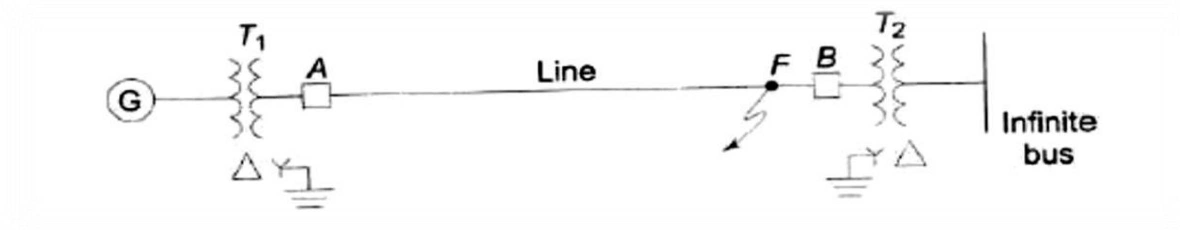
(e) Sustained short circuit current in breaker = $\dfrac{0.95}{0.8 + 0.0874} \times 0.183$

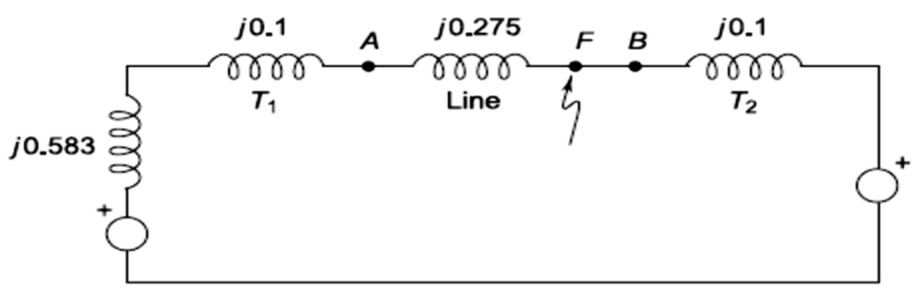
= **0.1959** k/a

**PROBLEM:19**

The system shown in Fig is delivering 50 MVA at 11kv, 0.8 lagging power factor into a bus which may regarded as infinite Particulars of various system components are:

Generator : 60 MVA, 12 kV, $X'_d$ = 0.35 Pu
Transformers( each): 80 MVA, 12/66kV, X = 0.08 pu
Line: Reactance12 ohms, resistance negligible
Calculate the symmetrical current that the circuit breakers A and B will be called upon to interrupt in the event of a three-phase fault occurring at F near the circuit breaker B




Base: 100 MVA; 12 kV (Gen. ckt), 66 kV (line)

Base current (gen ckt) = $100/(\sqrt{3} \times 12)$ = 4.81 k/A

Base current (line) = $100/(\sqrt{3} \times 66)$ = 0.875 kA

Component reactances

Gen: 0.35 × (100/60) = 0.583 pu;

$$\text{Line: } = \frac{12 \times 100}{(66)^2} = 0.275 \text{ pu}$$

Transformer: 0.08 × (100/80) = 0.1 pu

Load: 50/100 = 0.5 pu,

11/12 = 0.917 pu,

*pf* = 0.8 lag; $\angle -36.9°$

Load current = 0.5/0.917 = 0.545 pu

Thévenin voltage at *F* before fault, $V° = 0.917 \angle 0°$

$$\text{Current through breaker } A \text{ due to fault} = \frac{0.917}{j(0.583 + 0.1 + 0.275)}$$

$$= 0.957 \angle -90^\circ$$

$$\text{Post fault current through breaker } A = 0.957 \angle -90^\circ + 0.545 \angle -36.9^\circ$$

$$= 0.436 - j\,1.284 = 1.356 \text{ pu} = 6.522 \text{ kA}$$

$$\text{Current through breaker } B \text{ due to fault} = 0.917/j\,0.1 = 9.17 \angle -90^\circ$$

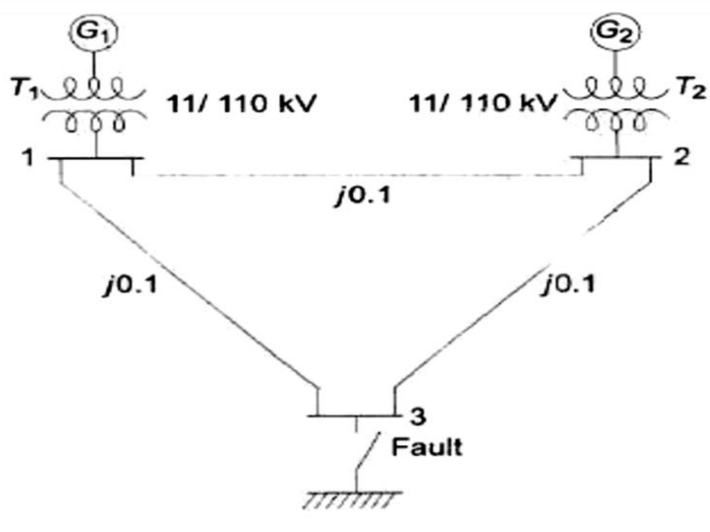
$$\text{Post fault current through breaker } B = 9.17 \angle -90^\circ + 0.545 \angle -36.9^\circ$$

$$= 0.436 - j\,9.497 = 9.507 \text{ pu}$$

$$= \mathbf{8.319} \text{ kA}$$

**PROBLEM:20**

Consider the 3-bus system of Fig. The generators are 100 MVA, with transient reactance 10% each. Both the transformers are 100 MVA with a leakage reactance of 5%t. The reactance of each of the lines to a base of 100 MVA, 110 KV is 10%. Obtain the short circuit solution for a three-phase solid short circuit on bus 3. Assume prefault voltages to be 1 pu and prefault currents to be zero.




$$Y_{BUS} = \begin{bmatrix} -j26.67 & j10 & j10 \\ j10 & -j26.67 & j10 \\ j10 & j10 & -j20 \end{bmatrix}$$

Inverting,

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} j\,0.0885 & j\,0.0613 & j\,0.0749 \\ j\,0.0613 & j\,0.0885 & j\,0.0749 \\ j\,0.0749 & j\,0.0749 & j\,0.1249 \end{bmatrix}$$

Using Eq. (9.26), $V_1^f = V_1^0 - (Z_{13}/Z_{23})\ V_3^0$

The prefault condition being no load, $V_1^0 = V_2^0 = V_3^0 = 1$ pu

$$\therefore \qquad V_1^f = 1.0 - \frac{j\,0.0749}{j\,0.1249} \times 1 = 0.4004 \text{ pu} \text{ // } ^{ly}$$

$$V_2^f = 0.4004;\ V_3^f = 0$$

From Eq. (9.25) $\quad I_f = 1.0/j\ 0.1249 = -\,j\ \mathbf{8.006}$ pu

S.C. current in line 1–3

$$I_{13}^f = \frac{V_1^f - V_3^f}{z_{13}} = \frac{0.4004 - 0}{j\,0.1} = -\,j\ \mathbf{4.094} \text{ pu}$$

The fault current for a fault on bus 1 (or bus 2) will be

$$I^f = \frac{1.00}{Z_{11}\,(\text{or } Z_{22})}$$

$$= \frac{1.00}{j\,0.0885} = -\,j\ 11.299 \text{ pu.}$$

## UNIT-IV
## UNSYMMETRICAL FAULT ANALYSIS

**PROBLEM:1**

Draw the zero sequence network for the system described in Assume zero sequence reactances for the generator and motors of 0.06 per unit. Current limiting reactors of 2.5 ohms each are connected in the neutral of the generator and motor No. 2. The zero sequence reactance of the transmission line is 300 ohms.

The zero sequence reactance of the transformer is equal to its positive sequence reactance. Hence Transformer zero sequence reactance : 0.0805 pu
Generator zero sequence reactances: 0.06 pu

$$\text{Zero sequence reactance of motor 1} = 0.06 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2$$
$$= 0.082 \text{ pu}$$

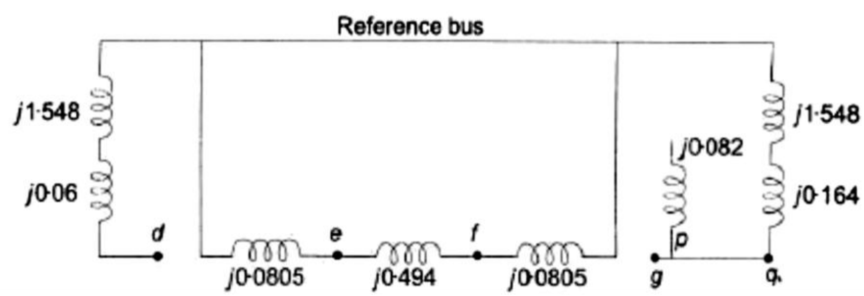
$$\text{Zero sequence reactance of motor 2} = 0.06 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2$$
$$= 0.164 \text{ pu}$$

$$\text{Reactance of current limiting reactors} = \frac{2.5 \times 25}{(11)^2} = 0.516 \text{ pu}$$

Reactance of current limiting reactor included in zero sequence network
$$= 3 \times 0.516 = 1.548 \text{ pu}$$

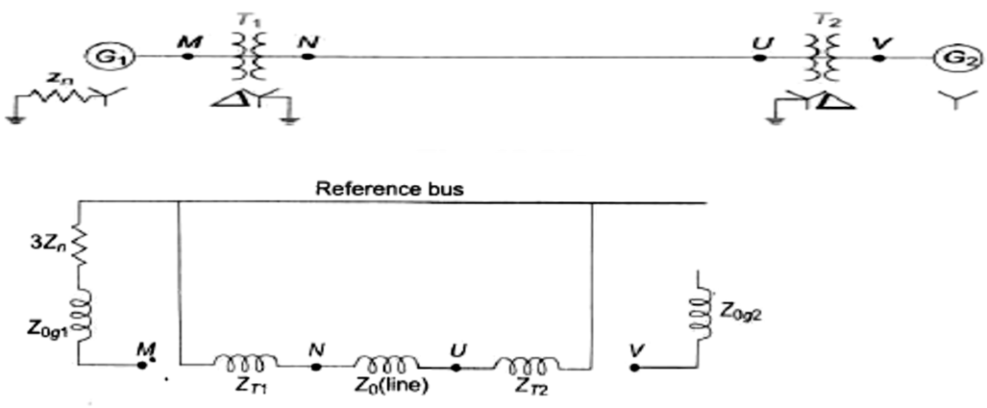
$$\text{Zero sequence reactance of transmission line} = \frac{300 \times 25}{(123.2)^2}$$
$$= 0.494 \text{ pu}$$

The zero sequence network is shown in Fig. 10.27.




**PROBLEM:2**

For the power system whose one-line diagram is shown in Fig, sketch the zero sequence network.




**PROBLEM:3**

A 25 MVA, 11 kV, three-phase generator has a sub transient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in the one-line diagram of Fig. The motors have rated inputs of 15 and 7.5 MVA, both 10 kV with 25% sub transient reactance. The three-phase Transformers are both rated 30 MVA, I0.8/121 kV, connection Δ-Y with leakage Reactance of 10% each. The series reactance of the line is 100 ohms. Draw the positive and negative sequence networks of the system with reactances marked in per unit. Assume that the negative sequence reactance of each machine is equal to its sub transient reactance. Omit resistances. Select generator rating as base in the generator circuit

$$\text{Transmission line voltage base} = 11 \times \frac{121}{10.8} = 123.2 \text{ kV}$$

$$\text{Motor voltage base} = 123.2 \times \frac{10.8}{121} = 11 \text{ kV}$$

The reactances of transformers, line and motors are converted to pu values on appropriate bases as follows:

$$\text{Transformer reactance} = 0.1 \times \frac{25}{30} \times \left(\frac{10.8}{11}\right)^2 = 0.0805 \text{ pu}$$

$$\text{Line reactance} = \frac{100 \times 25}{(123.2)^2} = 0.164 \text{ pu}$$

$$\text{Reactance of motor 1} = 0.25 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.345 \text{ pu}$$

$$\text{Reactance of motor 2} = 0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.69 \text{ pu}$$

**PROBLEM:4**

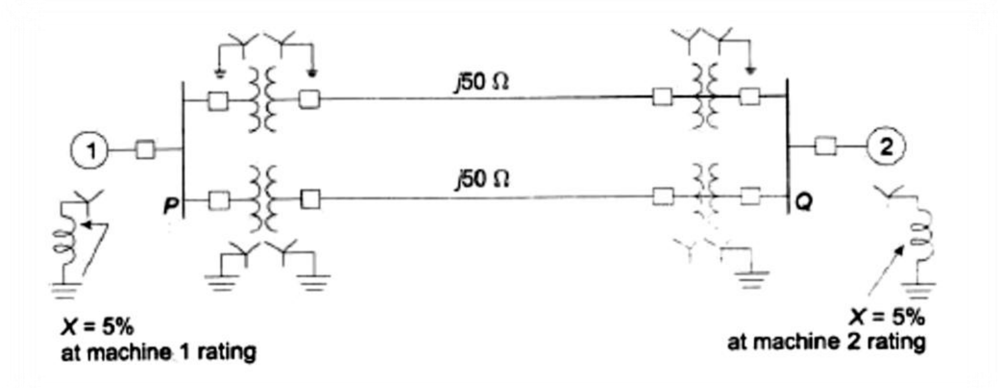
Draw the positive, negative and zero sequence impedance networks for the power system. Choose a base of 50 MVA, 220 kV in the 50Ω transmission lines, and mark all reactances in pu. The ratings of the generators and transformers are:

Generator1:25 MVA, 11 kV, X" = 20%

Generator2:25 MVA, 11 kV, X" = 20%

Three-phase transformer( each):20 MVA, 11Y/220Y kV, X = 15%

The negative sequence reactance of each synchronous machine is equal to its sub transient reactance the zero sequence reactance of each machine is 8%. Assume that the zero sequence reactances of lines are 250% of their positive sequence reactances.



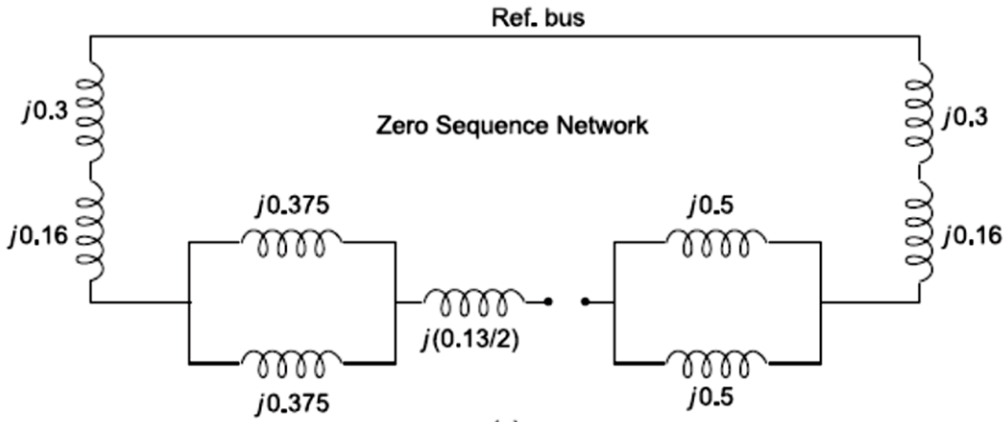

Base: 50 MVA, 220 kV (in line), 11 kV (Gen. 1 and 2)

$$X''_{g1} = 0.2 \times \frac{50}{25} = 0.4 \qquad X_0 \text{ (each m/c)} = 0.08 \times \frac{50}{25} = 0.16$$

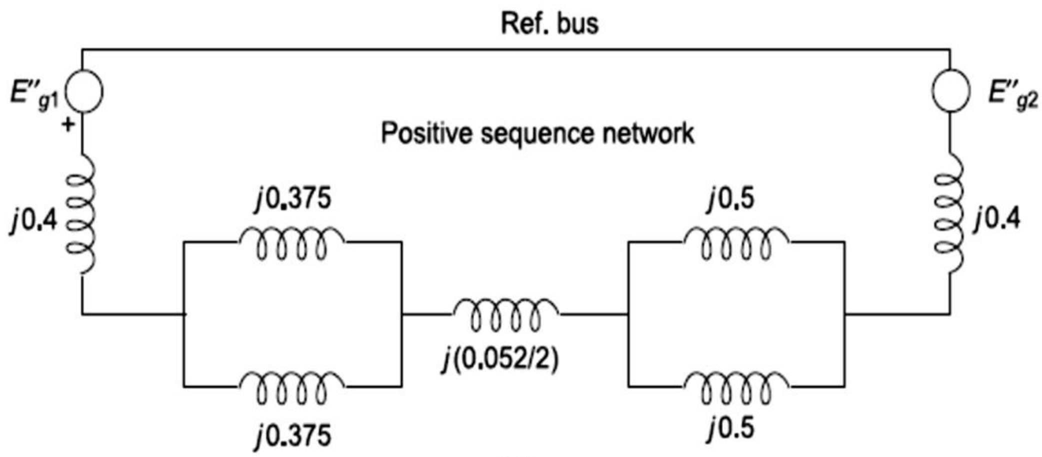
$$X''_{g2} = 0.4, \qquad X_T \text{ (each)} = 0.15 \times \frac{50}{25} = 0.375$$

$$X_L = \frac{50 \times 50}{(220)^2} = 0.052 \qquad X_{L0} = 0.052 \times 2.5 = 0.13$$

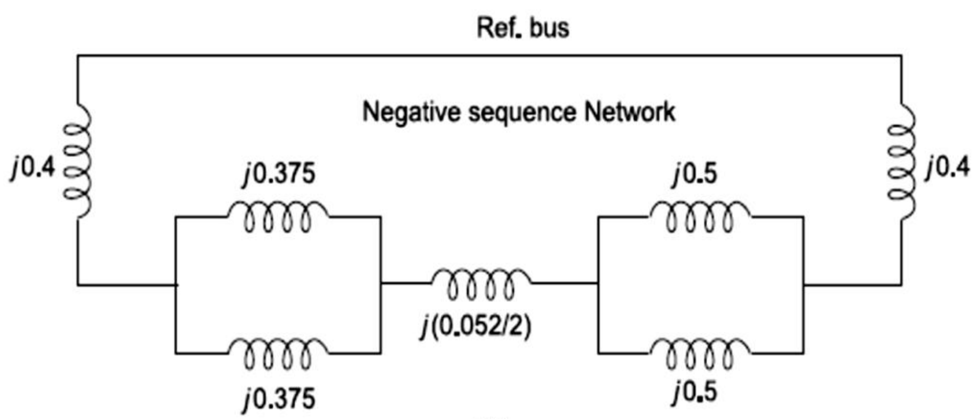
$$\text{Grounding reactance (each)} = 0.05 \times \frac{50}{25} = 0.1$$


Ref. bus
Zero Sequence Network
j0.3
j0.3
j0.16
j0.16
j0.375
j0.375
j0.5
j0.5
j(0.13/2)

(c)


Ref. bus
E″g1
E″g2
+
Positive sequence network
j0.4
j0.4
j0.375
j0.375
j0.5
j0.5
j(0.052/2)

(a)


Ref. bus
Negative sequence Network
j0.4
j0.4
j0.375
j0.375
j0.5
j0.5
j(0.052/2)

(b)

**PROBLEM:5**

For the power system draw the positive, negative and zero sequence networks. The generators and transformers are rated as follows:

Generator1: 25 MVA, 11 kV, X”=0.2, X2 = 0.15,Xo = 0.03 pu

Generator2: 15 MVA, 11 kV, X” =0.2, X2= 0.15,X 0 = 0.05 pu

Synchronous Motor 3: 25 MVA, 11 kV, X” = 0.2, X2= 0.2, Xo = 0.1 pu

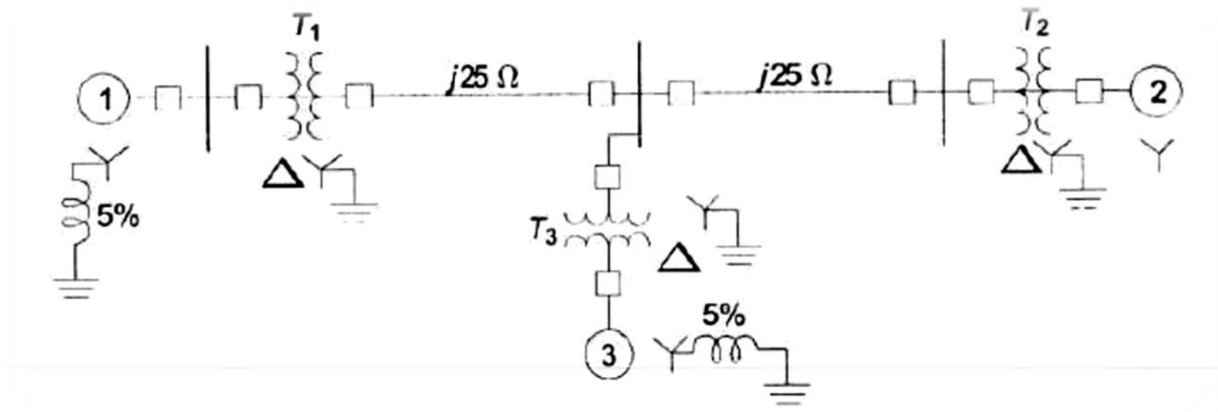
Transformer l: 25 MVA, 11Δ/120 Y kV, X = 10%

2: 12.5 MVA, 11Δ/120 Y kV, X = 10%

3: 10 MVA, 120Y/11 Y kV, X=10%

Choose a base of 50 MVA, 11 kV in the circuit of generator l.

