EE6002 POWER SYSTEM TRANSIENTS

UNIT-1

Introduction and Survey:-

Power system transient is the Outward manifestation of a sudden change in system Conditions as when a switch opens (or) Closes (or) a fault Occurs on a system

→ The transient period is very short and they Cause overvoltages on power system.

Sources and Effects of transients on power system.

ⓐ Internal Causes

Mainly due to the oscillations set-up by the sudden changes in the circuit due to breaker operation

↳ The circuit changes may be normal switching operation such as opening of a circuit breaker or it may be a fault condition.

(i) Switching surges

The making and breaking of the electric circuits with switch gear may result in abnormal transient Over voltages in power system having the Inductances and Capacitances.

Switching surges occurs in different situations

1. Interruption of low inductive currents by high speed circuit breaker.
2. Interruption of small capacitive currents
3. Ferro resonance condition
4. Energization of a loaded line

Current Chopping :-

→ Results in the production of high voltage transients across the contacts of air blast circuit breaker.

When breaking low currents (ie) unloaded transformer or reactor magnetizing current, the powerful deionizing effect of air blast causes the current abruptly to zero well before the natural current zero is reached. This phenomenon is called Current Chopping.

Prevention

Resistance Switching.

Switching Operation of unloaded line :-

→ Traveling waves are set up which produce transient overvoltage on the line.

→ On reaching the terminal point, the wave is reflected back to the supply end without change of sign. This causes voltage doubling (voltage on the line becomes twice the normal value).

It is because the line losses attenuate the wave in a very short time, the line will attain its normal supply voltage.

Ferro Resonance Condition:-

Resonance in an electrical system occurs

{Inductive reactance of the circuit} = {Capacitive Reactance}

Also Impedence of the circuit = Resistance of the circuit

Power factor is unity

⇒ Causes high transient voltage in the power system

→ In usual transmission lines, the capacitance is very small hence the resonance rarely occurs at normal freq.

→ If the generator emf wave is distorted, the trouble of resonance may occur due to 5th or higher harmonics

This phenomenon is referred as ferro Resonance.

Energization of a loaded line:-

→ Transients will be produced during the switching operations of a loaded line

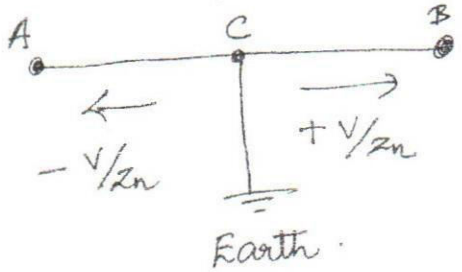
→ Loaded line is suddenly interrupted

→ Set up a voltage of $2 Z_n I$ across the break, where "I" is the instantaneous value of current and "Z_n" is the natural impedence of the line.

$$Z_n = \sqrt{L/C}$$

Insulation failure :-

Common case of transient over voltage in power system is the Insulation failure between line and earth, which cause high voltage in the system. Suppose a line at potential "V" is earthed at point 'C'



Earthing of line causes '2' equal voltages '-V' travel along CA and CB containing currents $-\frac{V}{Z_n}$ and $+\frac{V}{Z_n}$ respectively.

Both currents pass through 'C' to earth so that the current

Arcing ground :-

If the neutral of 3ϕ wires was not earthed in long high voltage transmission lines, a serious problem called arcing ground

→ Arcing ground produces severe oscillations of three to four times the normal voltage.

The phenomenon of intermittent arc taking place in the line to ground fault of a 3ϕ system with consequent production of transients is known as arcing ground.

Prevention :

Arcing ground can be prevented by earthing the neutral.

Natural Cause (Lighting).

→ Electric discharge between cloud and earth
→ Between clouds
→ Between the charge centres of the same cloud

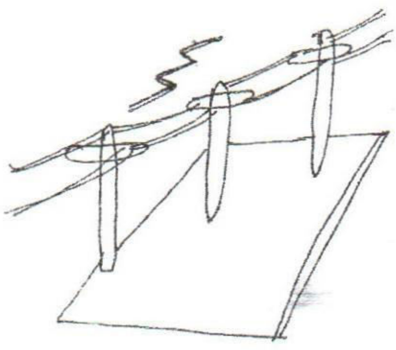
Lighting is a huge spark and takes place when clouds are charged to such a high potential with respect to earth (or) neighbouring cloud.

Lightning transients :-

→ Unavoidable event that affects power system through several mechanism.

→ Significant lightning parameters include waveforms, amplitude & frequency of occurrence.

<u>Direct Flash</u>

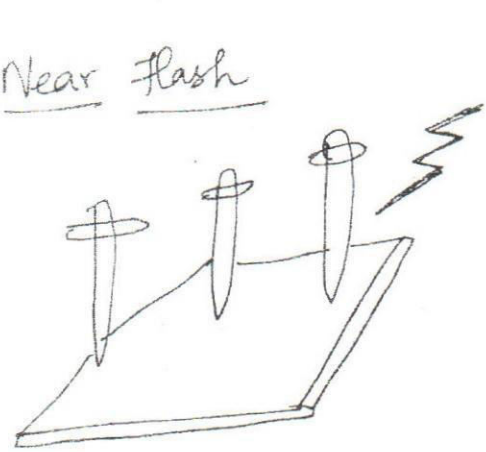


→ Flow of lightning current through the earthing Impedence resulting in Overvoltages.