Note: Zero sequence reactance of each line is 250% of its positive Sequence reactance




Base: 50 MVA, 11 kV (Gen 1, 2, Motor), 120 kV (line)

Gen 1: $X'' = 0.4$, $X_2 = 0.3$, $X_0 = 0.06$

Gen 2: $X'' = 0.67$, $X_2 = 0.5$, $X_0 = 0.17$

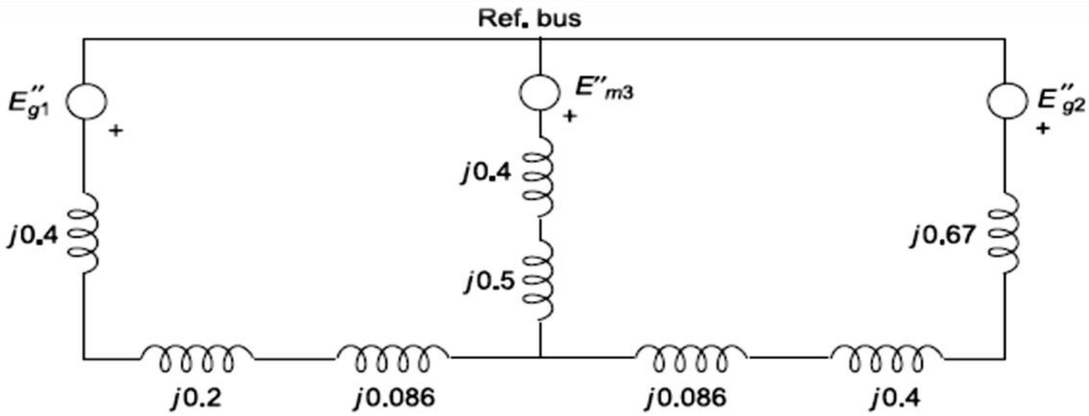
Mot. 3: $X'' = 0.4$, $X_2 = 0.4$, $X_0 = 0.2$

Transf. 1: $X = 0.2$, Transf. 2: $X = 0.4$, Transf. 3: $X = 0.5$

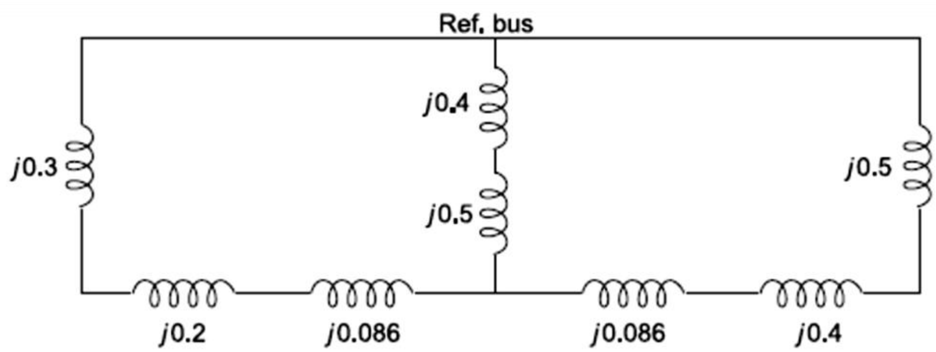
Line (each) $= 25 \times 50/(120)^2 = 0.086$,

$X_{LO} = 0.086 \times 2.5 = 0.215$

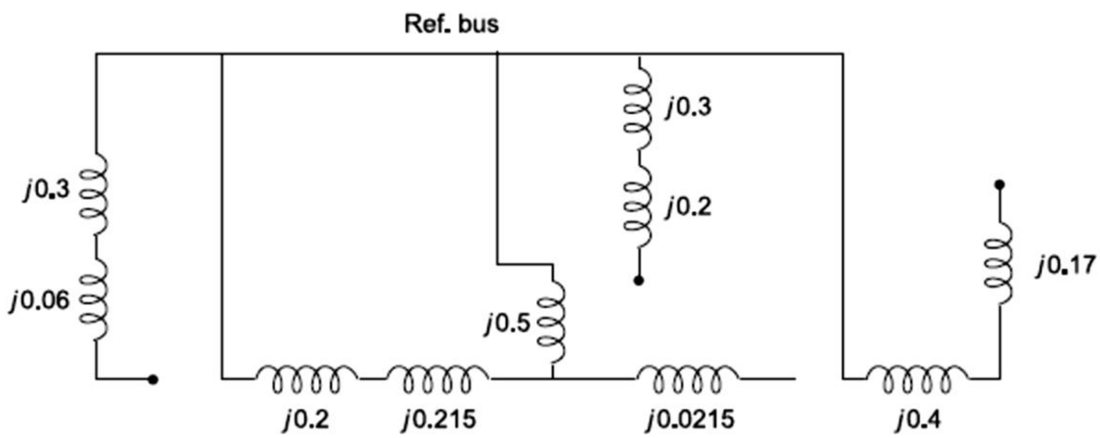
Neutral grounding reactance of $G_1$ and $M_3$ = 0.1 each




(a) Positive Sequence Network



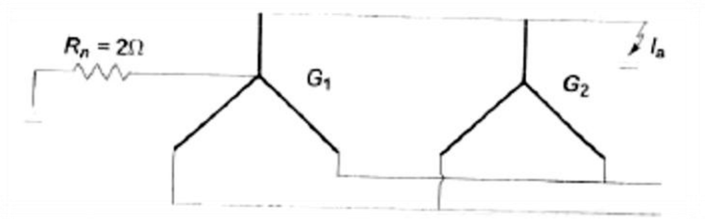

(b) Negative Sequence Network




(c) Zero Sequence Network

**PROBLEM:6**

Two 11 kV, 20 MVA, three-phase star connected generators operate in parallel As shown in Fig the positive, negative and-zero sequence reactances of each being, respectively j0.18, j0.15,j0.10 pu. The star point generators of one of the is isolated and that of the other is earthed through a 2.0 ohm resistor. A single line-to-ground fault occurs at the terminals of one of the generators. Estimate (i) the fault current, (ii) current in grounding resistor, and (iii) the voltage across grounding resistor.




*Solution* (*Note*: All values are given in per unit.)

Since the two identical generators operate in parallel,

$$X_{1eq} = \frac{j0.18}{2} = j0.09,\ X_{2eq} = \frac{j0.15}{2} = j0.075$$

Since the star point of the second generator is isolated, its zero sequence reactance does not come into picture. Therefore,

$$Z_{0eq} = j0.10 + 3R_n = j0.10 + 3 \times \frac{2\times 20}{(11)^2} = 0.99 + j0.1$$

For an LG fault, using Eq. (11.18), we get

$$I_f \text{ (fault current for LG fault)} = I_a = 3I_{a1} = \frac{3E_a}{X_{1eq} + X_{2eq} + Z_{0eq}}$$

**(a)** $I_f = \dfrac{3\times 1}{j0.09 + j0.075 + j0.1 + 0.99} = \dfrac{3}{0.99 + j0.265}$

$= 2.827 - j0.756$

**(b)** Current in the grounding resistor $= I_f = 2.827 - j0.756$

$$|I_f| = 2.926 \times \frac{20}{\sqrt{3}\times 11} = 3.07 \text{ kA}$$

**(c)** Voltage across grounding resistor $= \dfrac{40}{121}(2.827 - j0.756)$

$= 0.932 - j0.249$

$= 0.965 \times \dfrac{11}{\sqrt{3}} = 6.13$ kV

## PROBLEM:7

For Problem:6 , assume that the grounded generator is solidly grounded. Find
The fault current in each phase and voltage of the healthy phase for a double line to ground fault on terminals of the generators. Assume solid fault (Zf - 0).

$$I_{a1} = \frac{E_a}{X_{1eq} + X_{2eq}} = \frac{1}{j0.09 + j0.075} = -j6.06$$

Using Eq. (11.15), we have

$$I_f \text{ (fault current)} = I_b = -j\sqrt{3}I_{a1} = (-j\sqrt{3})(-j6.06) = -10.496$$

Now

$$V_{a1} = V_{a2} = E_a - I_{a1}X_{1eq} = 1.0 - (-j6.06)(j0.09)$$

$$= 0.455$$

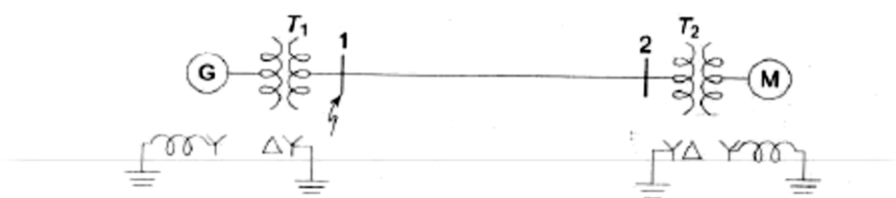
$$V_{a0} = -I_{a0}Z_0 = 0 \qquad (\because I_{a0} = 0)$$

Voltage of the healthy phase,

$$V_a = V_{a1} + V_{a2} + V_{a0} = 0.91$$

## PROBLEM:8

A single line to ground fault (on phase a) occurs on the bus I of the system of Fig




.

Find

(a) Current in the fault.

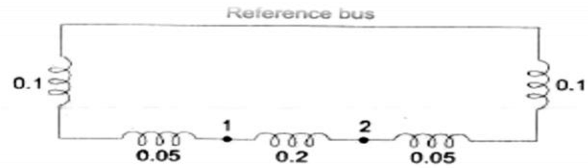
(b) sc current on the transmission line in all the three phases. :

(c) SC current in phase a of the generator.

(d) Voltage of the healthy phases of the bus 1.

Given: Rating of each machine 1200 kvA, 600 v with x’ = x2 = 10%, xo = 5%. Each three-phase transformer is rated 1200 kvA, 600 v – Δ/3300V-Y with leakage reactance of 5% The reactances of the transmission line are x1 = X2 = 20% and Xo = 40% on a base of 1200 kVA, 3300 V. The reactances of the neutral grounding reactors are 5% on the kVA and voltage base of the machine.

Note: Use Z bus, method.




Bus 1 to reference bus

$$Z_{1\text{-BUS}} = j[0.15]$$

Bus 2 to Bus 1

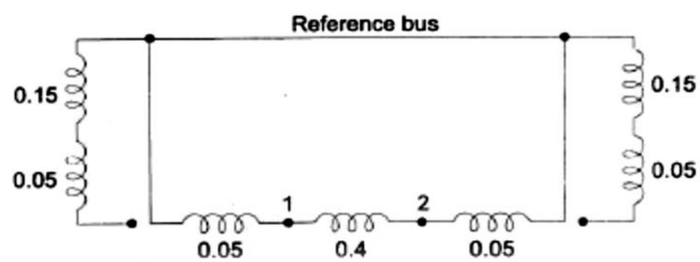
$$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.35 \end{bmatrix}$$

Bus 2 to reference bus

$$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.35 \end{bmatrix} - \frac{j}{0.35 + 0.15}\begin{bmatrix} 0.15 \\ 0.35 \end{bmatrix}[0.15 \quad 0.35]$$

or $$Z_{1\text{-BUS}} = j\begin{bmatrix} 0.105 & 0.045 \\ 0.045 & 0.105 \end{bmatrix} = Z_{2\text{-BUS}} \qquad \text{(i)}$$

Zero sequence network of the system is drawn in Fig. 11.29 and its bus impedance matrix is computed below.




Bus 1 to reference bus

$$Z_{0\text{-BUS}} = j\ [0.05]$$

Bus 2 to bus 1

$$Z_{0\text{-BUS}} = j\begin{bmatrix} 0.05 & 0.05 \\ 0.05 & 0.45 \end{bmatrix}$$

Bus 2 to reference bus

$$Z_{0\text{-BUS}} = j\begin{bmatrix} 0.05 & 0.05 \\ 0.05 & 0.45 \end{bmatrix} - \frac{j}{0.45 + 0.05}\begin{bmatrix} 0.05 \\ 0.45 \end{bmatrix}[0.05 \quad 0.45]$$

or

$$Z_{0\text{-BUS}} = j\begin{matrix} 1 \\ 2 \end{matrix}\begin{bmatrix} 0.045 & 0.005 \\ 0.005 & 0.045 \end{bmatrix} \qquad \text{(ii)}$$

$$I^f_{1-1} = \frac{V_1^0}{Z_{1-11} + Z_{2-11} + Z_{0-11} + 3Z^f}$$

But $V^o_1 = 1$ pu (system unloaded before fault)

Then

$$I^f_{1-1} = \frac{-j1.0}{0.105 + 0.105 + 0.045} = -\ j3.92 \text{ pu}$$

$$I^f_{1-1} = I^f_{2-1} = I^f \quad \ldots 2 \text{ pu}$$

(a) Fault current, $I^f_1 = 3I^f_{1-1} =$

(b) $V^f_{1-1} = V^o_{1-1} = Z_{1-11}\ I^f_{1-1}$

$= 1.0 - j0.105 \times -\ j3.92 = 0.588;\ V^o_{1-1} = 1$ pu

$V^f_{1-2} = V^o_{1-2} - Z_{1-21} I^f_{2-1};\ V^o_{1-2} = 1.0$ (system unloaded before fault)

$= 1.0 - j0.045 \times -\ j3.92 = 0.824$

$V^f_{2-1} = -\ Z_{2-11} I^f_{2-1}$

$= -\ j0.105 \times -\ j3.92 = 0.412$

$V^f_{2-2} = -\ Z_{2-21}\ I^f_{2-1}$

$= -\ j0.045 \times -\ j3.92 = -\ 0.176$

$V^f_{0-1} = -\ Z_{0-11} I^f_{0-1}$

$= -\ j0.045 \times -\ j3.92 = -\ 0.176$

$V^f_{0-2} = -\ Z_{0-21}\ I^f_{0-1}$

$= -\ j0.005 \times -\ j3.92 = -\ 0.02$

$I^f_{1-12} = y_{1-12}\ (V^f_{1-1} - V^f_{1-3})$

$$= \frac{1}{j0.2}\ (0.588 - 0.824) = j1.18$$

$I^f_{2-12} = y_{2-12}\ (V^f_{2-1} - V^f_{2-2})$

$$= \frac{1}{j0.2}(-\ 0.412 + 0.176) = j1.18$$

$I^f_{0-12} = y_{0-12}\ (V^f_{0-1} - V^f_{0-2})$

$$= \frac{1}{j0.4}\ (-\ 0.176 + 0.020) = j0.39$$

$$\begin{bmatrix} I^f_{a-12} \\ I^f_{b-12} \\ I^f_{c-12} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} I^f_{1-12} \\ I^f_{2-12} \\ I^f_{0-12} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} \begin{bmatrix} j1.18 \\ j1.18 \\ j0.39 \end{bmatrix}$$

$$I^f_{a-12} = j1.18 + j1.18 + j0.39 = j2.75$$

$$I^f_{b-12} = j1.18 \angle 240^\circ + j1.18 \angle 120^\circ + j0.39$$

$$= -j079$$

$$I^f_{c-12} = j1.18 \angle 120^\circ + j1.18 \angle 240^\circ + j0.39$$

$$= j0.79$$

(c) $I^f_{1\text{-}G} = \dfrac{1}{j0.15}(1 - 0.588) \angle -33^\circ$

$$= -1.37 - j2.38$$

$$I^f_{0\text{-}G} = \frac{1}{j0.15}[0 - (-0.412)] \angle 30^\circ$$

$$= 1.37 - j2.38$$

$I^f_{0\text{-}G} = 0$ (*see* Fig. 11.29)

$$\therefore \quad I^f_{a\text{-}G} = (-1.37 - j2.38) + (1.37 - j2.38)$$

$$= -j4.76$$

Current in phases *b* and c of the generator can be similarly calculated.

(d) $V^f_{b-1} = 2V^f_{1-1} + V^f_{2-1} + V^f_{0-1}$

$$= 0.588 \angle 240^\circ - 0.412 \angle 120^\circ - 0.176$$

$$= -0.264 - j0.866 = 0.905 \angle -107^\circ$$

$$V^f_{c-1} = V^f_{1-1} + V^f_{2-1} + V^f_{0-1}$$

$$= 0.588 \angle 120^\circ - 0.412 \angle 240^\circ - 0.176$$

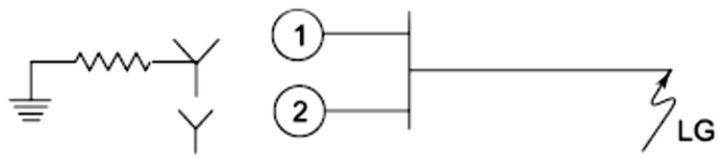
$$= -0.264 + j0.866 = 0.905 \angle 107^\circ$$

**PROBLEM:9**

Two 25 MVA, 11 kv synchronous generators are connected to a common bus bar which supplies a feeder. The star point of one of the generators is grounded through a resistance of 1.0 ohm, while that of the other generator is isolated. A line-to-ground fault occurs at the far end of the feeder. Determine: (a) the fault current; (b) the voltage to ground of the sound phases of the feeder at the fault point; and (c) voltage of the star point of the grounded generator with respect to ground.

The impedances to sequence currents of each generator and feeder are given below:

| | Generator (per unit) | Feeder (ohms/phase) |
|---|---|---|
| Positive sequence | $j0.2$ | $j0.4$ |
| Negative sequence | $j0.15$ | $j0.4$ |
| Zero sequence | $j0.08$ | $j0.8$ |




Base 25 MVA, 11 kV

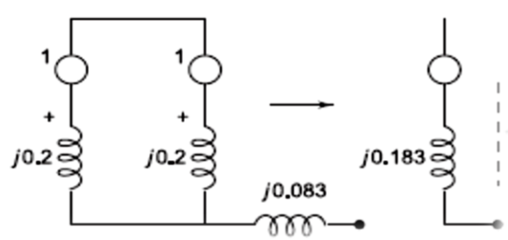
Feeder reactances: Pos. sequence $\frac{j\,0.4 \times 25}{121} = j\ 0.083$ pu

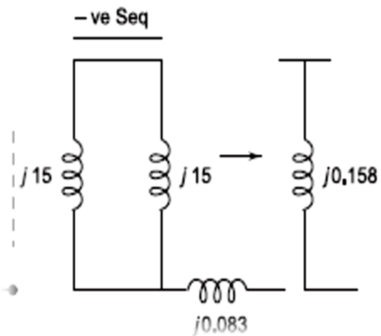
Neg. sequence $= j\ 0.083$ pu

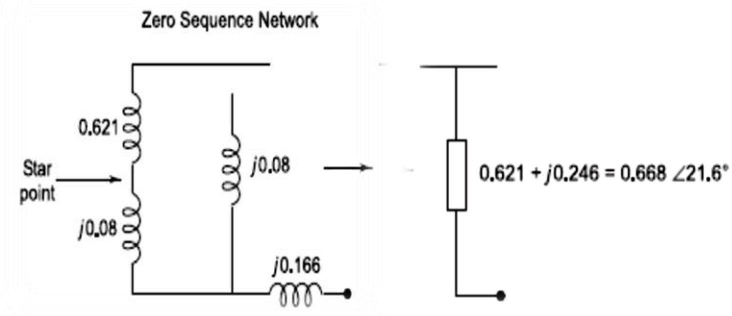
Zero sequence $= j\ 0.166$ pu

Grounding resistance $= \frac{1 \times 25}{121} = 0.207$ pu, $3R_n = 0.621$

Positive sequence network






LG fault at feeder end

(a) $I^f = I_a = \dfrac{3}{0.621 + j\,0.587}$ or $|I^f|$ = **3.51 pu**

(b) $I_{a1} = I_{a2} = I_{a0} = \dfrac{1}{0.621 + j\,0.587} = 1.17\ \angle -43.4^\circ$

$$V_{a1} = 1 - j\,0.183 \times 1.17\ \angle -43.4^\circ = 0.872\ \angle -10.3^\circ$$

$$V_{a2} = -\,j\,0.158 \times 1.17\ \angle -43.4^\circ = -\,0.184\ \angle 46.6^\circ$$

$$V_{a0} = -\,0.668\ \angle 21.6^\circ \times 1.17\ \angle -43.4^\circ = -\,0.782\ \angle -21.8^\circ$$

$$V_b = 0.872\ \angle -130.3^\circ - 0.184\ \angle 166.6^\circ - 0.782\ \angle -21.8^\circ$$

$$= \mathbf{1.19\ \angle -159.5^\circ}$$

$$V_c = 0.872\ \angle 109.7^\circ - 0.184\ \angle -73.4^\circ - 0.782\ \angle -21.8^\circ$$

$$= \mathbf{1.68\ \angle 129.8^\circ}$$

(e) Voltage of star point w.r.t. ground $= 3I_{a0} \times 0.207$

$$= 3 \times 1.17 \times 0.207$$

$$= \mathbf{0.726\ pu}$$

**PROBLEM:10**

Determine the fault currents in each phase following a double line-to-ground short circuit at the terminals of a star-connected synchronous generator operating initially on an open circuit voltage or 1.o pu. The positive, negative and zero sequence reactance of the generator are, respectively, j0.35, j0.25 and j0.20, and its star point is isolated from ground.