→ Effective Impedence of the Lightning channel is high (Few Thousand ohms)

→ Lightning current can be practically considered as an Ideal current source.

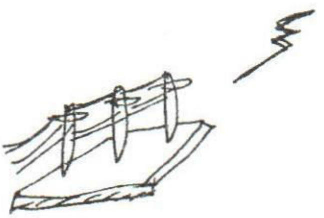
<u>Near Flash</u>



→ Immediate threat is voltage induced in the circuit loops

→ Which in turn produce surge currents.

Far Flash.



→ Threat is limited to indu voltages

→ Reflects the Characteristics of Coupling path such as Distance and nature of the system between the point of flash and the end-user facility, earthing practic and earth Connection Impedence and branching out of the distribution System.

Origin of Lightning Surges:

Current surges ⇒ Due to direct flashes to Overhead lines

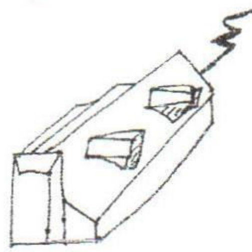
⇒ Including Flashover events

Induced transients (transmits) Overvoltages } ⇒ Due to flashes at some distance

⇒ Due to the resulting Surge Currents.

⇒ Caused by the resistive, inductive and Capacitive Coupling from the systems Carrying lightning Currents and resulting Surge Currents.

Direct flashes to Overhead lines :-



→ Effective impedence of the lightning channel is high.

→ Lightning current can practically be considered as an Ideal Current Source.

→ Resulting overvoltages are determined by the effective Impedence

→ The Impedence in the first moment is determined by the Characteristic Impedence of the line.

→ Typical values of characteristic impedences ranging from tens of ohms to 400 ohms, very high overvoltages occur that can be expected to cause flashover to earth long before the service entrance of a building becomes involved.

→ Lightning surges appearing at the service entrance, while reflecting the severity of lightning stroke and its distance bears no resemblences to the actual lightning current.

Induced transient overvoltages on Overhead lines.

→ Due to the Changes in electromagnetic field ca[used] by the lightning flash surges are induced in th[e] overhead lines of all kinds, even at the Considerab[le] distance from the flash.

→ These voltages have essentially the same value for all conductors because the phase seperation i[s] small Compared to the distance to the flash.

→ Highvoltage line with 10m Conductor height

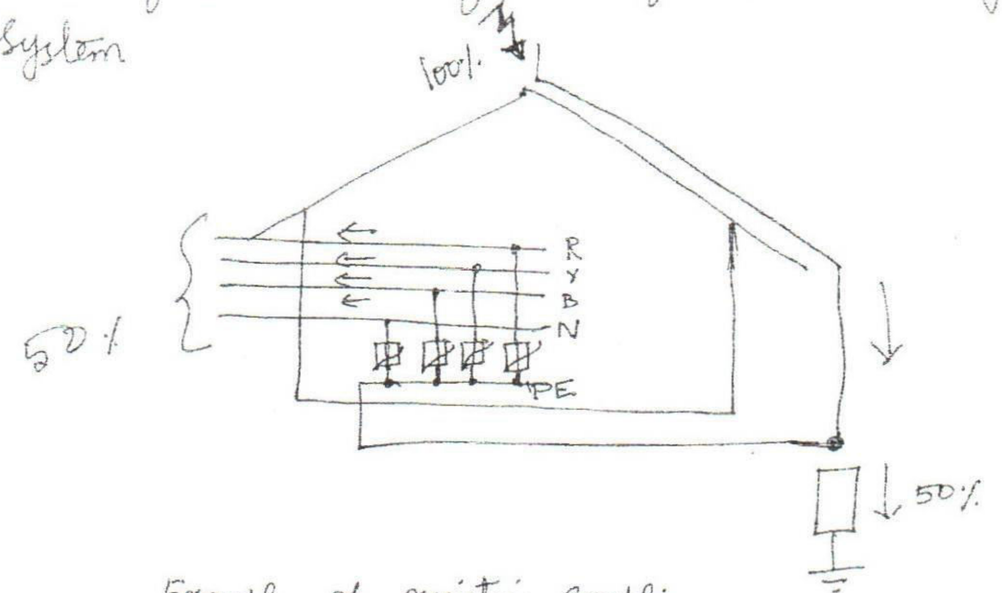
→ Lightning current of 20KA

→ Induced voltage is in the order of 100KV for a flash at 100m distance.

→ Low-voltage line with a height of 5m Current of 100KA, induce a voltage of about 2KV even at a distance of 10Km.

Transient overvoltages Caused by Coupling from other Systems:-

A lightning flash to earth or to a part of a system normally at ground potential can result in an earth potential of high value at the point of strike. This phenomenon will cause overvoltages in electrical systems using this point of earth as reference for their earthing System



Example of resistive coupling from lightning protection system.

Potential rise of the earthing system is determined by

(1) Lightning Current
(2) Effective earthing Impedence.

→ In the first moment, the potential of the earth electrode is determined by the local Impedence

→ This means that a high voltage is produced between the earthing System and electrical installations inside the building, with a high probability of Causing insulation breakdown.