Since the star point is isolated from ground LLG fault is just like LL fault.

$$I_b = -\,I_c = \frac{-j\sqrt{3} \times 1}{j\,0.35 + j\,0.25} = -\ \mathbf{2.887\ pu}$$

**PROBLEM:11**

A three-phase synchronous generator has positive, negative and zero sequence Reactance per phase respectively, of 1.0,0.8 and0.4 ohm. The winding resistances are negligible. The phase sequence of the generator is RYB with a no load voltage of 11kV between lines. A short circuit occurs between lines I and B and earth at the generator terminals. Calculate sequence currents in phase R and current in the earth return circuit, (a) if the generator neutral is solidly earthed and (b) if the generator neutral is isolated.
Use R phase voltage as reference.

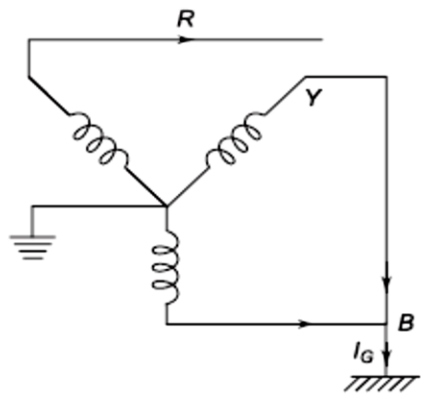



**Fig. S-11. 7**

$$V_{R1} = \frac{11}{\sqrt{3}} \angle 0° \text{ kV} = 6351 \text{ volts}$$

Neutral solidly grounded (See Fig. S-11.2 b)

$$I_{R1} = \frac{6{,}351}{j1 + (j0.8 || j0.4)} = -j\ 5{,}013 \text{ A}$$

$$I_{R2} = j\ 5{,}013 \times \frac{0.4}{1.2} = j\ 1{,}671$$

$$I_{R0} = j\,5{,}013 \times \frac{0.8}{1.2} = j\,3{,}342$$

$$I_Y = \alpha^2 I_{R1} + \alpha I_{R2} + I_{R0}$$
$$= 5013\ \angle 150° + 1671\ \angle -150° + j\,3{,}342 = -\,\mathbf{5.79} + \boldsymbol{j}\,\mathbf{5.01\ kA}$$

$$I_B = \alpha\, I_{R1} + \alpha^2 I_{R2} + I_{R0} = 5013\ \angle 30° + 1671\ \angle -30° + j\,3{,}342$$
$$= \mathbf{5.79 + j\ 5.01\ kA}$$

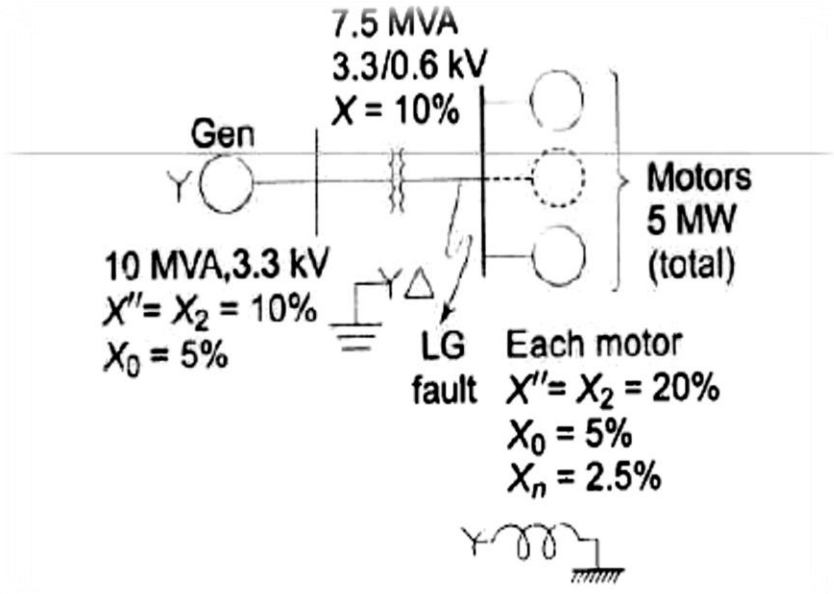
$$I_G = I_Y + I_B = j\ \mathbf{10.02\ kA};\ \boldsymbol{I_R} = \mathbf{0}$$

(b) This is equivalent to LL case

$$I_B = -\,I_Y = (-\,j\sqrt{3} \times 6{,}351)/j1.8 = -\ \mathbf{6.111\ kA}$$
$$I_G = 0\ \text{A}.$$

**PROBLEM:12**

A generator supplies a group of identical motors as shown in Fig. The motors are rated 600 V, 9O% efficiency at full load unity power factor with sum of their output ratings being 5MW. The motors are sharing equally a load of 4MW at rated voltage 0.8 power factor lagging and 90% efficiency when an LG fault occurs on the low voltage side of the transformer. Specify completely the sequence networks to simulate the fault so as to include the effect of prefault current. The group of motors can be treated as a single equivalent motor find the subtransient line currents in all parts of the system with prefault current ingnored




Base: 10 MVA, 3.3 kV (gen and line), 0.6 kV (motors)

Motor MVA = $\frac{5}{0.9}$ = 5.56 (Total). Let there be $n$ motors.

$\therefore$ Rating of each motor = $\frac{5.56}{n}$ MVA, 0.6 kV;

$$X'' = X_2 = 20\%,\ X_0 = 5\%.$$

Rating of eqv. motor = 5.56 MVA, 0.6 kV, $X'' = X_2 = \frac{20}{n} \times \frac{5.56}{\frac{5.56}{n}} = 20\%$

Motor reactance to base of 10 MVA

$$X_0 = 5\%\ X_n = 2.5\% \text{ on eqv. motor rating}$$

$$X'' = X_2 = 0.2 \times \frac{10}{5.56} = 0.36 \text{ pu;}$$

$$X_0 = 0.05 \times \frac{10}{5.56} = 0.09 \text{ pu}$$

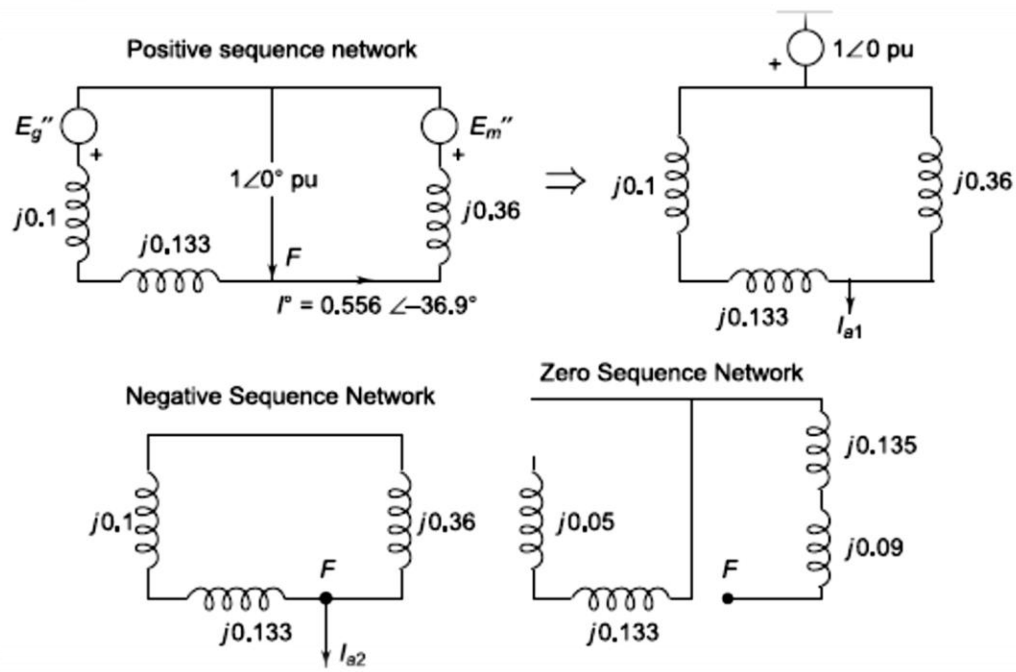
$$X_n = 0.025 \times \frac{10}{5.56} = 0.045$$

Motor load: 4/10 = 0.4 pu (MW): 1 pu voltage, 0.8 lag pf

$$\text{Prefault motor current} = \frac{0.4}{0.9 \times 0.8 \times 1} = 0.556\ \angle -36.9^\circ \text{ pu}$$

Generator reactance $X'' = X_2 = 0.1$ pu, $X_0 = 0.05$ pu

Transformer reactance $X = 0.1 \times 10/7.5 = 0.133$ pu



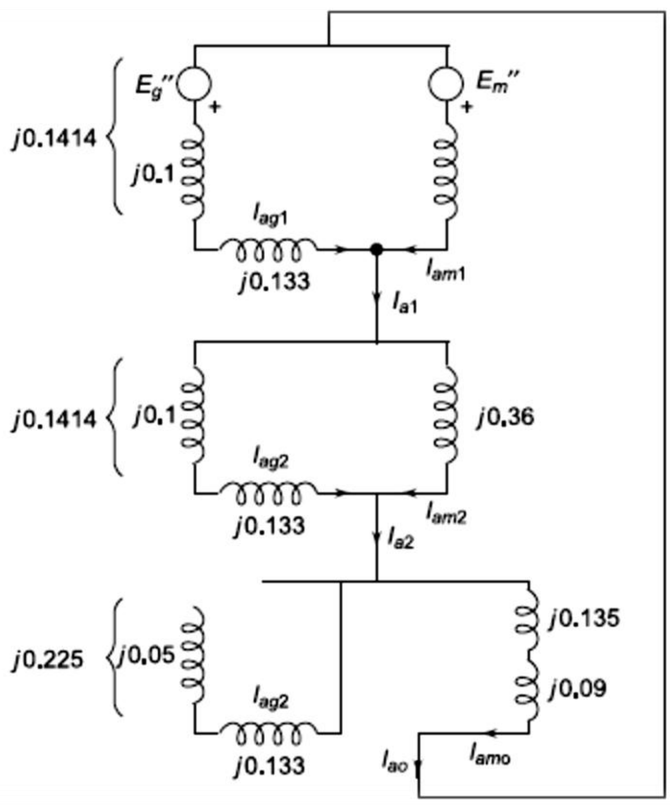

$$E_g'' = 1 + j\ 0.233 \times 0.556\ \angle -36.9^\circ$$
$$= 1 + 0.13\ \angle 53.1^\circ = 1.08\ \angle 5.5^\circ$$
$$E_m'' = 1 - j\ 0.36 \times 0.556\ \angle -36.9^\circ = 0.89\ \angle -10.3^\circ$$

Connection of sequence networks to simulate the fault (LG) It immediately follows from sequence network connection that

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{j(0.1414 + 0.1414 + 0.225)}$$
$$= -j\ 1.97$$
$$I^f = 3 \times -j\ 1.97 = -j\ 5.91 \text{ pu}$$
$$I_{ag1} = -j\ 1.97 \times \frac{0.36}{0.593} = -j\ 1.20$$
$$I_{ag2} = -j\ 1.2;\ I_{ag0} = 0$$

Positive sequence and negative sequence currents on star side are shifted by + 90° and – 90° respectively from delta side.

$$I_{ag1} = 1.20\quad I_{ag2} = -1.2,\ I_{ag0} = 0$$
$$I_{am1} = -j\ 1.97 \times \frac{0.233}{0.593} = -j\ 0.77$$



$I_{am2} = -j\ 0.77;\ I_{am0} = -j\ 1.97$

$I_{am} = -j$ **3.51 pu**

$I_{bm} = (\alpha^2 + \alpha)(-j\ 0.77) - j\ 1.97 = -j$ **1.20 pu**

$I_{cm} = (\alpha + \alpha^2)(-j\ 0.77) - j\ 1.97 = -j$ **1.20 pu**

$I_{ag} = $ **0 pu**

$I_{bg} = (\alpha^2 - \alpha) \times 1.2 = -j$ **2.08 pu**

$I_{cg} = (\alpha - \alpha^2) \times 1.2 = j$ **2.08 pu**

**PROBLEM:13**

A double line-to-ground fault occurs on lines b and c at point F in the system. Find the sub transient current in phase c of machine 1, assuming prefault currents to be zero. Both machines are rated 1200k vA,600 v with reactances of x"= x2=10% and xo= 5%. each three-phase transformer is rated 1200kVA. 600 V-Δ/3300 V-Y with leakage reactance of 5%.The reactances of the transmission line are X1=X2=20% and Xo= 40% on a base of 1200 kVA, 3300V. The reactances of the neutral grounding reactors are 5% on the KvA base of the machines.


Equivalent seq. reactances are

$$X_1 = j\ 0.105 \text{ pu}$$
$$X_2 = j\ 0.105 \text{ pu}$$
$$X_0 = j\ \frac{0.05 \times 0.45}{0.5} = j\ 0.045 \text{ pu}$$
$$I_{a1} = \frac{1}{j0.105 + (j0.105 \parallel j0.045)}$$
$$= -j\ 7.33$$

$$I_{a2} = j\,7.33 \times \frac{0.045}{0.15} = j\,2.20$$

$$I_{a0} = j\,5.13$$

$$I_{a1}^{1} = -j\,7.33 \times \frac{0.35}{0.5} = -j\,5.131$$

$$I_{a2}^{1} = j\,2.2 \times \frac{0.35}{0.5} = j\,1.54;\; I_{a0}^{1} = 0$$

In the generator

$$I_{a1}^{1} = j\,(-j\,5.131) = 5.131;\; I_{a2}^{1} = -j\,(j\,1.54) = 1.54$$

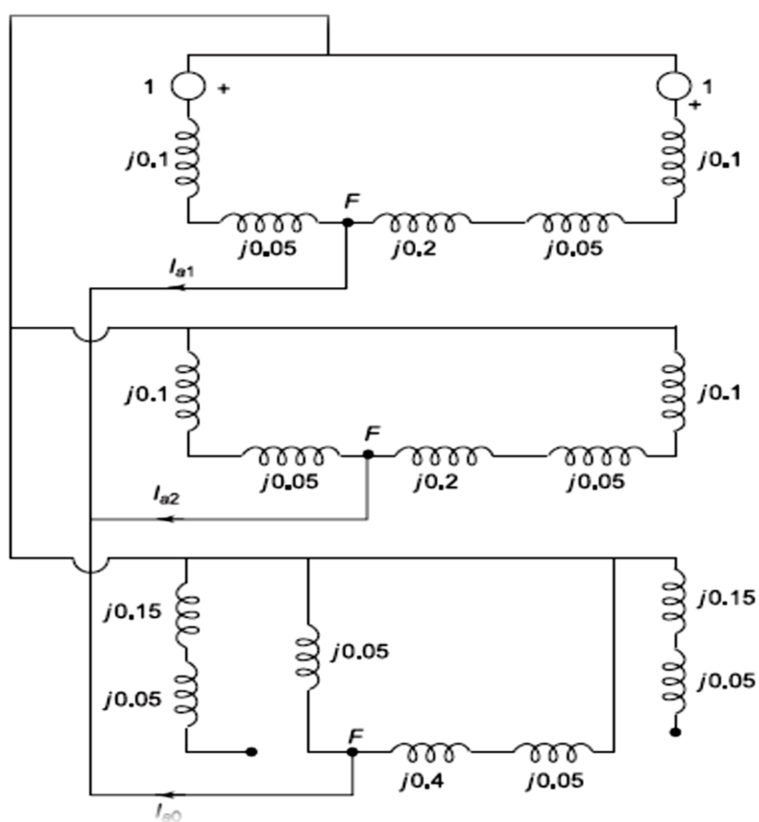
$$I_{c1}^{1} = \alpha\, I_{a1}^{1} = -2.566 + j\,4.443;$$

$$I_{c2}^{1} = \alpha^2\, I_{a2}^{1} = -0.77 - j\,1.333$$

$$\therefore \quad I_{c}^{1} = I_{c1}^{1} + I_{c2}^{1} = -3.336 + j\,3.11$$

$$\therefore \quad |I_{c}^{1}| = 4.56 \text{ pu; Base current } = \frac{1200 \times 1000}{\sqrt{3} \times 600} = 1{,}155\ A$$

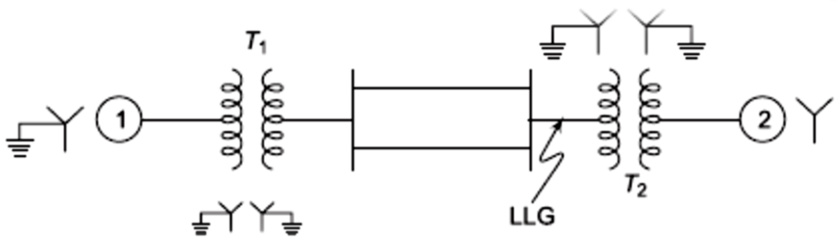
$$\therefore \quad |I_{c}^{1}| = 4.56 \times 1{,}155 = \mathbf{5{,}266}\ A$$



## PROBLEM:14

A synchronous machine1 generating 1 pu voltage is connected through a Y/Y transformer of reactance 0.1 pu to two transmission lines in parallel. The other ends of the lines are connected through a Y/Y transformer of reactance 0.1 pu to a machine 2 generating 1 pu voltage For both transformers X1 = X2 = Xo Calculate the current fed into a double line-to-ground fault on the line 'side terminals of the transformer fed from machine 2. The star point of machine 1 and of the two transformers. are solidly grounded. The reactances of the machines and lines referred to a common base are

| | $X_1$ | $X_2$ | $X_0$ |
|---|---|---|---|
| Machine 1 | 0.35 | 0.25 | 0.05 |
| Machine 2 | 0.30 | 0.20 | 0.04 |
| Line (each) | 0.40 | 0.40 | 0.80 |




Equivalent Sequence reactances are

$$X_1 = \frac{0.65 \times 0.4}{1.05} = 0.248;$$

$$X_2 = \frac{0.55 \times 0.3}{0.85} = 0.194; \; X_0 = j\ 0.55$$

$$I_{a1} = \frac{1}{j0.248 + (j0.194 \,||\, j0.55)} = -\ j\ 2.55$$

$$I_{a2} = j\ 2.55 \times \frac{0.55}{0.744}$$

$$= j\ 1.885$$

$$I_{a0} = j\ 0.665$$

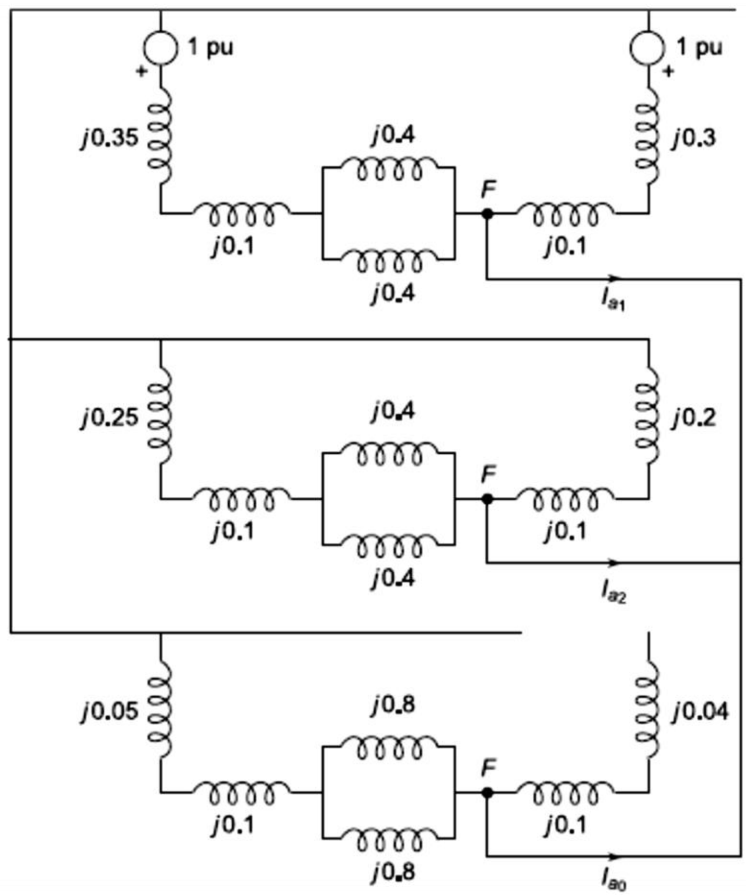
$$I_b = 2.55\ \angle 150° + 1.885\ \angle -150° + j\ 0.665$$

$$= -\ 3.84 + j\ 1.0$$

$$I_c = 2.55\ \angle 30° + 1.885\ \angle -30° + j\ 0.665$$

$$= 3.84 + j\ 1.0$$

$$I_f = I_b + I_c = j\ 2.0 \text{ pu}$$



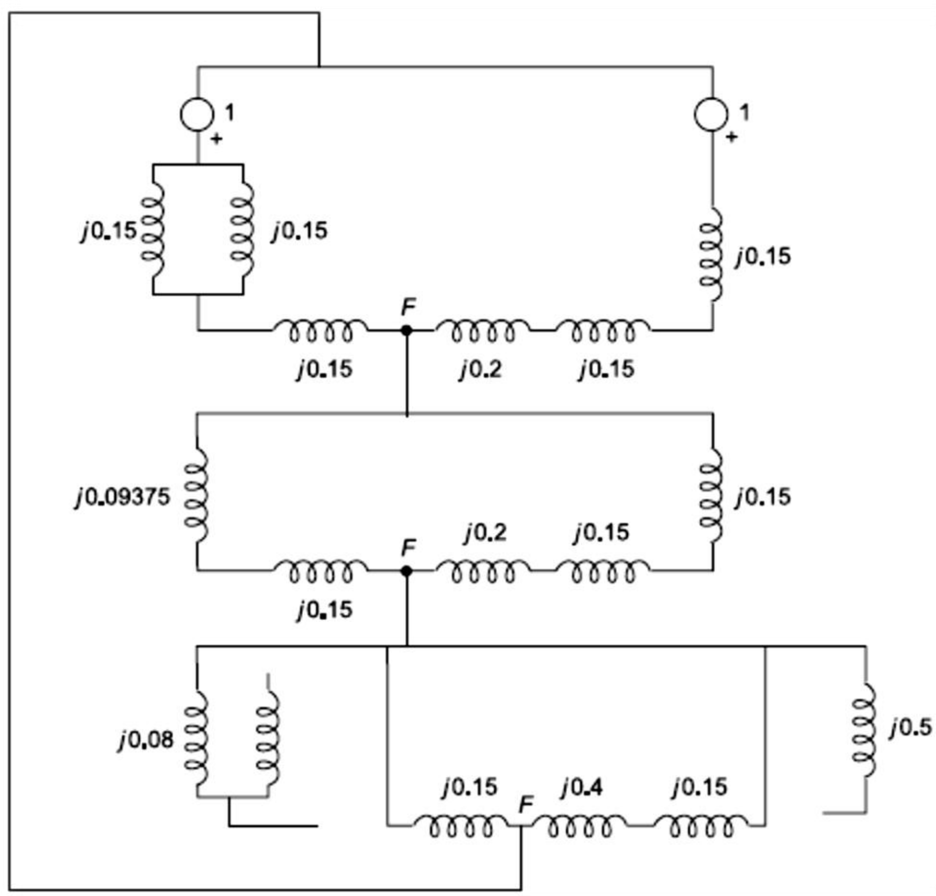

**PROBLEM:15**

Figure shows a power network with two generators connected in parallel to a transformer feeding a transmission line. The far end of the transmission line connected to an infinite bus through another transformer. Star point of each transformer, generator 1 and infinite bus are solidly grounded. The positive, negative and zero sequence reactances of various components in per unit on a common base are:

| | Positive | Negative | Zero |
|---|---|---|---|
| Generator 1 | 0.15 | 0.15 | 0.08 |
| Generator 2 | 0.25 | 0.25 | ∞(i.e. neutral isolated) |
| Each transformer | 0.15 | 0.15 | 0.15 |
| Infinite bus | 0.15 | 0.15 | 0.05 |
| Line | 0.20 | 0.20 | 0.40 |

(a) Draw the sequence networks of the power system.