Due to the high electromagnetic fields caused by the lightning current, inductive and capacitive Coupling to electrical systems that are close to lightning path can also cause over-voltages of Concern especially on electronic and data systems, causing failures and/or malfunctions.

Lightning surge transients from MV systems:-

The propagation of the surge through the MV system and the transfer rate to the LV system depends on the physical construction of the system

Switching Transients:-

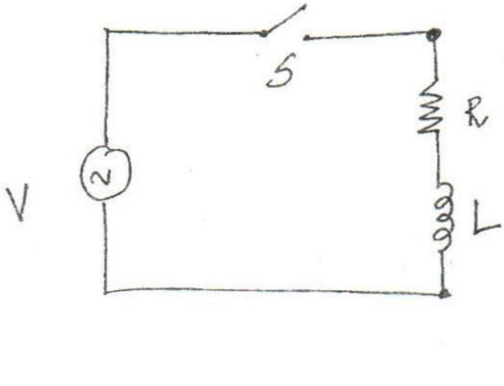
Switching transient is initiated whenever there is a sudden change of circuit conditions.

The transient is most frequently developed due to switching operations

→ Closing of a switch (or) circuit breaker to energies a load

→ The opening of a circuit breaker to clear a fault

RL Circuit transient with sine wave excitation:-



The load is represented by a series combination of resistance and Inductance which has a steady state power factor given by

$$\cos\varphi = \frac{R}{|z|} = \frac{R}{\sqrt{(R^2+\omega^2L^2)}} = \frac{R}{\left[(R^2+\omega^2L^2)\right]^{1/2}} \rightarrow (1)$$

The source is assumed to have negligible impedence compared with the load.

$$V = RI + L.\frac{dI}{dt} \rightarrow (2)$$

$$V = V_m \sin(\omega t+\theta) \rightarrow (3)$$

Equating (2) and (3) we get

$$RI + L\frac{dI}{dt} = V_m \sin(\omega t+\theta) \rightarrow (4)$$

$$= V_m\left[\sin\omega t \cos\theta + \cos\omega t \sin\theta\right]$$

Taking Laplace Transform of eq (4) on both sides

$$Ri(s) + Lsi(s) - Li(0) = V_m\left[\frac{\omega}{s^2+\omega^2}.\cos\theta + \frac{s}{s^2+\omega^2}\sin\theta\right] \rightarrow (5)$$

In this circuit $I(0)=0$

The solution for the current is

$$i(s) = \frac{V_m}{L}\cdot\frac{1}{s+(R/L)}\left[\frac{\omega.\cos\theta}{s^2+\omega^2} + \frac{s\sin\theta}{s^2+\omega^2}\right] \rightarrow (6)$$

$$i(s) = \frac{A}{(s+\alpha)(s^2+\omega^2)} + \frac{Bs}{(s+\alpha)(s^2+\omega^2)}$$

Detailed Steps

RL CIRCUIT

$$i(s) = \frac{A}{(s+\alpha)(s^2+\omega^2)} + \frac{Bs}{(s+\alpha)(s^2+\omega^2)}.$$

$$\frac{A+Bs}{(s+\alpha)(s^2+\omega^2)} = \frac{a}{(s+\alpha)} + \frac{bs+c}{(s^2+\omega^2)}$$

$$A+Bs = a(s^2+\omega^2) + bs+c(s+\alpha) \rightarrow (1)$$

Find the values of a, b, & c

Sub $s = -\alpha$ in eq (1) we get

$$A+B(-\alpha) = a((-\alpha)^2+\omega^2) + 0$$

$$A - B\alpha = a(\alpha^2+\omega^2)$$

$$\Rightarrow \boxed{a = \frac{A-B\alpha}{a(\alpha^2+\omega^2)}}$$

Equating the Co-efficient of "s^2" in eq (1) we get

$$0 = a+b \Rightarrow a = -b \Rightarrow b = -a$$

$$\boxed{b = \frac{-A+B\alpha}{a(\alpha^2+\omega^2}}$$

Equating the Constants in eq (1)

$$C = \frac{A}{\alpha} - \frac{a\omega^2}{\alpha}$$

$$C = \frac{A}{\alpha} - \left[\frac{A+B\alpha}{\alpha^2+\omega^2}\right]\left[\frac{\omega^2}{\alpha}\right]$$

Sub the values of a, b & c

$$= \frac{a}{S+\alpha} + \frac{bS+c}{(S^2+\omega^2)}$$

$$= \frac{a}{S+\alpha} + \frac{bS}{S^2+\omega^2} + \frac{c}{S^2+\omega^2}$$

$$= \frac{A-B\alpha}{(S+\alpha)\,(\alpha^2+\omega^2)} + \frac{(-A+B\alpha)\,S}{(\alpha^2+\omega^2)(S^2+\omega^2)} + \frac{A}{\alpha(S^2+\omega^2)}$$

$$- \frac{(A+B\alpha)}{\alpha(S^2+\omega^2)}\frac{\omega^2}{(\alpha^2+\omega^2)}$$

Seperating the Constants A & B.