(b) With both generator and infinite bus operating at 1.0pu voltage on no load, a line to ground fault occurs at one of the terminals of the star connected winding of the transformer A. Calculate the currents flowing (i) in the fault and (ii) through the transformer A.


Equivalent sequence reactances are:

$$X_1 = 0.1638$$
$$X_2 = 0.1638$$
$$X_0 = 0.118$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{j0.4456}$$
$$= -j\,2.244$$
$$\therefore \qquad I^f = 3I_{a1} = -j\,\mathbf{6.732}$$

Sequence currents through transformer *A*

$$I_{a1}\,(A) = I_{a2}\,(A) = -j\,2.244 \times \frac{0.5}{0.744} = -j\,1.508$$
$$I_{a0}\,(A) = -j\,2.244 \times \frac{0.55}{0.7} = -j\,1.763$$
$$I_a\,(A) = -j\,1.508 - j\,1.508 - j\,1.763 = -j\,\mathbf{4.779}\text{ pu}$$
$$I_b\,(A) = 1.508\;\angle 150^\circ + 1.508\;\angle 30^\circ - j\,1.763 = -j\,\mathbf{0.225}\text{ pu}$$
$$I_c\,(A) = 1.508\;\angle 30^\circ + 1.508\;\angle 150^\circ - j\,1.763 = -j\,\mathbf{0.255}\text{ pu}$$

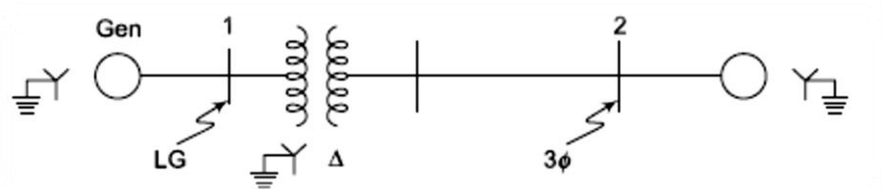
**PROBLEM:16**

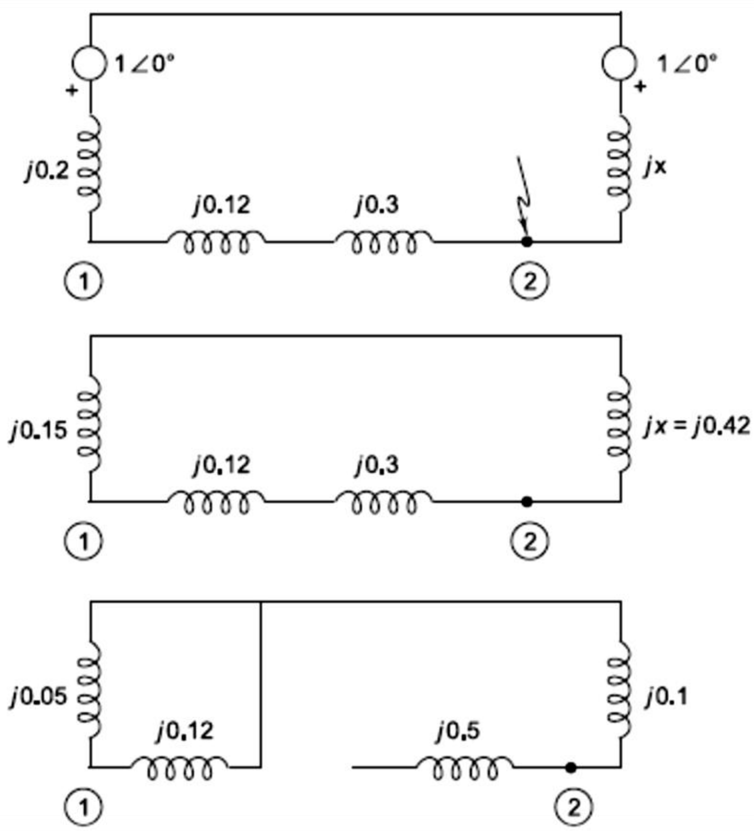
A star connected synchronous generator feeds bus bar 1 of a power system. Bus bar 1 is connected to bus bar 2 through a star/delta transformer in series with a transmission connected line. The power network connected to bus bar 2 can be equivalently represented by a star- connected generator with equal positive and negative sequences reactances. All star points are solidly connected to ground. The per unit reactances of various components are given below:

| | Positive | Negative | Zero |
|---|---|---|---|
| Generator | 0.20 | 0.15 | 0.05 |
| Transformer | 0.12 | 0.12 | 0.12 |
| Transmission line | 0.30 | 0.30 | 0.50 |
| Power network | X | X | 0.10 |

Under no load condition with 1.0 pu voltage at each bus bar, a current of 4.0 pu is fed to a three-phase short circuit on bus bar 2. Determine the positive sequence reactance X of the equivalent generator of the power network.

For the same initial conditions, find the fault current for single line- to-ground fault on bus bar l.






3 phase short at bus 2 ($F$):

$$\frac{1}{0.62} + \frac{1}{X} = 4$$

$$X = \mathbf{0.42\ pu}$$

LG fault at bus 1:

Equivalent sequence reactance are:

$$X_1 = \frac{0.2 \times 0.84}{1.04} = 0.1615$$

$$X_2 = \frac{0.15 \times 0.84}{0.99} = 0.1273$$

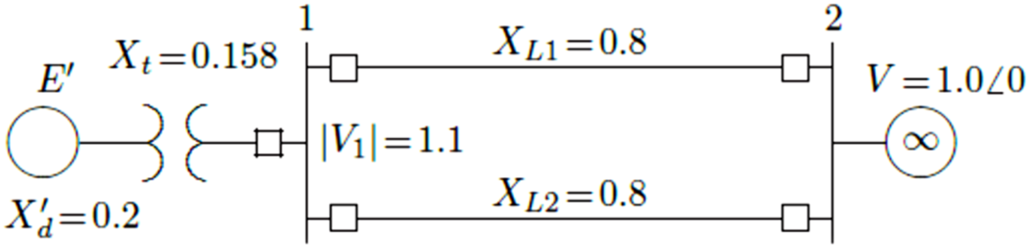
$$X_0 = \frac{0.05 \times 0.12}{0.17} = 0.0353$$

$$I_f = 3\ I_{a1} = \frac{3 \times 1}{j\,0.3241} = -\,j\ \mathbf{9.256\ pu}$$

# UNIT-V
# STABILITY ANALYSIS

## PROBLEM.1

A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage $V = 1.06\angle 0^0$ per unit. Determine the generator excitation voltage and obtain the swing equation




$$P = \frac{|V_1||V_2|}{X_L}\sin\delta_1$$

$$0.77 = \frac{(1.1)(1.0)}{0.4}\sin\delta_1$$

$$\delta_1 = 16.26^\circ$$

$$I = \frac{V_1 - V_2}{jX_L} = \frac{1.1\angle 16.26^\circ - 1.0\angle 0^\circ}{j0.4} = 0.77 - j0.14$$

$$= 0.7826\angle -10.305^\circ \text{ pu}$$

The total reactance is $X$ = 0.2+0.158+0.4 = 0.758, and the generator excitation voltage is

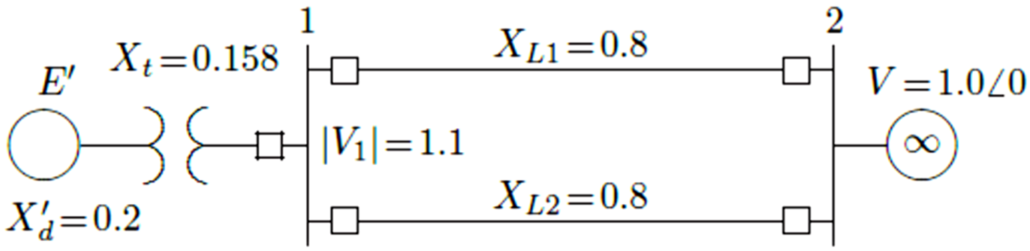
$$E' = 1.0 + j0.758(0.77 - j0.14) = 1.25\angle 27.819^\circ$$

the swing equation with $\delta$ in radians is

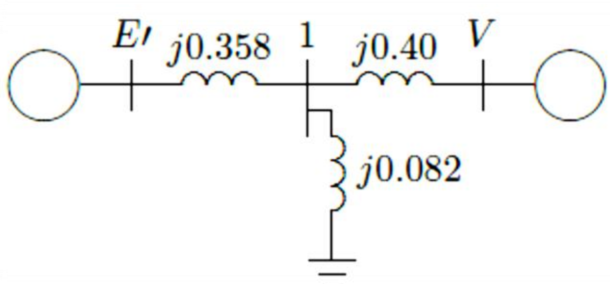
$$\frac{5.66}{60\pi}\frac{d^2\delta}{dt^2} = 0.77 - \frac{(1.25)(1)}{0.758}\sin\delta$$
$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 1.65\sin\delta$$

**PROBLEM.2**

A three-phase fault occurs on the system at the sending end of the transmission lines. The fault occurs through an impedance of 0.082 per unit. Assume the generator excitation voltage remains constant at *E'* = 1.25 per unit. Obtain the swing equation during the fault.




The impedance network with fault at bus 1, and with with $Z_f = j0.082$ is shown in Figure




Transforming the Y-connected circuit in Figure into an equivalent Δ, the transfer reactance between *E'* and *V* is

$$X = \frac{(0.358)(0.082) + (0.358)(0.4) + (0.4)(0.082)}{0.082} = 2.5 \text{ pu}$$

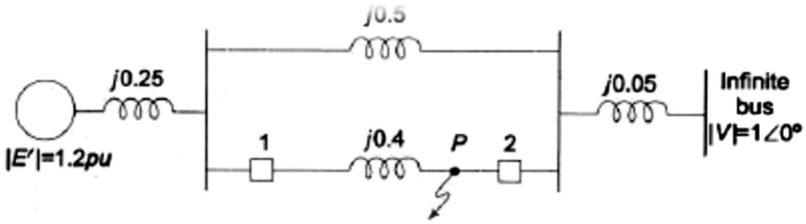
$$P_{2\,max} = \frac{(1.25)(1)}{2.5} = 0.5$$

Therefore, the swing equation during fault with ± in radians is

$$0.03\frac{d^2\delta}{dt^2} = 0.77 - 0.5\sin\delta$$

**PROBLEM:3**

For the system given in Fig a three-phase fault is applied at the point P




Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers I and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 pu power at the instant preceding the fault.

***I. Normal operation (prefault)***

$$X_I = 0.25 + \frac{0.5\times 0.4}{0.5+0.4} + 0.05$$

$$= 0.522 \text{ pu}$$

$$P_{eI} = \frac{|E'||V|}{X_I}\sin\delta = \frac{1.2\times 1}{0.522}\sin\delta$$

$$= 2.3 \sin\delta$$

Prefault operating power angle is given by

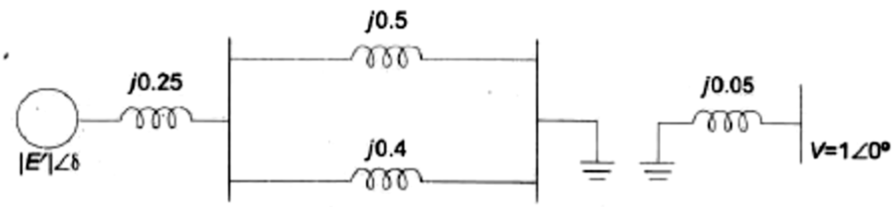
$$1.0 = 2.3 \sin \delta_0$$

or $\delta_0 = 25.8° = 0.45$ radians

***II. During fault***

It is clear from Fig. 12.31 that no power is transferred during fault, i.e.,

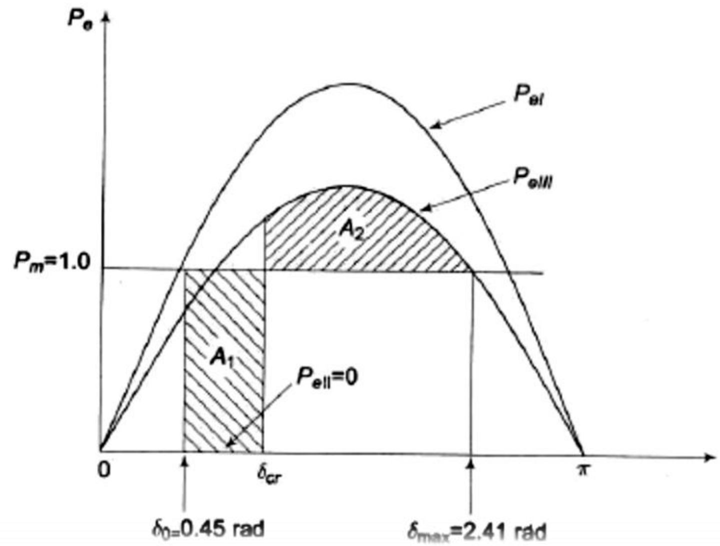
$$P_{eII} = 0$$



***III. Post fault operation (fault cleared by opening the faulted line)***

$$X_{III} = 0.25 + 0.5 + 0.05 = 0.8$$

$$P_{eIII} = \frac{1.2 \times 1.0}{0.8} \sin \delta = 1.5 \sin \delta \qquad \text{(iii)}$$




The maximum permissible angle $\delta_{max}$ for area $A_1 = A_2$ (see Fig. 12.35) is given by

$$\delta_{max} = \pi - \sin^{-1} \frac{1}{1.5} = 2.41 \text{ radians}$$

Applying equal area criterion for critical clearing angle $\delta_c$

$$A_1 = P_m (\delta_{cr} - \delta_0)$$

$$= 1.0 (\delta_{cr} - 0.45) = \delta_{cr} - 0.45$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\, d\delta$$

$$= \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1)\, d\delta$$

$$= - 1.5 \cos \delta - \delta \Big|_{\delta_{cr}}^{2.41}$$

$$= -1.5(\cos 2.41 - \cos \delta_{cr}) - (2.41 - \delta_{cr})$$

$$= 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

Setting $A_1 = A_2$ and solving

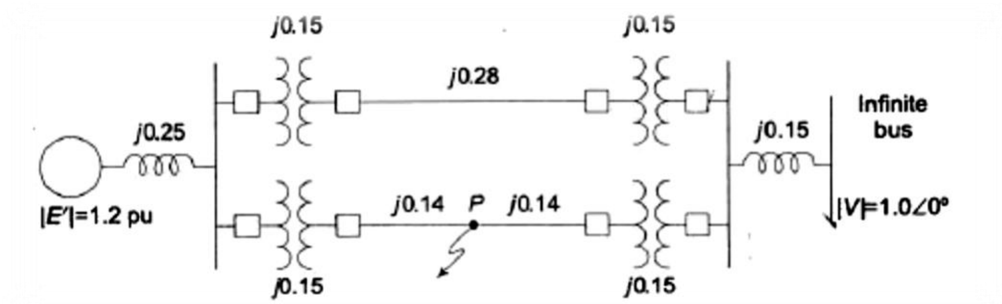
$$\delta_{cr} - 0.45 = 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

or $\qquad \cos \delta_{cr} = 0.843/1.5 = 0.562$

or $\qquad \delta_{cr} = 55.8°$

**PROBLEM:4**

Find the critical clearing angle for the system shown in Fig. for a three phase fault at the point P. The generator is delivering 1.0 pu power under prefault conditions.




***I. Prefault operation*** Transfer reactance between generator and infinite bus is

$$X_I = 0.25 + 0.17 + \frac{0.15 + 0.28 + 0.15}{2} = 0.71$$

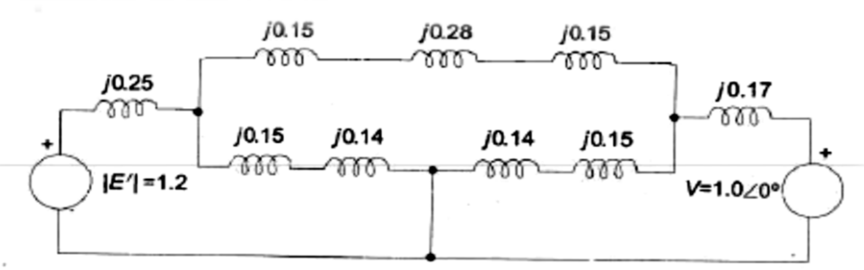
$$\therefore \qquad P_{eI} = \frac{1.2 \times 1}{0.71} \sin \delta = 1.69 \sin \delta \qquad \text{(i)}$$

The operating power angle is given by

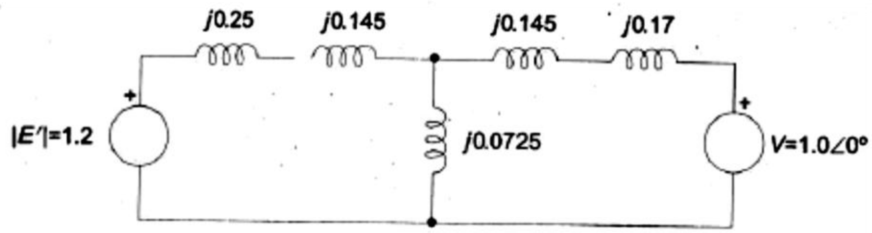
$$1.0 = 1.69 \sin \delta_0$$

or $\qquad \delta_0 = 0.633$ rad

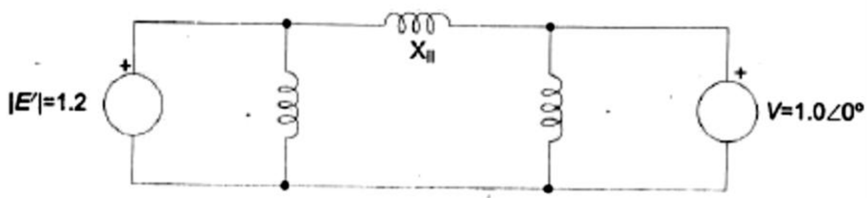
***II. During fault*** The positive sequence reactance diagram during fault is




(a) Positive sequence reactance diagram during fault




(b) Network after delta-star conversion




(c) Network after star-delta conversion

$$X_{II} = \frac{\dfrac{(0.25+0.145)\,0.0725+(0.145+0.17)\,0.0725+(0.25+0.145)}{(0.145+0.17)}}{0.075}$$

$$= 2.424$$

$$\therefore \quad P_{eII} = \frac{1.2\times 1}{2.424}\sin\delta = 0.495\sin\delta \qquad \text{(ii)}$$

***III. Postfault operation (faulty line switched off)***

$$X_{III} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

$$P_{eIII} = \frac{1.2\times 1}{1}\sin\delta = 1.2\sin\delta \qquad \text{(iii)}$$

With reference to Fig. 12.30 and Eq. (12.66), we have

$$\delta_{max} = \pi - \sin^{-1}\frac{1}{1.2} = 2.155 \text{ rad}$$

To find the critical clearing angle, areas $A_1$ and $A_2$ are to be equated.

$$A_1 = 1.0\,(\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta \max} 1.2 \sin \delta \, d\delta - 1.0\,(2.155 - \delta_c)$$

Now

$$A_1 = A_2$$

or

$$\delta_{cr} = 0.633 - \int_{0.633}^{\delta cr} 0.495 \sin \delta \, d\delta$$

$$= \int_{\delta cr}^{2.155} 1.2 \sin \delta \, d\delta - 2.155 + \delta_{cr}$$

$$\text{or } - 0.633 + 0.495 \cos \delta \Big|_{0.633}^{\delta_{cr}} = - 1.2 \cos \delta \Big|_{\delta_{cr}}^{2.155} - 2.155$$

$$\text{or } - 0.633 + 0.495 \cos \delta_{cr} - 0.399 = 0.661 + 1.2 \cos \delta_{cr} - 2.155$$

$$\text{or } \cos \delta_{cr} = 0.655$$

$$\text{or } \delta_{cr} = 49.1°$$

**PROBLEM:5**

A generator operating at 50 Hz delivers 1 pu power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.5 pu where as before the fault, this power was 2.0 pu and after the clearance of the fault, it is 1.5 pu. By the use of equal area criterion, determine the critical clearing angle.