$$= \frac{A}{(S+\alpha)\,(\alpha^2+\omega^2)} - \frac{AS}{(\alpha^2+\omega^2)(S^2+\omega^2)} + \frac{A}{\alpha(S^2+\omega^2)} - \frac{A\omega^2}{\alpha(S^2+\omega^2)(\alpha^2+\omega^2)}$$

$$- \frac{B\alpha}{(S+\alpha)(\alpha^2+\omega^2)} + \frac{B\alpha S}{(\alpha^2+\omega^2)(S^2+\omega^2)} - \frac{B\alpha\omega^2}{\alpha(S^2+\omega^2)(\alpha^2+\omega^2)}$$

$$\frac{A}{\alpha^2+\omega^2}\left[e^{-\alpha t}-\cos\omega t-\frac{\omega}{\alpha}\sin\omega t+\frac{\alpha^2+\omega^2}{\alpha\omega}\cdot\sin\omega t\right]$$

$$+\frac{B}{\alpha^2+\omega^2}\left[-\alpha e^{-\alpha t}+\alpha\cos\omega t+\sin\omega t\cdot\omega\right]$$

$$\frac{A}{\alpha^2+\omega^2}\left[e^{-\alpha t}-\cos\omega t-\frac{\omega}{\alpha}\sin\omega t+\frac{\omega}{\alpha}\sin\omega t+\frac{\alpha}{\omega}\sin\omega t\right]$$

$$+\frac{B}{\alpha^2+\omega^2}\left[-\alpha e^{-\alpha t}+\alpha\cos\omega t+\omega\cdot\sin\omega t\right]$$

$$\Rightarrow\frac{A}{\alpha^2+\omega^2}\left[e^{-\alpha t}-\cos\omega t+\frac{\alpha}{\omega}\sin\omega t\right]+\frac{B}{\alpha^2+\omega^2}\left[-\alpha e^{-\alpha t}+\alpha\cos\omega t+\omega\cdot\sin\omega t\right]$$

b A & B values in the above eqn

$$\frac{V_m}{L(\alpha^2+\omega^2)}\left[\omega\cos\theta\left(e^{-\alpha t}-\cos\omega t+\frac{\alpha}{\omega}\sin\omega t\right)+\sin\theta\left(\alpha\cos\omega t+\omega\sin\omega t-\alpha e^{-\alpha t}\right)\right]$$

where $A = \frac{V_m}{L} \omega . \cos\theta$

$B = \frac{V_m}{L} \sin\theta$

$\alpha = R/L$

The inverse Laplace transform of the equation (6)

Sub the values of A, B and α.

$$I(t) = \frac{V_m}{L[\alpha^2+\omega^2]} \Big[\omega\cos\theta\left(e^{-\alpha t} - \cos\omega t + \frac{\alpha}{\omega} \sin\omega t\right) + \sin\theta\left[\alpha . \cos\omega t + \omega \sin\omega t - \alpha e^{-at}\right]\Big] \rightarrow (7)$$

Sub

$$\tan\phi = \frac{\omega L}{R} = \frac{\omega}{\alpha}$$

$$\sin\phi = \frac{\omega}{(\alpha^2+\omega^2)^{1/2}}$$

$$\cos\phi = \frac{\alpha}{(\alpha^2+\omega^2)^{1/2}}$$

$$I(t) = \frac{V_m}{L(\alpha^2+\omega^2)^{1/2}} \left[-\sin(\theta-\phi) e^{-at} + \sin(\omega t + \theta - \phi) \right]$$

$$I(t) = \frac{V_m}{(R^2+\omega^2 L^2)^{1/2}} \left[\sin\omega t + (\theta-\phi) - \sin[\theta-\phi] e^{-at} \right]$$

In this equation, the first term is the steady state final value. Its amplitude is V_m/Z and it has a phase angle $-\phi$ w.r.t. voltage

The second term is the transient term

Case 1:-

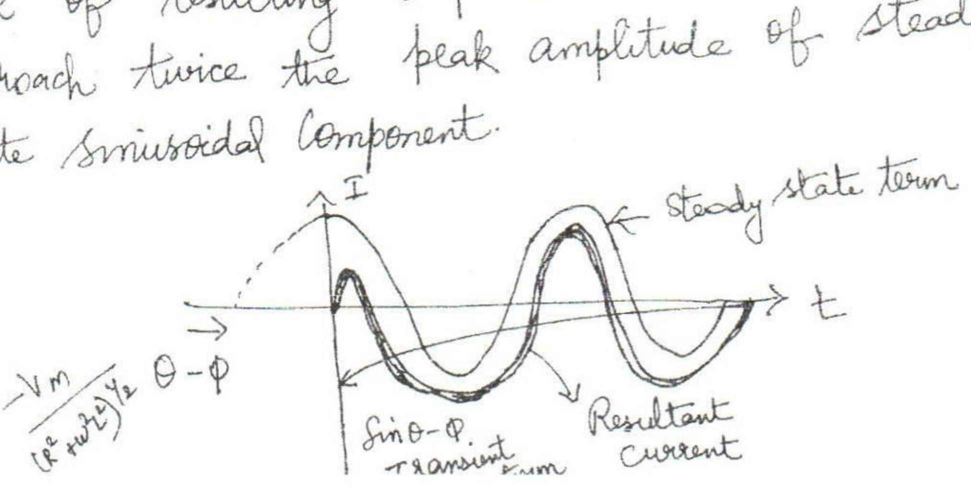
When $t=0$, the transient term is equal & opp to the steady state term & hence the current starts from zero.