Here $P_{\max I} = 2.0$ pu, $P_{\max II} = 0.5$ pu and $P_{\max III} = 1.5$ pu

Initial loading $P_m = 1.0$ pu

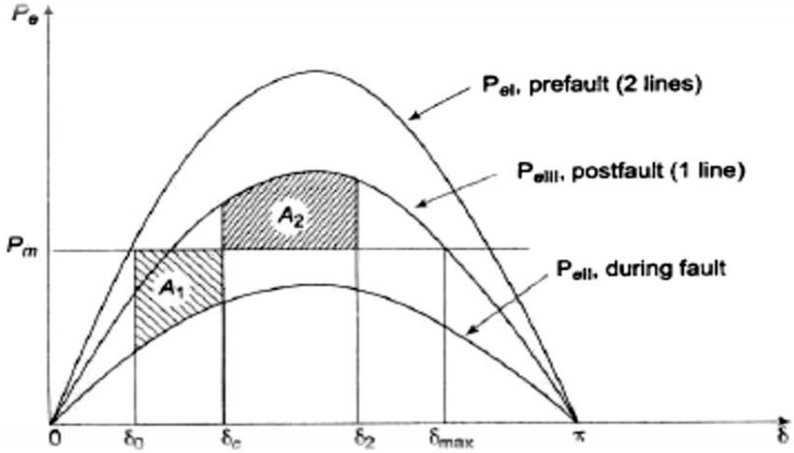
$$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{\max I}}\right) = \sin^{-1}\frac{1}{2} = 0.523 \text{ rad}$$

$$\delta_{\max} = \pi \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right)$$

$$= \pi - \sin^{-1}\frac{1}{1.5} = 2.41 \text{ rad}$$

$$\cos \delta_{cr} = \frac{1.0(2.41 - 0.523) - 0.5 \cos 0.523 + 1.5 \cos 2.41}{1.5 - 0.5} = 0.337$$

$$\text{or } \delta_{cr} = 70.3°$$




**PROBLEM:6**

A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator transient reactance is $X''_d = 0.35$ pu. Each transmission circuit has R = 0 and a reactance of 0.2 pu on a 20 MVA base. $|E'| = 1.1$ pu and infinite bus voltage $V = 1.0\angle 0^0$. A three-phase short circuit occurs at the mid point of one of the transmission lines. Plot swing curves with fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles and 6.25 cycles after the occurrence of fault. Also plot the swing curve over the period of 0.5 s if the fault is sustained.

Base MVA = 20

Inertia constant, $M(\text{pu}) = \dfrac{H}{180 f} = \dfrac{1.0 \times 2.52}{180 \times 50}$

$$= 2.8 \times 10^{-4}\ s^2/\text{elect degree}$$

I *Prefault*

$$X_I = 0.35 + \frac{0.2}{2} = 0.45$$

$$\therefore \quad P_{eI} = P_{maxI} \sin \delta$$

$$= \frac{1.1 \times 1}{0.45} \sin \delta = 2.44 \sin \delta$$

Prefault power transfer $= \dfrac{18}{20} = 0.9$ pu

Initial power angle is given by

$$2.44 \sin \delta_0 = 0.9$$

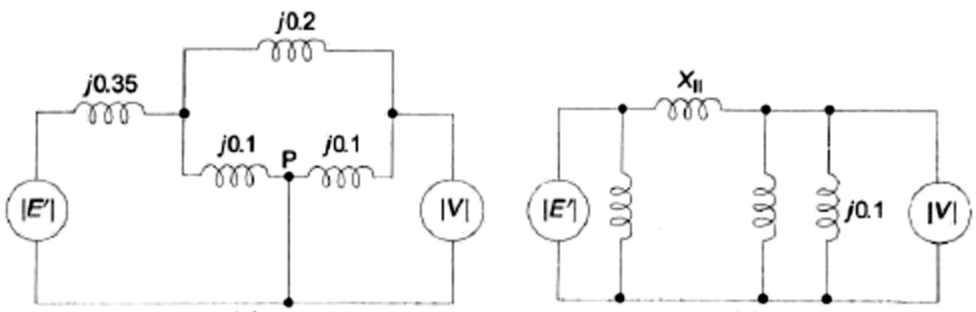
or $\delta_0 = 21.64°$

II During fault A positive sequence reactance diagram is shown in Fig.(a). Converting star to delta, we obtain the network of Fig.(b), in which

$$X_{II} = \frac{0.35\times 0.1 + 0.2\times 0.1 + 0.35\times 0.2}{0.1} = 1.25 \text{ pu}$$

$$\therefore \quad P_{eII} = P_{maxII} \sin \delta$$

$$= \frac{1.1\times 1}{1.25} \sin \delta = 0.88 \sin \delta \qquad \text{(ii)}$$



(a) (b)

III *Postfault* With the faulted line switched off,

$$X_{III} = 0.35 + 0.2 = 0.55$$

$$\therefore \quad P_{eIII} = P_{maxIII} \sin \delta$$

$$= \frac{1.1\times 1}{0.55} \sin \delta = 2.0 \sin \delta \qquad \text{(iii)}$$

Let us choose $\Delta t = 0.05$ s

The recursive relationships for step-by-step swing curve calculation are reproduced below.

$$P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1} \qquad \text{(iv)}$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \qquad \text{(v)}$$

$$\delta_n = \delta_{n-1} + \Delta\delta_n \qquad \text{(vi)}$$

Since there is a discontinuity in $P_e$ and hence in $P_a$, the average value of $P_a$ must be used for the first interval.

$$P_a(0_-) = 0 \text{ pu and } P_a(0_+) = 0.9 - 0.88 \sin 21.64° = 0.576 \text{ pu}$$

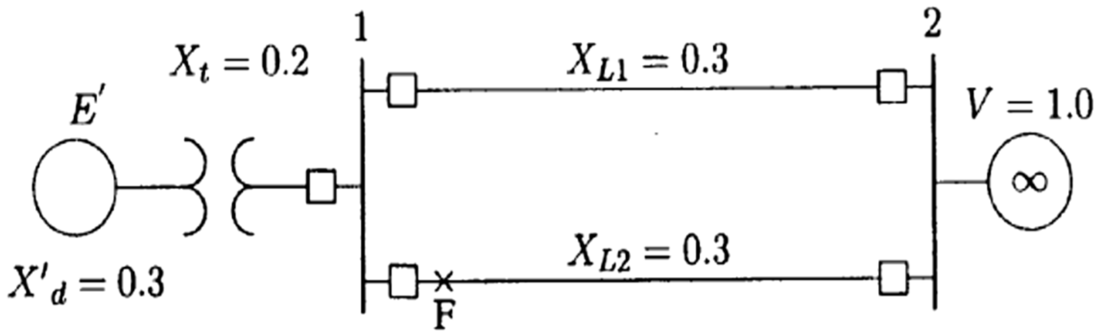
$$P_a(0_{average}) = \frac{0 + 0.576}{2} = 0.288 \text{ pu}$$

**PROBLEM:7**

A 60HZ Synchronous generator having inertia constant H=5 MJ/MVA and a direct axis transient reactance $X'_d$=0.3pu is connected to an infinite bus through a purely reactive circuit as shown. Reactances are marked on the diagram on a common system base. The generator is delivering real power $P_e$=0.8pu and Q=0.074pu to the infinite bus at a voltage of V=1pu.

(a) A temporary three-phase fault occurs at the sending end of the line at point F. When the fault is cleared both lines are intact. Determine the critical clearing angle and the critical fault clearing time.

(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle.




The current flowing into the infinite bus is

$$I = \frac{S^*}{V^*} = \frac{0.8 - j0.074}{1.0\angle 0°} = 0.8 - j0.074 \text{ pu}$$

The transfer reactance between internal voltage and the infinite bus before fault is

$$X_1 = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The transient internal voltage is

$$E' = V + jX_1 I = 1.0 + (j0.65)(0.8 - j0.074) = 1.17\angle 26.387° \text{ pu}$$

(a) Since both lines are intact when the fault is cleared, the power-angle equation before and after the fault is

$$P_{max} \sin\delta = \frac{(1.17)(1.0)}{0.65} \sin\delta = 1.8 \sin\delta$$

The initial operating angle is given by

$$1.8 \sin \delta_0 = 0.8$$

or

$$\delta_0 = 26.388^\circ = 0.46055 \text{ rad}$$

$$\delta_{max} = 180^\circ - \delta_0 = 153.612^\circ = 2.681 \text{ rad}$$

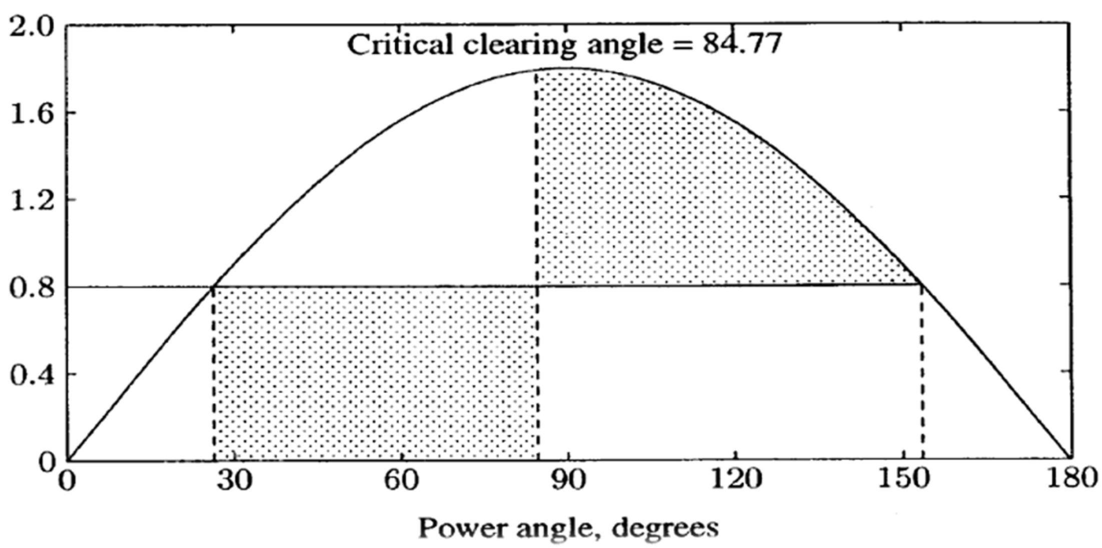
$$\cos \delta_c = \frac{0.8}{1.8}(2.681 - 0.46055) + \cos 153.61^\circ = 0.09106$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(0.09106) = 84.775^\circ = 1.48 \text{ rad}$$

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}} = \sqrt{\frac{(2)(5)(1.48 - 0.46055)}{(\pi)(60)(.8)}} = 0.26 \text{ second}$$

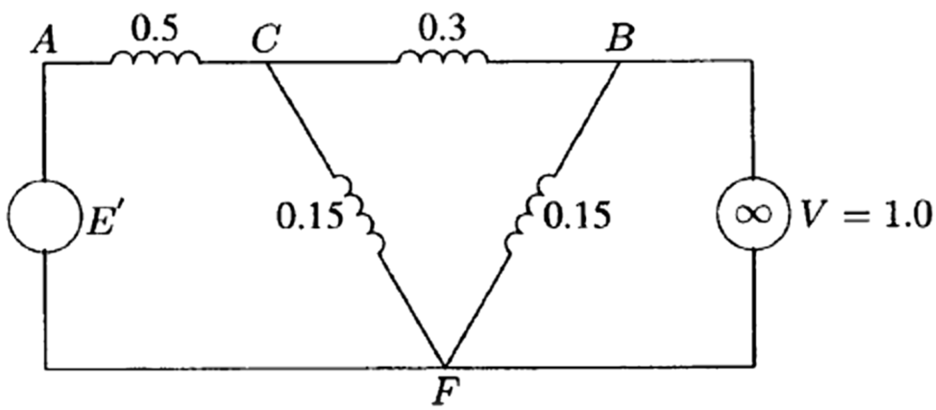
Application of equal-area criterion to a critically cleared system

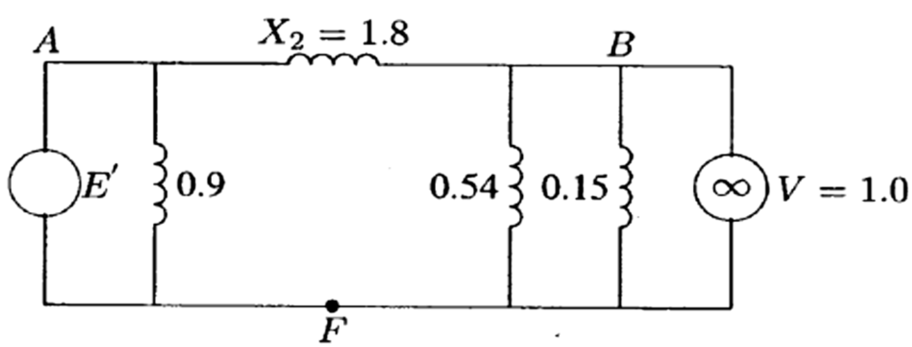


(b) The power-angle curve before the occurrence of the fault is the same as before, given by

$$P_{1\,max} = 1.8 \sin \delta$$

and the generator is operating at the initial power angle $\delta_0 = 26.4° = 0.4605$ rad. The fault occurs at point $F$ at the middle of one line, resulting in the circuit shown




The equivalent reactance between generator and the infinite bus is

$$X_2 = \frac{(0.5)(0.3) + (0.5)(0.15) + (0.3)(0.15)}{0.15} = 1.8 \text{ pu}$$

Thus, the power-angle curve during fault is

$$P_{2\,max} \sin \delta = \frac{(1.17)(1.0)}{1.8} \sin \delta = 0.65 \sin \delta$$

When fault is cleared the faulted line is isolated. Therefore, the postfault transfer reactance is

$$X_3 = 0.3 + 0.2 + 0.3 = 0.8 \text{ pu}$$

and the power-angle curve is

$$P_{3\,max} \sin\delta = \frac{(1.17)(1.0)}{0.8} \sin\delta = 1.4625 \sin\delta$$

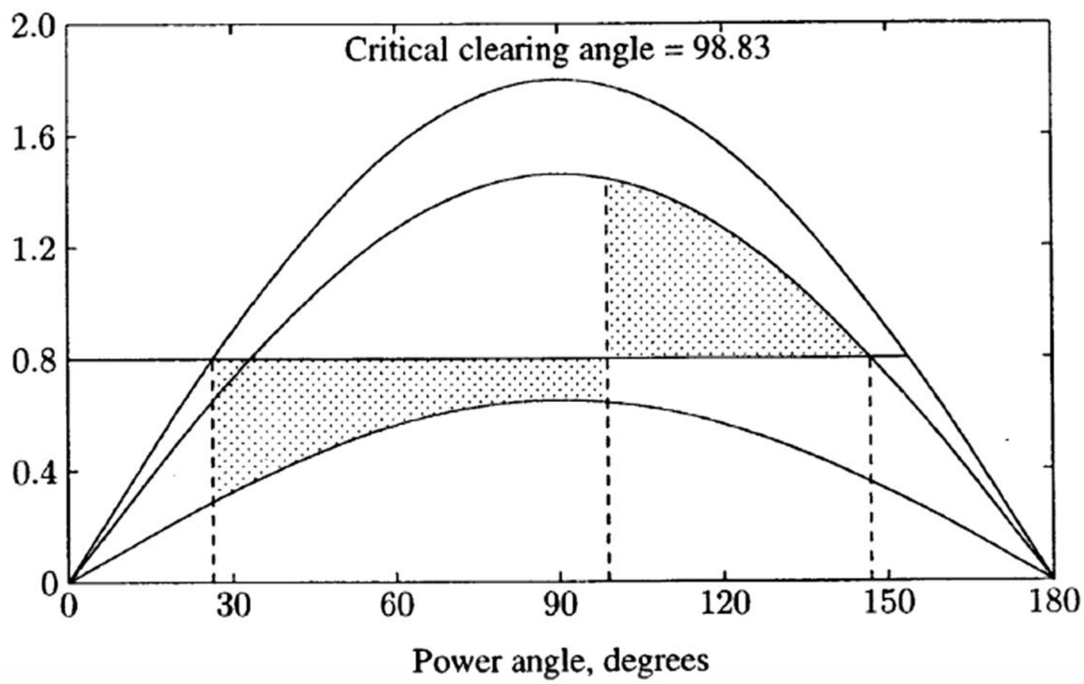
$$\delta_{max} = 180° - \sin^{-1}\left(\frac{0.8}{1.4625}\right) = 146.838° = 2.5628 \text{ rad}$$

$$\cos\delta_c = \frac{0.8(2.5628 - 0.46055) + 1.4625\cos 146.838° - 0.65\cos 26.388°}{1.4625 - 0.65} = -0.15356$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(-0.15356) = 98.834°$$

Application of equal-area criterion to a critically cleared system



**PROBLEM:8**

The single line diagram of a system is shown in figure, There are four identical generators of rating 555MVA,24KV,60HZ supplying power infinite bus bar through two transmission circuits. The reactance shown in figure are in pu on 2220MVA, 24KV base (refer to the LV side of the transformer) Resistances are assumed to be negligible. The initial operating conditions with quantities expressed in pu on 2220MVA,24KV base is as follows

P=0.9, Q=0.436(over excited) $\bar{E}_t$=1.0∟28.34 $\bar{E}_B$=0.90081∟0

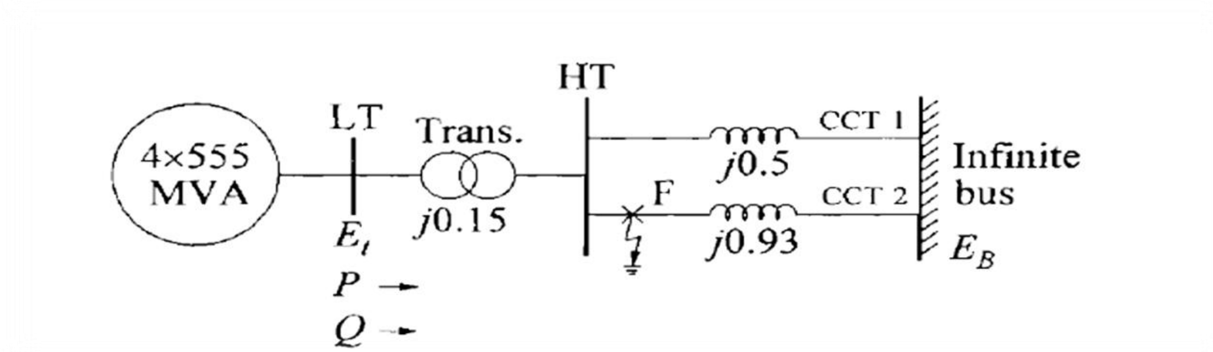
The generators are modeled as a single equivalent generator represented by the classical model with the following parameters expressed in per unit on 2220MVA, 24KV base

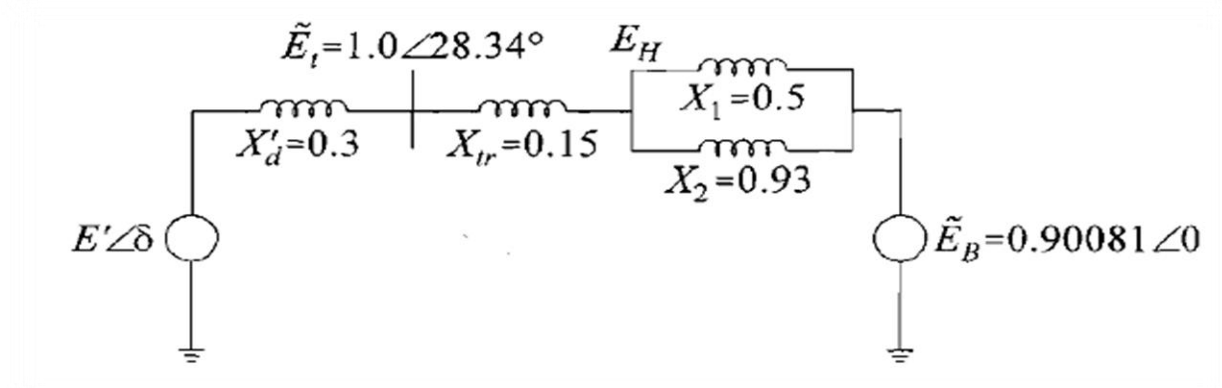
$X'_d$=0.3, H=3.5MW/MVA $K_D$=0

Circuit 2 experiences a solid 3Ø fault at point F and the fault is cleared by isolating the fault circuit.