Case 2:-

When the switch closes at $\theta=\phi$, the transient term will be zero and the current wave will be symmetrical.

Case 3

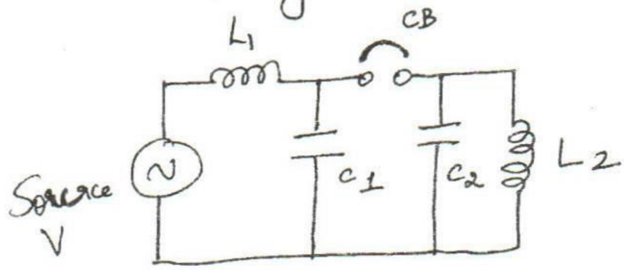
When the closes at $(\theta-\phi) = \pm \pi/2$, the transient term attains its maximum amplitude and the first peak of resulting composite current wave will approach twice the peak amplitude of steady state sinusoidal component.



The opening and closing forces are proportional to the square of the current.
Thus if the current is doubled, the forces are increased four fold.

If a short circuit occurs, assymmetrical current can flow through the contacts of a closed breaker

Double Frequency transients :-



The double freq transient is initiated by opening the circuit breaker as shown in the above circuit

Here $L_1 \Rightarrow$ Inductance on the source side
$C_1 \Rightarrow$ Stray Capacitance on the source side
$L_2 \Rightarrow$ Inductive Load
$C_2 \Rightarrow$ Stray Capacitance on the Load side

When the switch operates in the circuit, it completely divorces the load from the supply.
Thus the two halves of the circuit behave independently.

Before the Switch Opening, the voltage will divide in proportion to the inductances

The Voltage of the Capacitors will be $\left(\frac{L_2}{L_1 + L_2}\right)(V)$

If $L_2 \gg L_1$, the regulation would be very poor

C_1 & C_2 are charged to little less than instantaneous voltage

When current reaches zero, the voltage will be at its peak. The Natural freq will be

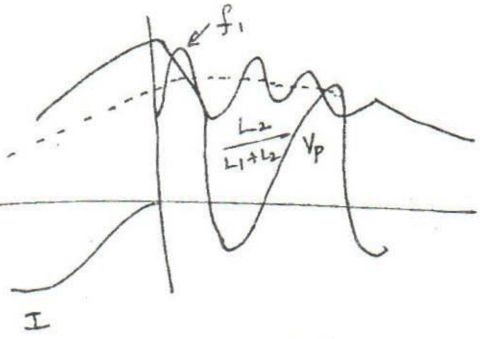
$$f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

Mean while, 'c' free to take up the source potential will oscillate about that value until the losses of the system damp out the disturbance

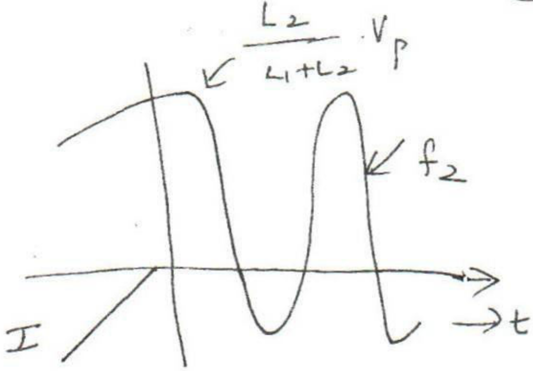
Frequency of oscillation will be

$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

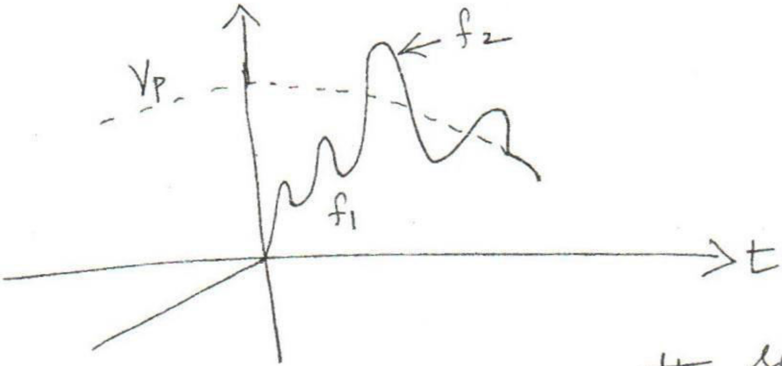
$\left.\begin{array}{l}\text{Recovery voltage across}\\\text{the C.B Contacts}\end{array}\right\} = \left\{\begin{array}{l}\text{Source side}\\\text{transients}\end{array}\right\} - \left\{\begin{array}{l}\text{Load}\\\text{side}\\\text{transients}\end{array}\right\}$