(a)Determine the critical clearing time and critical clearing angle by computing the time response of the rotor angle using numerical integration.

(b) Check the above value of critical clearing angle using the equal area criterion



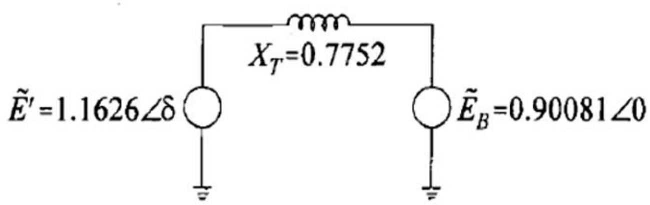



For the initial operating condition, the voltage behind $X'_d$ is

$$\tilde{E}' = \tilde{E}_t + jX'_d\tilde{I}_t$$

$$= 1.0\angle 28.34^\circ + \frac{j0.3(0.9 - j0.436)}{1.0\angle -28.34^\circ}$$

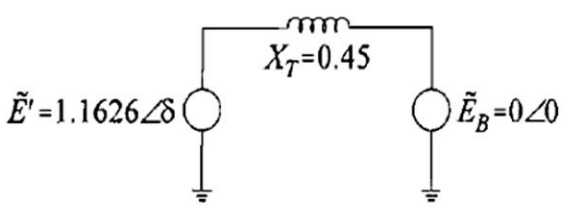
$$= 1.1626\angle 41.77^\circ$$




$$P_e = \frac{1.1626 \times 0.90081}{0.7752}\sin\delta$$

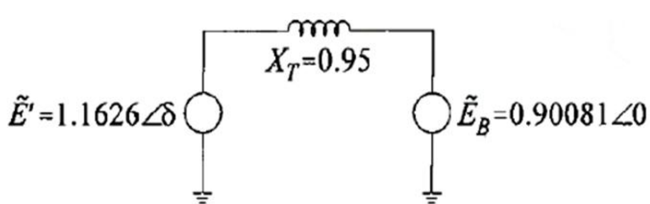
$$= 1.351\sin\delta$$

(a) Prefault




$$P_e = 0$$

(b) During fault




$$P_e = \frac{1.1626 \times 0.90081}{0.95}\sin\delta$$

$$= 1.1024\sin\delta$$

(c) Postfault

(a) Time response using numerical integration

$$p(\Delta\omega_r) = \frac{1}{2H}(P_m - P_{max}\sin\delta)$$

$$= \frac{1}{7.0}(0.9 - P_{max}\sin\delta)$$

$$p(\delta) = \omega_0 \Delta\omega_r$$

$$= 377\,\Delta\omega_r$$

where

$$P_{max} = \begin{cases} 1.351 & \text{before the fault} \\ 0 & \text{during the fault} \\ 1.1024 & \text{after the fault} \end{cases}$$

The initial values of $\delta$ and $\Delta\omega_r$ are 41.77° and 0 pu, respectively.

$$(\Delta\omega_r)_{n+1} = (\Delta\omega_r)_n + \frac{k_1' + k_2'}{2}$$

$$\delta_{n+1} = \delta_n + \frac{k_1'' + k_2''}{2}$$

$$t_{n+1} = t_n + \Delta t$$

where

$$k_1' = \left[0.1286 - \frac{P_{max}}{7.0}\sin(\delta)_n\right]\Delta t$$

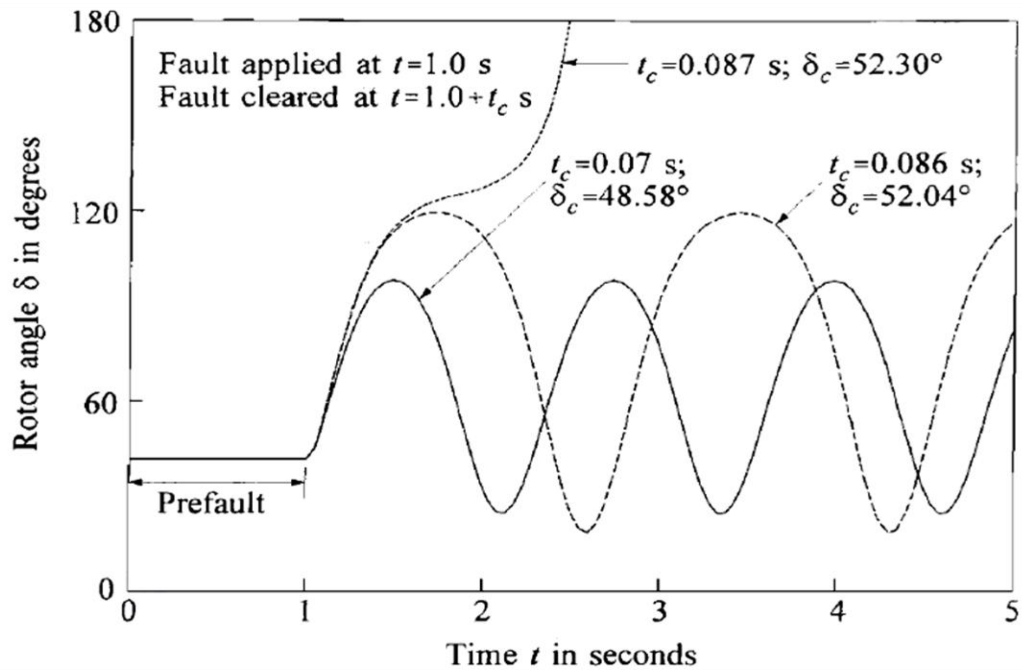
$$k_1'' = [377(\Delta\omega_r)_n]\Delta t$$

$$k_2' = \left[0.1286 - \frac{P_{max}}{7.0}\sin(\delta_n + k_1'')\right]\Delta t$$

$$k_2'' = \{377[(\Delta\omega_r)_n + k_1']\}\Delta t$$

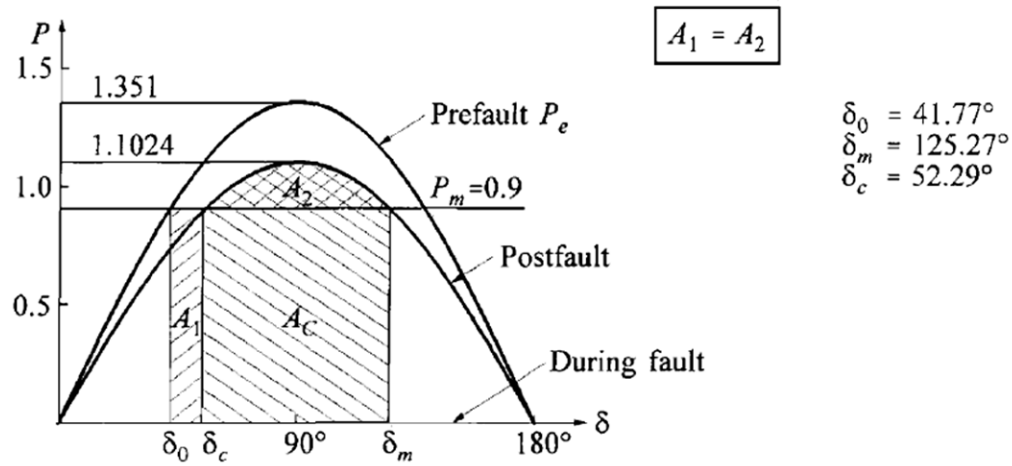
clearing time ($t_c$): 0.07 s, 0.086 s, and 0.087 s. The corresponding values of clearing angle ($\delta_c$) are 48.58°, 52.04°, and 52.30°, respectively. These results were computed using a time step ($\Delta t$) of 0.05 s throughout the solution. The time step was, however, adjusted near the fault-clearing time so as to give the exact switching instant.

From the results, we see that the system is stable with $t_c$=0.086 s ($\delta_c$=52.04°), and is unstable with $t_c$=0.087 s ($\delta_c$=52.30°); the critical clearing time is, therefore, 0.0865±0.0005 s, and the critical clearing angle is 52.17°±0.13°.



(b) Equal area criterion

$1.1024 \sin\delta_m = 0.9$ hence, $\delta_m = 125.27°$




$$A_1 = A_2$$

or

$$A_1 + A_c = A_2 + A_c$$

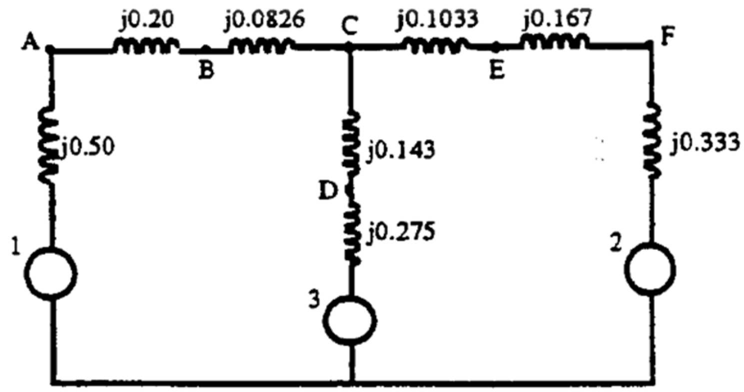
Therefore, from the plots, we have

$$0.9(125.27 - 41.77)\frac{\pi}{180} = \int_{\delta_c}^{125.27} 1.1024 \sin\delta \, d\delta$$

$$1.3116 = 1.1024(\cos\delta_c + 0.5781)$$

Thus the critical clearing angle is

$$\delta_c = 52.29°$$




$$\text{Gen 1: } X'' = 0.2 \times \frac{50}{20} = 0.50 \text{ per unit}$$

$$3\phi \text{ rating } T_2 = 220/18 \text{ kV, } 30 \text{ MVA}$$

$$\text{Base in trans. line: } 220 \text{ kV, } 50 \text{ MVA}$$

$$\text{Base for Gen 2} = 18 \text{ kV}$$

$$\text{Gen 2: } X'' = 0.2 \times \frac{50}{30} = 0.333 \text{ per unit}$$

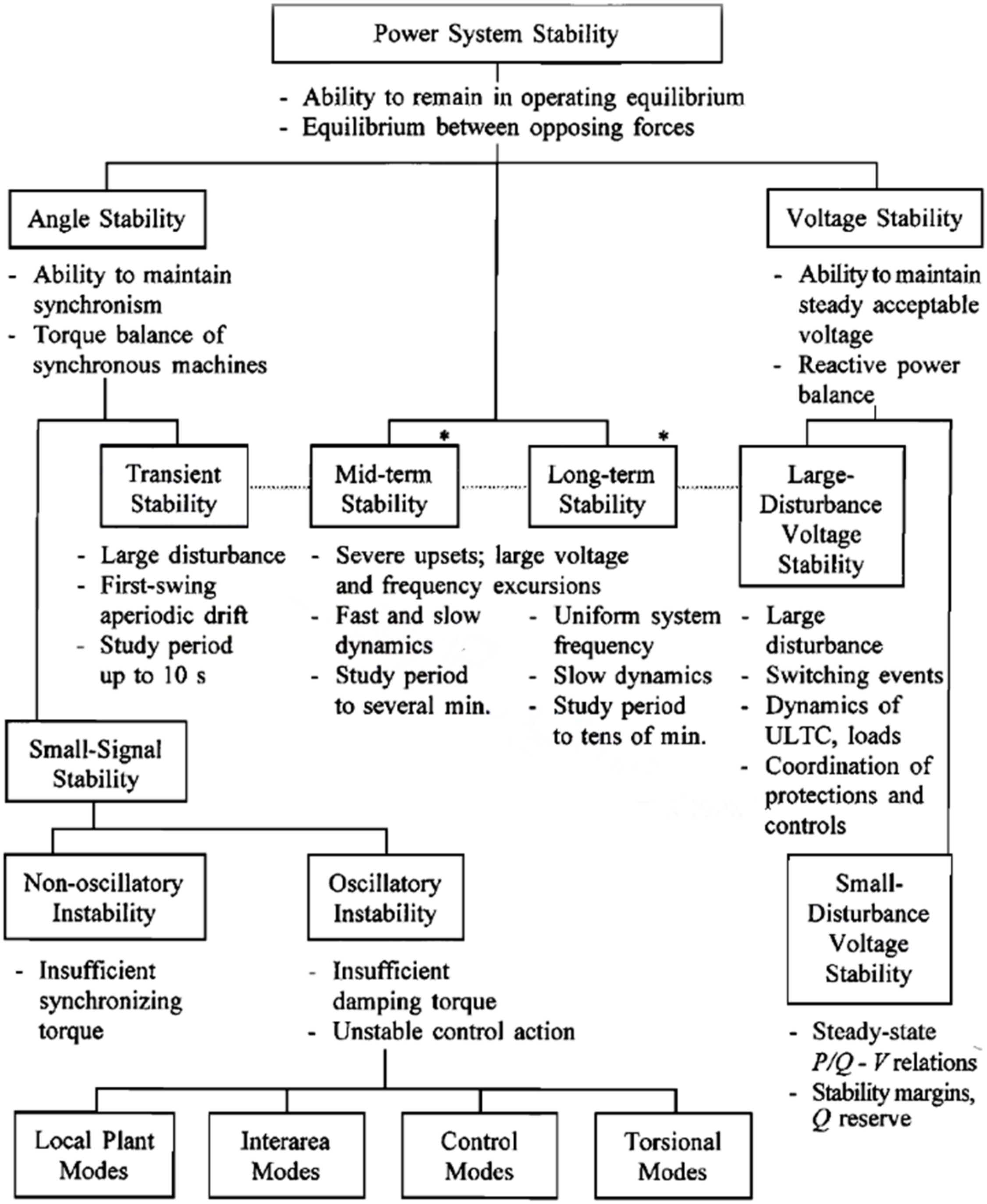
$$\text{Base for Gen 3} = 22 \text{ kV}$$

$$\text{Gen 3: } X'' = 0.2 \left(\frac{20}{22}\right)^2 \times \frac{50}{30} = 0.275 \text{ per unit}$$

$$\text{Transformer } T_1\text{: } X = .01 \times \frac{50}{25} = 0.20 \text{ per unit}$$

$$\text{Transformer } T_2\text{: } X = .01 \times \frac{50}{30} = 0.167 \text{ per unit}$$

$$\text{Transformer } T_3\text{: } X = .01 \times \frac{50}{35} = 0.143 \text{ per unit}$$




* With availability of improved analytical techniques providing unified approach for analysis of fast and slow dynamics, distinction between mid-term and long-term stability has become less significant.

## SWING EQUATION

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the *power angle* or *torque angle*. During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air gap mmf, and a relative motion begins. The equation describing this relative motion is known as the *swing equation*. If, after this oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance does not involve any net change in power, the rotor returns to its original position. If the disturbance is created by a change in generation, load, or in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field.

In order to understand the significance of the power angle we refer to the combined phasor/vector diagram of a two-pole cylindrical rotor generator illustrated in Figure 3.2. From this figure we see that the power angle $\delta_r$ is the angle between the rotor mmf $F_r$ and the resultant air gap mmf $F_{sr}$, both rotating at synchronous speed. It is also the angle between the no-load generated emf $E$ and the resultant stator voltage $E_{sr}$. If the generator armature resistance and leakage flux are neglected, the angle between $E$ and the terminal voltage $V$, denoted by $\delta$, is considered as the power angle.

Consider a synchronous generator developing an electromagnetic torque $T_e$ and running at the synchronous speed $\omega_{sm}$. If $T_m$ is the driving mechanical torque, then under steady-state operation with losses neglected we have

$$T_m = T_e$$

A departure from steady state due to a disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque $T_a$ on the rotor.

$$T_a = T_m - T_e$$

If $J$ is the combined moment of inertia of the prime mover and generator, neglecting frictional and damping torques, from law's of rotation we have

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

where $\theta_m$ is the angular displacement of the rotor with respect to the stationary reference axis on the stator. Since we are interested in the rotor speed relative to synchronous speed, the angular reference is chosen relative to a synchronously rotating reference frame moving with constant angular velocity $\omega_{sm}$, that is

$$\theta_m = \omega_{sm}t + \delta_m$$

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt}$$

and the rotor acceleration is

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

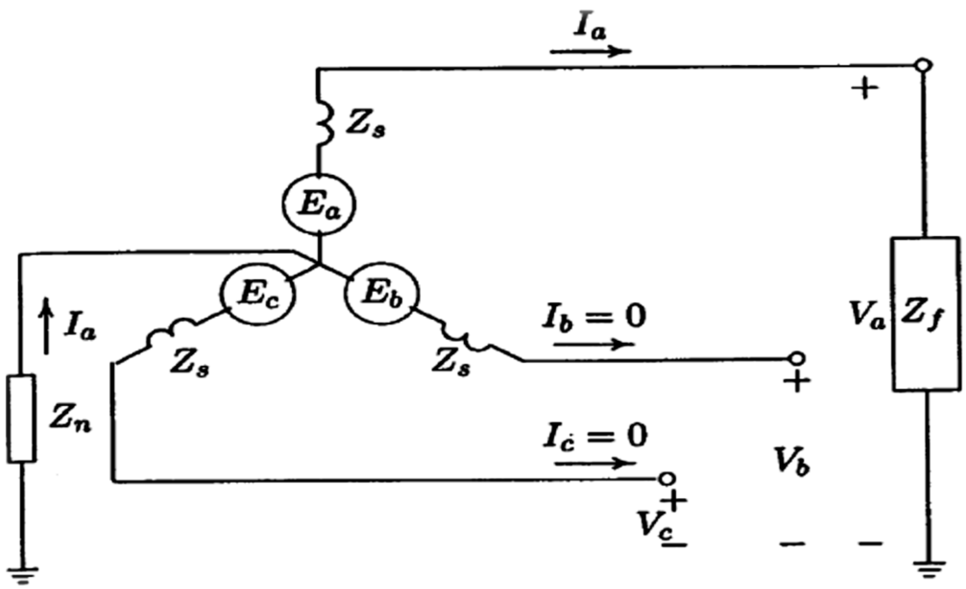
Multiply by $\omega_m$ on both sides

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e$$

Since angular velocity times torque is equal to the power, we write the above equation in terms of power

$$J\omega_m \frac{d^2\delta_m}{dt^2} = P_m - P_e$$

**Single line to ground fault**




Suppose a line-to-ground fault occurs on phase $a$ through impedance $Z_f$. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_a = Z_f I_a$$
$$I_b = I_c = 0$$

Substituting for $I_b = I_c = 0$, the symmetrical components of currents from are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From the above equation, we find that

$$I_a^0 = I_a^1 = I_a^2 = \frac{1}{3} I_a$$

Phase $a$ voltage in terms of symmetrical components is

$$V_a = V_a^0 + V_a^1 + V_a^2$$

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}$$

$$V_a^0 = 0 - Z^0 I_a^0$$
$$V_a^1 = E_a - Z^1 I_a^1$$
$$V_a^2 = 0 - Z^2 I_a^2$$

$$V_a = E_a - (Z^1 + Z^2 + Z^0) I_a^0$$

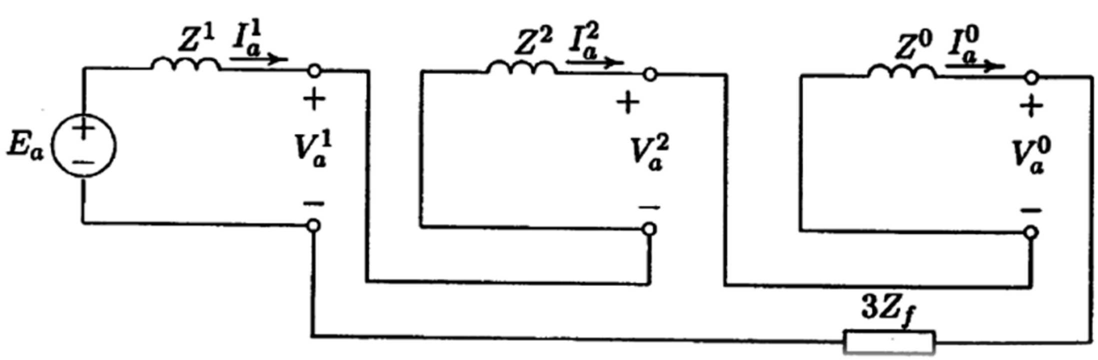
$$3Z_f I_a^0 = E_a - (Z^1 + Z^2 + Z^0) I_a^0$$

or

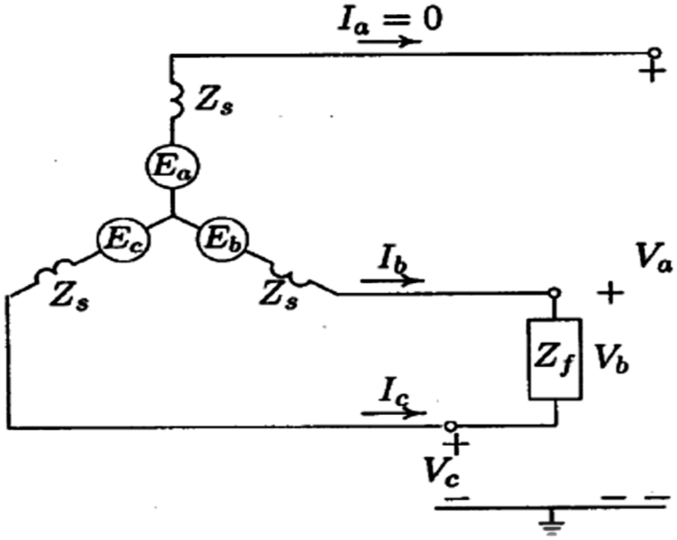
$$I_a^0 = \frac{E_a}{Z^1 + Z^2 + Z^0 + 3Z_f}$$