Source Side transient



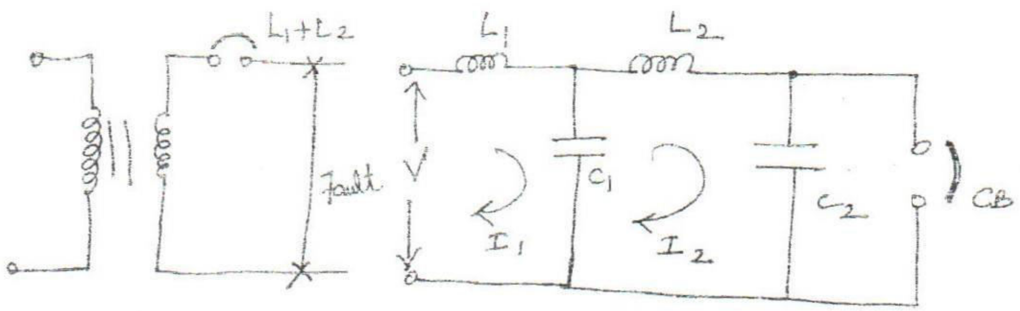
Load Side transient



Recovery voltage across the switch

There are many double frequency circuits
Consider a circuit showing a circuit breaker clearing a short circuit on the secondary side of transformer

Let $L_1 \Rightarrow$ Inductance upto the transformer
$L_2 \Rightarrow$ Leakage inductance of the transformer
C_1 & $C_2 \Rightarrow$ Inherent Capacitance on either side of the transformer



The initial voltage on the Capacitor c_1 in the above fig

$$V_{C_1}(0) = \frac{L_2}{L_1+L_2} \cdot V \rightarrow (1)$$

Now

$$V_{C_1} = V - L_1 \cdot \frac{dI_1}{dt} = V_{C_1}(0) + \frac{1}{C_1}\int (I_1 - I_2)\,dt \rightarrow (2)$$

$$V_{C_2} = \frac{1}{C_2}\int I_2\, dt \rightarrow (3)$$

$$V_{C_2} = V - L_1 \frac{dI_1}{dt} - L_2 \cdot \frac{dI_2}{dt} \rightarrow (4)$$

On transforming this eq$\underline{n}$

$$\frac{V}{S} - L_1 S i_1(S) - L_1 I_1(0) = \frac{V_{C_1}(0)}{S} + \frac{1}{C_1 S}\left[i_1(S) - i_2(S)\right]$$

from eq (3) taking L.T

$$V_{C_2}(S) = \frac{i_2(S)}{S C_2}$$

Taking L.T of eq (4)

$$V_{C_2}(S) = \frac{V}{S} - L_1 S i_1(S) + L_1 I_1(0) - L_2 S i_2(S) + L_2 I_2(0)$$

Time is measured from the instant when the switch clears at Cott Current zero (10)

$I_1(0)$ and $I_2(0) = 0$

$$i_1(s) = \frac{V}{L_1 s^2}\left[\frac{L_2 C_2 s^2 + 1}{L[s]}\right] V_{C_2}[s] \rightarrow (8)$$

Sub (5), (6) and (7) in (8) eqn

$$\left[\left[\frac{L_2 C_2 s^2 + 1}{L_1 s}\right] V_{C_2}[s] - \frac{V}{L_1 s_2}\right] L_1 s + \frac{1}{C_1 s} + \frac{C_2}{C s} V_{C_2}(s)$$

$$= \frac{V_{C_1}(0) - V}{s} \rightarrow (9)$$

Eq. (9) Can be rewritten as

$$s^4 + s^2\left[\frac{1}{L_1 C_1} + \frac{1}{L_2 C_2} + \frac{1}{L_2 C_1}\right] + \frac{1}{L C_1 L_2 C_2}\Bigg] V_{C_2}(s)$$

$$= V\left[\frac{s}{(L_1 + L_2) C_2} + \frac{1}{L_1 C_1 L_2 C_2 s}\right]$$

Different types of power system transients :-

Electrical power systems contain energy storage elements in the form of Inductances and Capacitances of electrical Components and inertia of rotating machines

(I) Depending upon the duration of transients

- Ultra-fast transient
- Medium fast transient
- Slow transient

Ultra-fast transient

→ Caused by either lightning or by the abrupt but normal network Changes resulting from normal switching operation.

→ These transients are entirely electrical in nature

→ They generally last only for few milli seconds

→ These transients give rise to high voltage which provides the basis of insulation design in the system

Meduim Fast transients :-

→ These transients occurs due-to abrupt short Circuits in the System

→ It Causes abnormal structural changes in the system.

→ These transients are also entirely electric in nature and are responsible for excessive currents in the system.

→ Short circuit transient may be present in the system for longer period

Slow transients :-

→ They are electro mechanical in nature
→ It causes mechanical oscillations of rotors of synchronous machines
→ Causes instability of the interconnected power system.
→ This occurs by putting some or all the machines out of synchronism.

I) Depending on its Nature :-

- Impulsive
- Oscillatory

Impulsive Low-frequency transient :-

→ Rises in 0.1 ms and lasts more than 1 ms
→ Contains freq components upto 5 KHz.
→ Most common type of transients recorded on power system.
→ They are not only easily propagated but they can also be amplified by a power-system resonance phenomenon

→ Measurements of these types of transients should be useful for all classes of applications like benchmarking, legal, trouble shooting & laboratory.