The fault current is

$$I_a = 3I_a^0 = \frac{3E_a}{Z^1 + Z^2 + Z^0 + 3Z_f}$$



**Line to line fault**




$$V_b - V_c = Z_f I_b$$
$$I_b + I_c = 0$$
$$I_a = 0$$

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

From the above equation, we find that

$$I_a^0 = 0$$
$$I_a^1 = \frac{1}{3}(a - a^2)I_b$$
$$I_a^2 = \frac{1}{3}(a^2 - a)I_b \qquad I_a^1 = -I_a^2$$

$$V_b - V_c = (a^2 - a)(V_a^1 - V_a^2)$$
$$= Z_f I_b$$

$$(a^2 - a)[E_a - (Z^1 + Z^2)I_a^1] = Z_f I_b$$

$$E_a - (Z^1 + Z^2)I_a^1 = Z_f \frac{3I_a^1}{(a - a^2)(a^2 - a)}$$

Since $(a - a^2)(a^2 - a) = 3$, solving for $I_a^1$ results in

$$I_a^1 = \frac{E_a}{Z^1 + Z^2 + Z_f}$$

The phase currents are

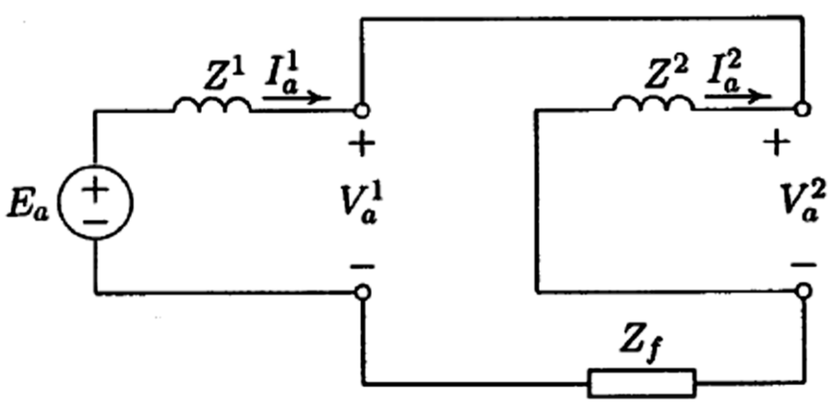
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^1 \\ -I_a^1 \end{bmatrix}$$

The fault current is

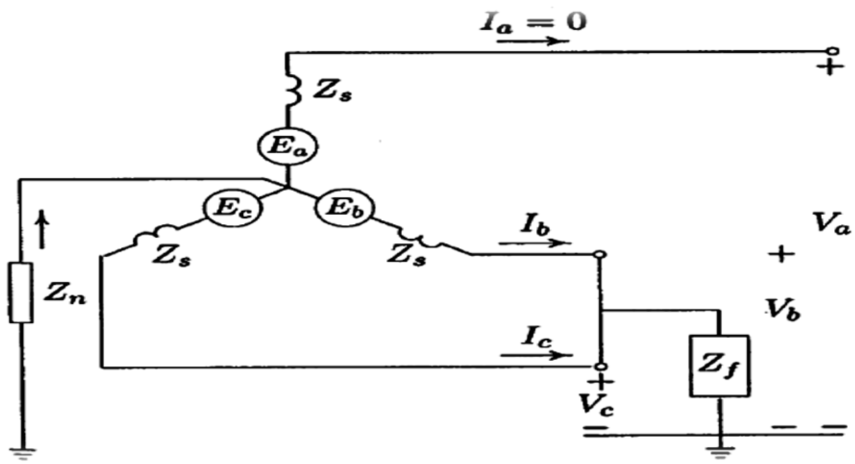
$$I_b = -I_c = (a^2 - a)I_a^1$$

or

$$I_b = -j\sqrt{3}\, I_a^1$$



**Double line to ground fault**




$$V_b = V_c = Z_f(I_b + I_c)$$
$$I_a = I_a^0 + I_a^1 + I_a^2 = 0$$

$$V_b = V_a^0 + a^2V_a^1 + aV_a^2$$
$$V_c = V_a^0 + aV_a^1 + a^2V_a^2$$

Since $V_b = V_c$, from above we note that

$$V_a^1 = V_a^2$$

$$\begin{aligned} V_b &= Z_f(I_a^0 + a^2I_a^1 + aI_a^2 + I_a^0 + aI_a^1 + a^2I_a^2) \\ &= Z_f(2I_a^0 - I_a^1 - I_a^2) \\ &= 3Z_fI_a^0 \end{aligned}$$

$$\begin{aligned} 3Z_fI_a^0 &= V_a^0 + (a^2 + a)V_a^1 \\ &= V_a^0 - V_a^1 \end{aligned}$$

Substituting for the symmetrical components of voltage from and solving for $I_a^0$, we get

$$I_a^0 = -\frac{E_a - Z^1 I_a^1}{Z^0 + 3Z_f}$$

Also, substituting for the symmetrical components of voltage

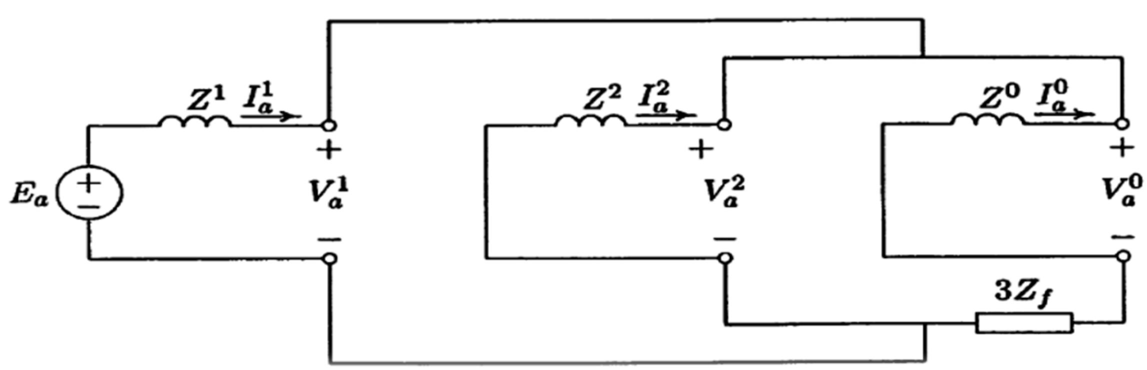
$$I_a^2 = -\frac{E_a - Z^1 I_a^1}{Z^2}$$

**Substituting the values of $Ia^0$ and $Ia^2$ in the equation**

$$I_a = I_a^0 + I_a^1 + I_a^2 = 0$$

$$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0+3Z_f)}{Z^2+Z^0+3Z_f}}$$

$$I_f = I_b + I_c = 3I_a^0$$



# NEED FOR SYSTEM PLANNING AND OPERATIONAL STUDIES

Need for power system analysis in planning and operation of power system, operational planning covers the whole period ranging from the incremental stage of system development. The system operation engineers at various points like area, space, regional and national load despatch deals with the despatch of power.

Power balance equation is

$$P_D = \sum_{i=1}^{N} P_{Gi}, \quad i = 1, 2, \ldots\ldots, N \qquad \ldots (1.1)$$

Total demand = Sum of the real power generation

*i.e.*, The generation should be such a way that to meet out the required demand.

When this relation is satisfied, it gives good economy and security.

The operation of a power system must be reliable and uninterrupted. The reliability of power supply implies more than availability of power. The loads must be fed at constant voltage and frequency.

Electrical areas are large in size. So planning for future expansion of a power system is essential. More network data must be collected for planning a power system network. For planning of power system, power engineers use computer programming.

Importance of power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities.

The need for planning and operational analysis is explained in the Fig.1.2 shown.

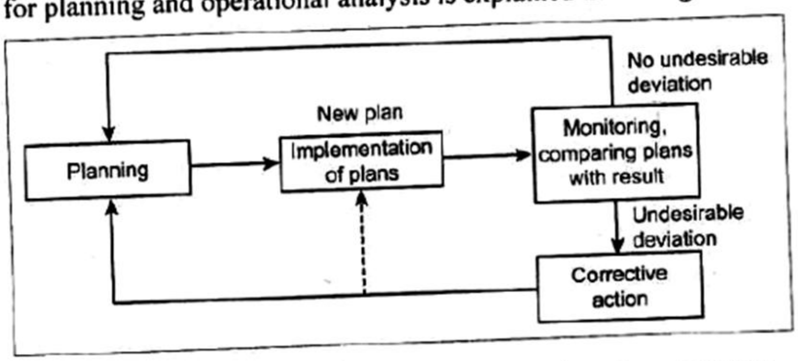



***Fig. 1.2. Block diagram for planning and operation of power system***

***Steps to be followed:***

- Planning of power system.
- Implementation of the plans.
- Monitoring the system.
- Compare with the results.

- If no undesirable deviation occurs, then directly go to planning of system.
- If undesirable deviation occurs, then take corrective action (*i.e.*, increase or decrease) and then go to planning of the system.

For planning and operation of power system, the following analysis are more important.

They are – Load flow analysis,

Short circuit analysis, and

Transient analysis.

To identify the potential deficiencies of the proposed system, the cause of the equipment failure and malfunction can be determined through a system study.

## 1. Load Flow Analysis

Normally, electrical power systems operate in their steady-state mode and the basic calculation required to determine the characteristics of this state is called as load flow.

Power flow studies are used to determine the voltage, current, active and reactive power flows in a given power system. A number of operating conditions can be analyzed including contingencies such as loss of generator, loss of a transmission line, loss of a transformer or a load. These conditions may cause equipment overloads or unacceptable voltage levels. The study results can be used to determine the optimum size and location of the capacitors for power factor improvement. The results of the power flow analysis are the starting point for the stability analysis.

Load flow study is done during the planning of a new system or the extension of an existing one. This is also needed to evaluate the effect of different loading conditions of an existing system.

## 2. Short Circuit Studies

Short circuit in any part of a power system causes as manifold increase in current and creates an abnormal or faulty condition in the system.

The short circuit studies are performed to determine the magnitude of the current flowing throughout the power system at various time intervals after a fault. The magnitude of the current flowing through the power system after a fault varies with time until it reaches a steady state conditions.

The objective of short circuit analysis is to precisely determine the currents and voltages at different locations of the system corresponding to different types of faults, such as three phase to ground fault, line to ground fault, line to line fault, double line to ground fault and open conductor fault. The data is used to select fuses, protective relays and circuit breakers to rescue the system from the abnormal condition. The symmetrical components and sequence networks are used in the analysis of unsymmetrical faults.

## 3. Transient Stability Analysis

The ability of the power system consisting of two or more generators to continue to operate after a change occurs on the system is a measure of the stability.

In power system, the stability depends on the power flow pattern, generator characteristics, system loading level and the line parameters, *etc.*

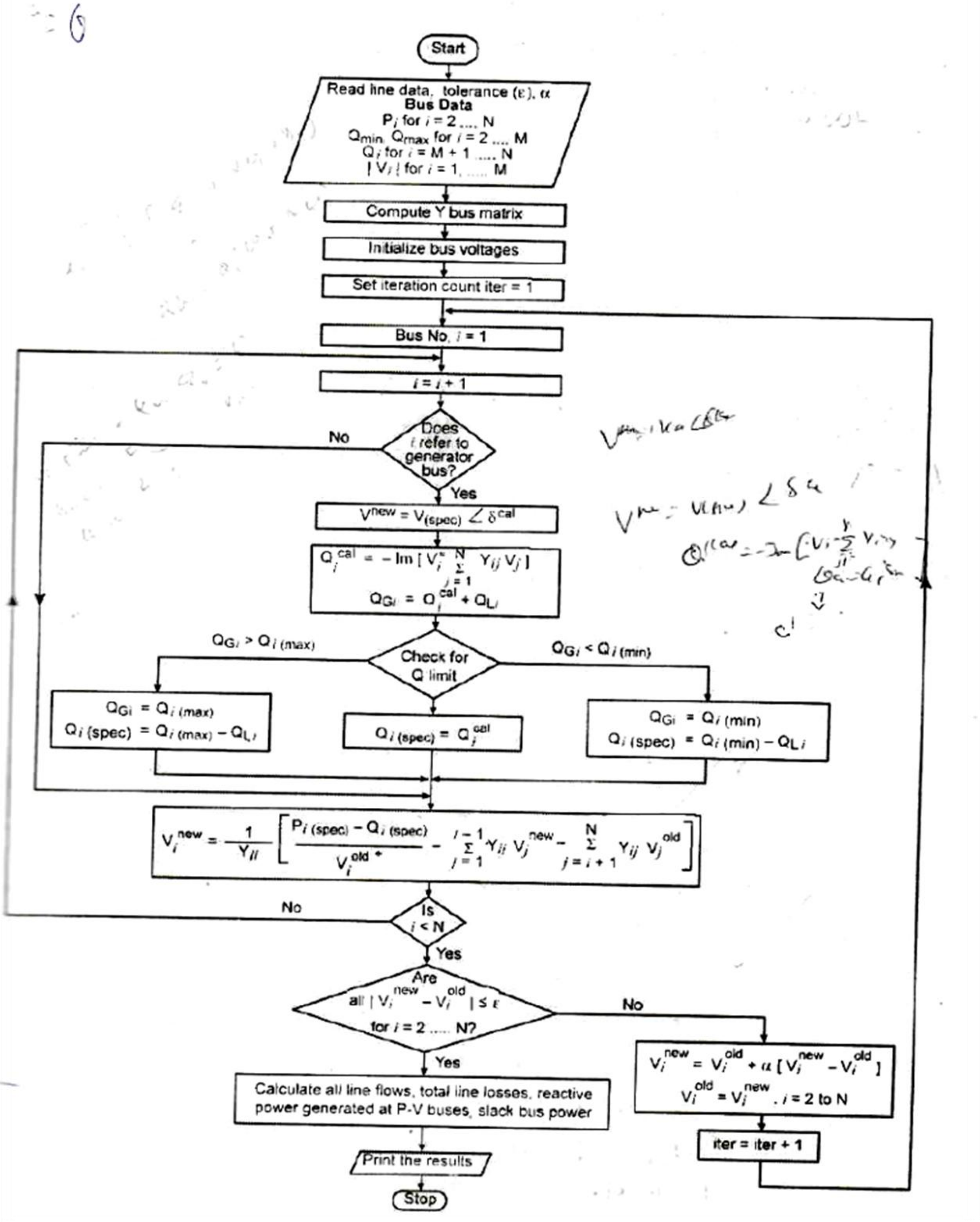
There are two forms of instability in power system such as the loss of synchronism between synchronous machines and the stalling of asynchronous loads. Stability may be divided into steady state and transient stability.

The steady state stability is defined as the ability of the power system to remain in synchronism following relatively slow load change or continual changes in generation and the switching out of lines.

Transient stability is defined as the ability of the power system to remain in synchronism under large disturbance conditions, such as fault and switching operations. The maximum power transfer limit is less than that of the steady state condition.

Transient stability studies are conducted when new generating and transmitting facilities are planned. The studies are helpful in determining the nature of the relaying system needed, critical clearing time of circuit breakers, voltage level and transfer capability between systems, *etc.*

## 5.7.6. FLOW CHART FOR GAUSS-SEIDEL METHOD INCLUDING PV BUS ADJUSTMENT



**FLOW CHART :**

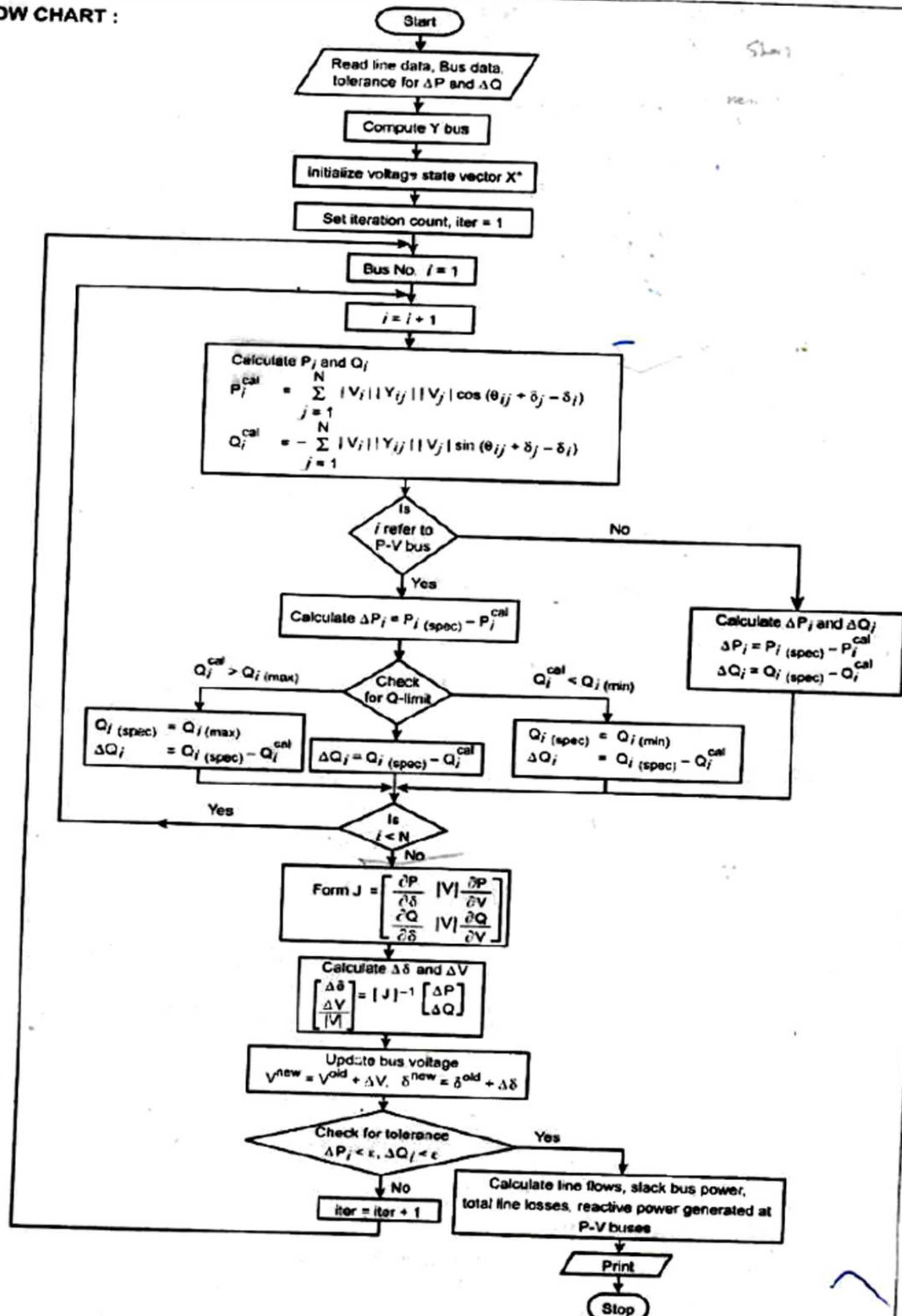
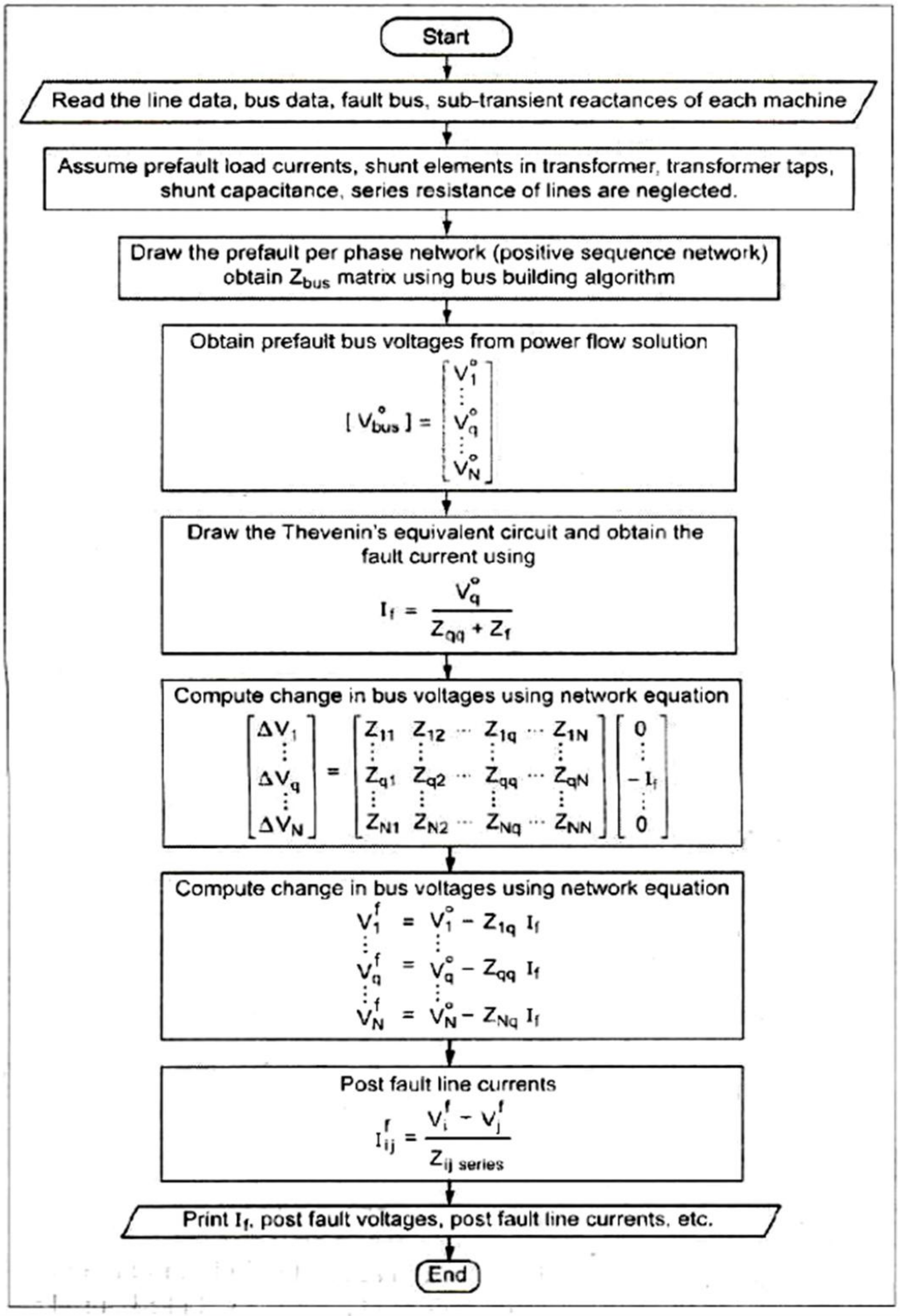



*Fig. 5.8.*

## Symmetrical Fault Analysis using $Z_{bus}$ (Flow chart)

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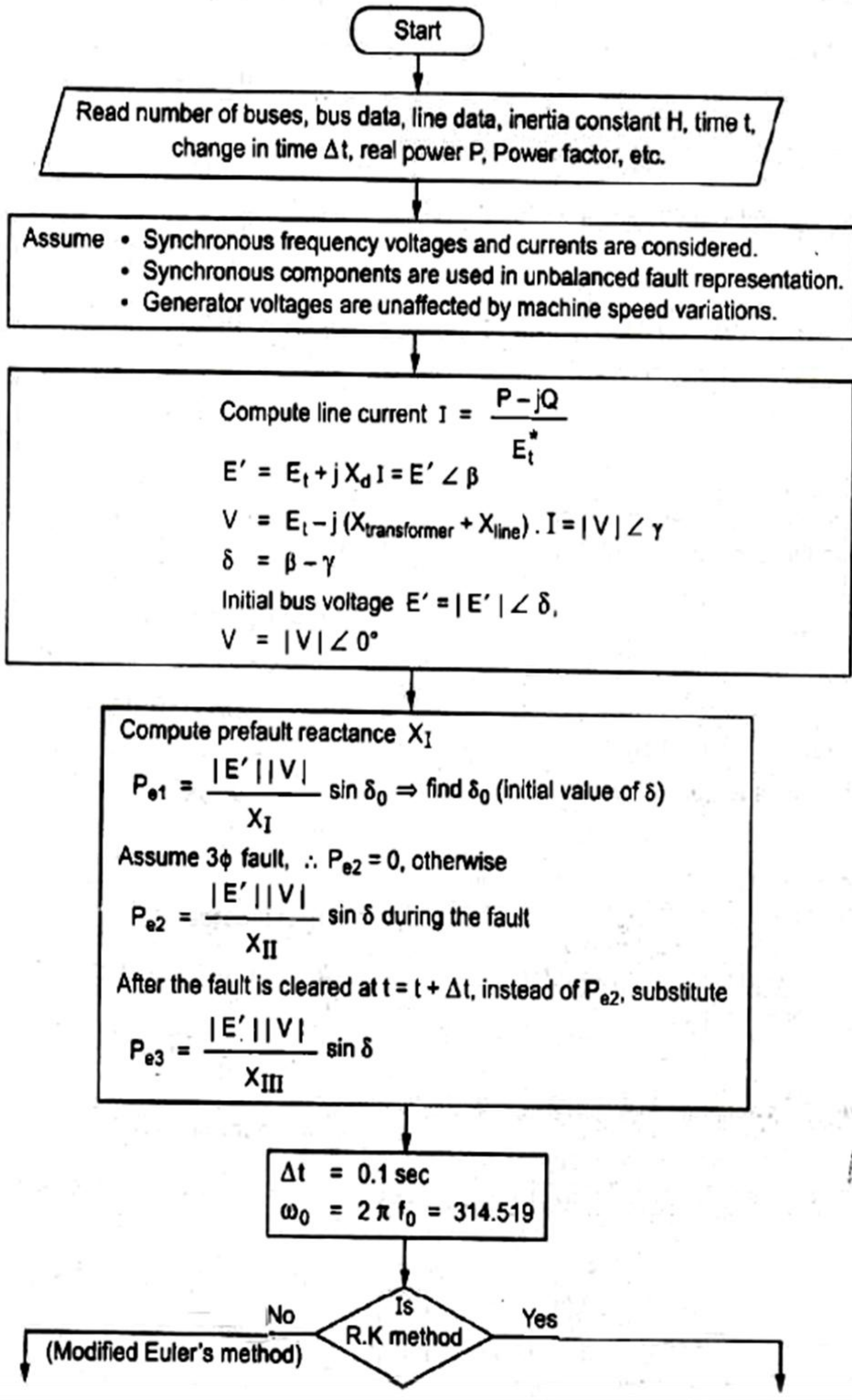
## Flow Chart for Modified Euler and Runge Kutta Method



State variable form

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate $t_1 = t_0 + \Delta t$,

$$\delta^P_{i+1} = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} . \Delta t$$

$$\Delta\omega^P_{i+1} = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} . \Delta t$$

Compute the derivatives: Using the predicted values $\delta^P_{i+1}$ and $\Delta\omega^P_{i+1}$, determine the derivatives at the end of iteration

$$\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega^P_{i+1}} = \Delta\omega^P_{i+1}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta^P_{i+1}} = \left.\frac{\pi f}{H}\right|_{\delta^P_{i+1}}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega^P_{i+1}}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta^P_{i+1}}}{2}$$

Compute the final estimate (corrected value),

$$\delta^C_{i+1} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega^P_{i+1}}}{2}\right]\Delta t$$

$$\Delta\omega^C_{i+1} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta^P_{i+1}}}{2}\right]\Delta t$$

**I Estimates:**

$$K_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$$

$$l_2 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \times \Delta t$$

$$= \frac{\pi f}{H}\,[P'_m - P_{e(\delta_i)}] \times \Delta t$$

**II Estimates:**

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$$

$$l_2 = \frac{\pi f}{H}\,[P'_m - P_{e\,(\delta_i + (K_1(2))}] \times \Delta t$$

**III Estimates:**

$$K_3 = \left[\Delta\omega_i + \frac{l_2}{2}\right] \times \Delta t$$

$$l_3 = \frac{\pi f}{H}\,[P'_m - P_{e\,(\delta_i + (K_2(2))}] \times \Delta t$$

**III Estimates:**

$$K_4 = (\Delta\omega_i \times l_3) \times \Delta t$$

$$l_4 = \frac{\pi f}{H}\,[P'_m - P_{e\,(\delta_i + (K_3)}] \times \Delta t$$

**Find estimates at $t = t_1$:**

$$\delta_{i+1} = \delta_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

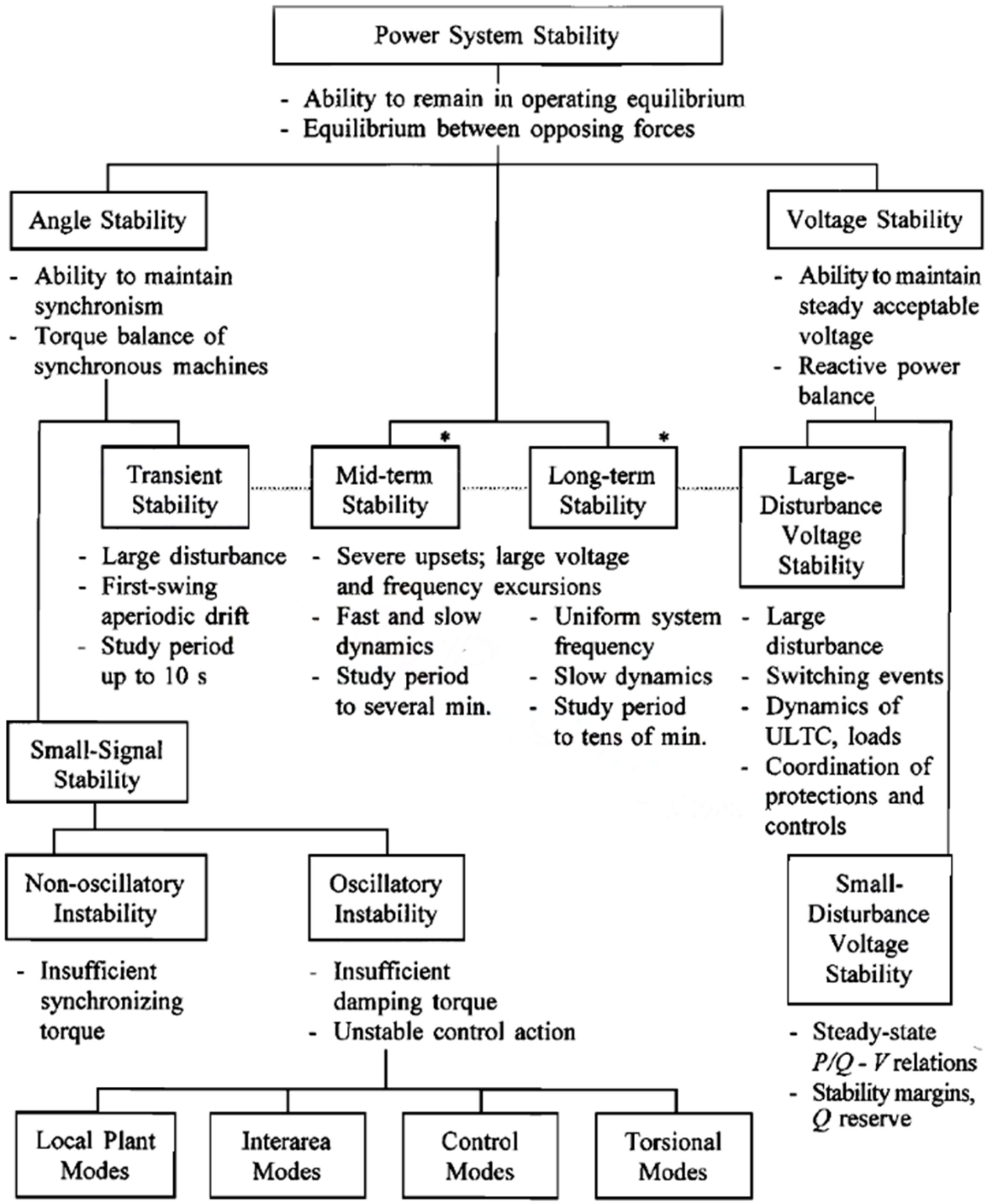
$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

Is $\delta_{i+1} - \delta_i = 0$, $\Delta\omega_{i+1} - \Delta\omega_i = 0$?

No → iter = iter + 1; iter = 0

Yes → Print critical clearing angle δ and the corresponding critical clearing time t

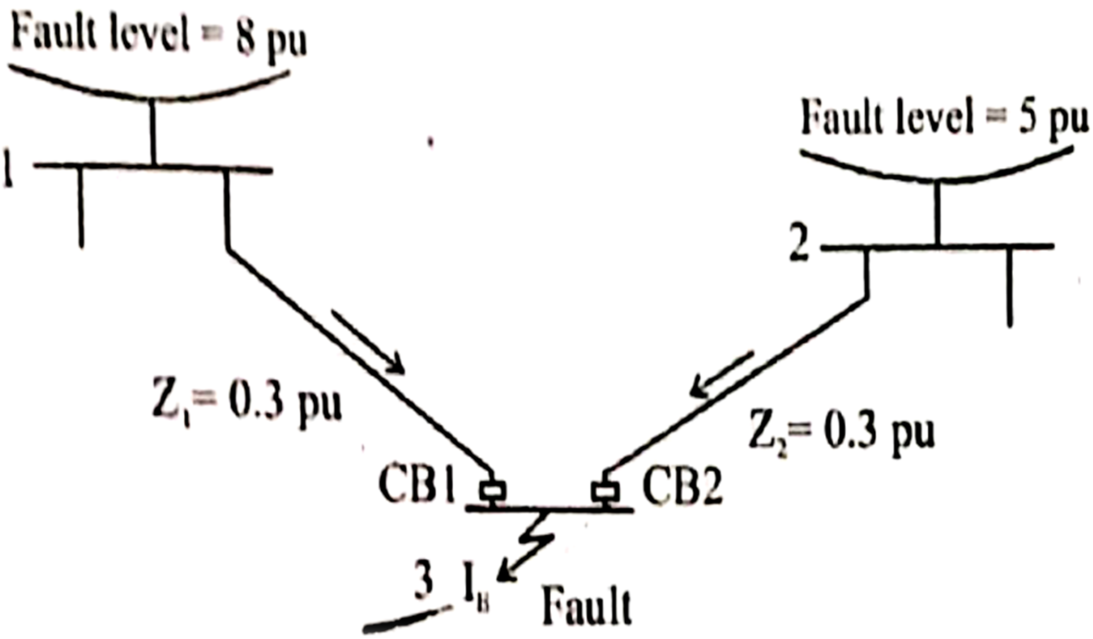
End




* With availability of improved analytical techniques providing unified approach for analysis of fast and slow dynamics, distinction between mid-term and long-term stability has become less significant.

## EXAMPLE

**Figure shows a part of power system, where the rest of the system at two points of coupling have been represented by their thevenin's equivalent circuit ( or by a voltage source of 1 pu together its fault level which corresponds to the per unit value of the effective thevenin's impedance )**




With CB1 and CB2 open, short circuit capacities are

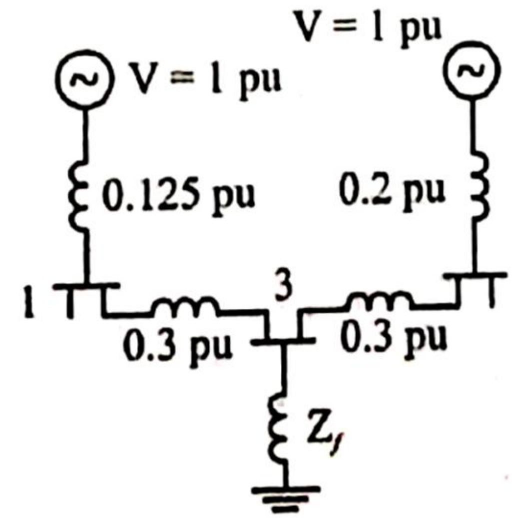
SCC at bus 1 = 8 pu. gives $Z_{g1}$ = 1/8 = 0.125 pu

SCC at bus 2 = 5 pu gives $Z_{g2}$ = 1/5 = 0.20 pu

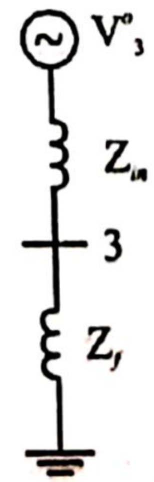
Each of the lines are given to have a per unit impedance of 0.3 pu.

$Z_1 = Z_2 = 0.3$ pu.

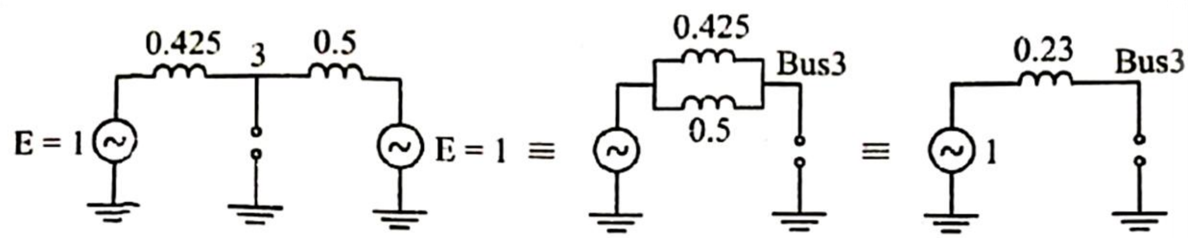
Determine the fault current at bus 3.




*System equivalent circuit*




*Thevenin's equivalent 3*

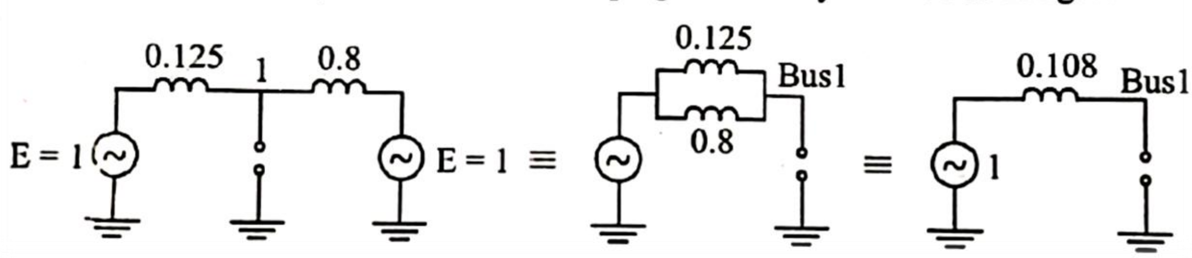



Thus, the equivalent input impedance is given by to give $Z_{in}$ as 0.23 pu at bus 3,

So that the short circuit capacity at busbar 3 is given as

|SCC3| = 1/0.23 = 4.35 p.u

The network may also be reduced keeping the identity of Bus 1 as in figure



Thus, the equivalent input impedance is given by to give Zin as 0.108 pu at bus 1, so that the short circuit capacity at busbar 1 is given as

|SCC| = 1/0.108 = 9.25 p.u

This is a 16% increase on the short circuit capacity of bus 1 with the circuit breakers open.

The network may also be reduced keeping the identity of Bus 2. This would yield a value of $Z_{in}$ as 0.157 pu, giving the short circuit capacity at busbar 2 as |SCC2| = 1/0.157 = 6.37 p.u

This is a 28% increase on the short circuit capacity of bus 2 with the circuit breakers open

**EXAMPLE**

**An alternator of negligible resistance, with solidly grounded neutral having rated voltage at no load condition is subjected to different types of fault at its terminals. The per unit values of the magnitude of the fault currents are (i) Three phase fault=4.0p.u (ii) Line to ground fault =4.2857p.u (iii) Line to line fault= 2.8868p.u. Determine the p.u values of the sequence reactance's of the machine**

(i) Three-phase fault, $I_f = 4.0$ p.u.

$$E_a = E'_g = E''_g = 1.0 \text{ p.u.}$$

$$|I_f| = \frac{|E_a|}{X'_d}$$

$$X'_d = \frac{|E_a|}{|I_f|} = \frac{1.0}{4.0} = 0.25 \text{ p.u.}$$

$$Z_1 = X'_d = j0.25 \text{ p.u.}$$

**(ii) Line to line fault, $I_f = 2.8868$ p.u.**

$$I_f = (-j\sqrt{3})\frac{E_a}{Z_1 + Z_2}$$

$$I_f = (-j\sqrt{3})\frac{1.0}{j0.25 + jZ_2}$$

$$|I_f| = \frac{\sqrt{3}}{0.25 + Z_2}$$

$$2.8868 = \frac{\sqrt{3}}{0.25 + Z_2}$$

$$0.25 + Z_2 = \frac{\sqrt{3}}{2.8868}$$

$$0.25 + Z_2 = 0.5999$$

$$Z_2 = 0.5999 - 0.25 = 0.35$$

$$Z_2 = j0.35 \text{ p.u.}$$

(iii) Single line to ground fault, $I_f = 4.2857$ p.u.

$$I_f = \frac{3E_a}{Z_0 + Z_1 + Z_2}$$

$$Z_0 + Z_1 + Z_2 = \frac{|3E_a|}{|I_f|}$$

$$Z_0 + 0.25 + 0.35 = \frac{3}{4.2857}$$

$$Z_0 = 0.7 - 0.25 - 0.35 = 0.1$$

$$Z_0 = j0.1 \text{ p.u.}$$