Medium - Frequency Impulse transient :

→ Lasts between 50ns to 1ms

→ Oscillatory transients between 5 and 500 kHz

→ These transients may not propagate as easily as low-frequency types

→ Causes arcing faults on the power distribution system which results in voltage sag on many of the user power systems

→ Applications → Trouble shooting & Lab classes.

High frequency types

→ Duration below 50ns

→ Frequency ranges between 0.5 and 5MHz

→ Applications Lab & trouble shooting.

Oscillatory transients :

Low Frequency transients :

→ Caused when a discharged power - factor correction capacitor is switched on across the line (8)

(12)

Capacitor resonates with the inductance of the distribution System

Peak of this waveform can't exceed twice the peak Voltage of the sine wave and is more typically 120% - 140% of the sine peak.

→ In some specific circumstances, there can be multiplication of the transient by resonance with the other power factor Correction Capacitors

High frequency transients :-

→ Caused by lightning & when inductive loads cut-off.

→ Typical peak voltages for end-use application are hundreds of volts to a few thousand volts

→ Several thousand amps of current may be available

Extremely fast transients (or) EFT's

→ Have rise and fall times in nano second region

→ Caused by arcing faults such as bad brushes in motors and are rapidly damped out by even a few meters of distribution wiring.

→ The standard line filters included on almost all electronic equipments remove EFTs.

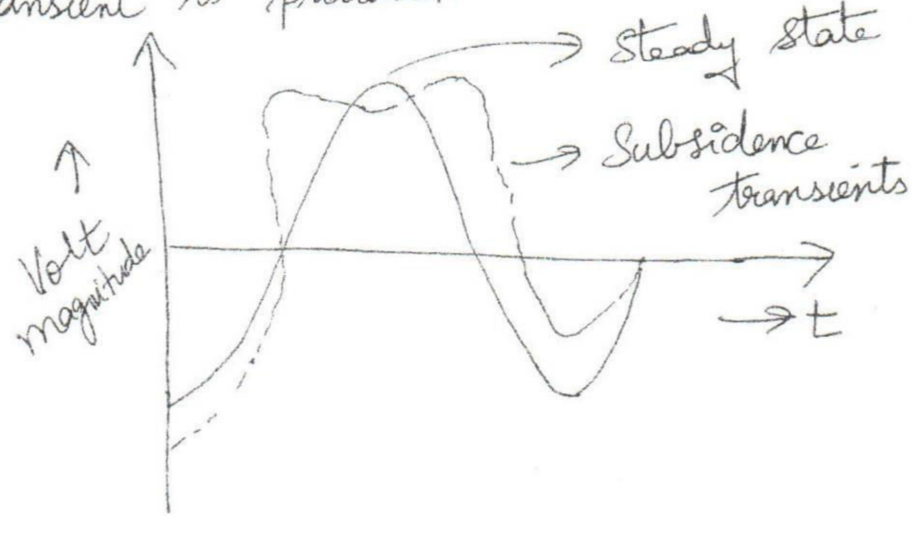
Subsidence transients :

→ In Coupling capacitor, voltage transformers and bushing Capacitor voltage transformer, the elements L & C contain stored energy.

→ When the disturbance such as fault occurs on the primary, then the Subsidence transient is produced.

→ Due to this sudden reduction of voltage produced on the primary, the voltage may be oscillatory or may be unidirectional

→ Due to this severe reduction, the secondary transient is produced.



Depending upon the Control of the transients produced in power System

→ There are '3' types

(a) Single transients under our Control

→ In this type, we are in a position to Open (or) close the switch at our discretion & are therefore able to anticipate the Consequences.

→ (b) Recurrent transients :-

→ The transients occurring regularly as Commutation transients in Converting equipments.

(c) Random transients :-

The transients generated by extraneous Operations beyond our Control which appear in an unpredictable random manner on our System.

Effects of Lightning transients :-

→ A direct (or) Indirect lightning stroke on a transmission line produces a steep fronted voltage wave on the line.

→ The voltage of this wave rise from zero to peak value in about 1 μs.

→ The travelling wave produced due to lightning transient will shatter the insulators and may even wreck poles.

→ If the travelling waves produced due to lightning hit the windings of the transformer or generator, it may cause considerable damage

→ The inductance of the windings opposes any sudden passage of electric charge through it.

→ The electric charges "piles up" against the transformer

→ This induces such an excessive pressure between the windings that the insulation may breakdown, resulting in the production of arc.

→ The line voltage is sufficient to maintain the arc long enough to severly damage the machine.

→ If the arc is initiated in any part of the power system by lightning stroke, this arc will set up very disturbing oscillations in the line

→ This may damage other equipment connected to the line.

Importance of study of transients in planning:-

→ Transients are system problems

→ The disturbance created in one location will permeate through out the system, after causing difficulties at points quite remote from its origin.

→ The study of switching transients in integrated system specially relates to EHV field.

→ For long line, the most serious problems are voltage surges in the power systems which are the consequence of lightning. Since the lightning produce the highest voltage.

→ As system voltages continue to increase, "switching transients" which in magnitude geared to system voltage causes more serious trouble to power system.

→ In EHV transmission system, for the economic reasons we have to limit & control switching surges.

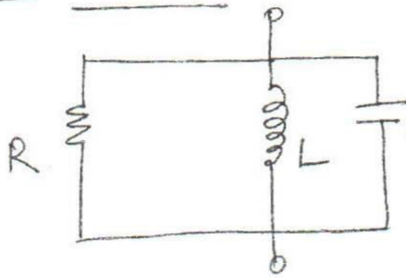
→ The line insulation required determines the cost of such transmission system.

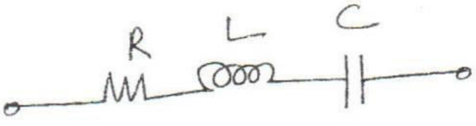
→ If more insulators are added to a string, then the benefits obtained / unit decreases progressively as the String becomes longer.

→ This is due to the poor distribution of voltage due to Capacitance effects.

→ Sudden reversal of voltage polarity which can occur with the travelling waves from switching surges causes a marked reduction in the flash over voltage of certain equipment

RLC Circuit :-





For parallel circuit, depending upon the Input drive F(t) Can be written as

$$F(t) = \frac{d^2\phi}{dt^2} + \frac{1}{RC} \cdot \frac{d\phi}{dt} + \frac{\phi}{LC}$$

'ϕ' can be the current in any of the branches (or) voltage across the circuit

$$F(t) = \frac{d^2\psi}{dt^2} + \frac{R}{L} \cdot \frac{d\psi}{dt} + \frac{\phi}{LC}$$

Component (or)

V is the voltage across any

ent through the Circuit.

Time constant for parallel circuit $T_P = RC$

Time Constant for series Circuit $T_S = L/R$

he product of '3' time Constants is the square

the angular period of the undamped circuit

which is given by

$$T_P T_S = LC = T^2$$

Here $$\eta = \frac{R}{Z_0} = R\left(\frac{C}{L}\right)^{1/2}$$

The parameter 'η' is given by the ratio of the resistance "R" to the surge Impedance Z_0.

The ratio of the parallel circuit time Constant to the series circuit time constant is equated to η^2

$$\frac{T_P}{T_S} = \frac{R^2 C}{L} = \eta^2$$

The relationship leads to a duality in the analysis of the series & parallel circuits.

(34)

While the basic phenomena in these two circuits are simple, the solutions to the equations can be complicated.

Step 1 :- Find the L.T that appears regularly in the Operational Solutions for problems.

Step 2 :- Determine their inverse transform.

Step 3 :- Plot the inverse transforms in dimensionless curves using 2.

Step 4 :- Extract the solutions from this curves.

Basic transform of RLC Circuit :-

(a) PARALLEL RLC CIRCUIT :-

To determine the Solution for the inductor in parallel circuit.

When a Switch is closed in the Capacitor branch allowing 'C' to discharge through 'R' & 'L'.

Let the current through the inductor is "I_L"

The current from the capacitor must be equal to the sum of the Currents in the other '2' branches.

$$-C \cdot \frac{dV_c}{dt} = I_L + \frac{V_c}{R} \rightarrow (1)$$

(4)

$$V_C = L \cdot \frac{dI_L}{dt} \rightarrow (2)$$

Elimination 'V_c' from eq (1) (Sub (1) in (2))

$$-C \cdot L \cdot \frac{d^2 I_L}{dt^2} = I_L + \frac{L}{R} \cdot \frac{dI_L}{dt}$$

Rewriting the above eqn.

$$\frac{d^2 I_L}{dt^2} + \frac{1}{T_p} \cdot \frac{dI_L}{dt} + \frac{I_L}{LC} = 0 \rightarrow (3)$$

Where $T_p = RC$

$LC = T^2$

By taking LT of eq (3) we get

$$\left(S^2 + \frac{S}{T_p} + \frac{1}{T^2}\right) i_L(S) = \left(S + \frac{1}{T_p}\right) I_L(0) + I_L'(0) \rightarrow (4)$$

When $I_L(0) = 0$, $I_L'(0) = \frac{V_C(0)}{L}$

Eq (4) becomes

$$i_L(S) = \frac{V_C(0)}{L} \frac{1}{\left(S^2 + \left(S/T_p\right) + \left(1/T^2\right)\right)}$$

The roots of the equation

$S^2 + \frac{S}{T_P} + 1/T^2 = 0$ are

$a=1, \quad b = \frac{1}{T_p}, \quad c = 1/T^2$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-\frac{1}{T_p} \pm \sqrt{\left(1/T_p\right)^2 - 4(1)\left(1/T^2\right)}}{2(1)}$$

The two roots

$$s_1 = -\frac{1}{2T_p} + \frac{1}{2}\left(\frac{1}{T_p^2} - \frac{4}{T^2}\right)^{1/2}$$

$$s_2 = -\frac{1}{2T_p} - \frac{1}{2}\left(\frac{1}{T_p^2} - \frac{4}{T^2}\right)^{1/2}$$

Then,

$$i_L(s) = \frac{V_c(0)}{L\left(\frac{1}{T_p^2} - \frac{4}{T^2}\right)^{1/2}} \left[\frac{1}{(s-s_1)} - \frac{1}{(s-s_2)}\right]$$

Take Inverse L.T.

$$i_L(t) = \frac{V_c(0)}{L\left(\frac{1}{T_p^2} - \frac{4}{T^2}\right)} \left(e^{+s_1 t} - e^{s_2 t}\right) \quad (3)$$

The solution for this circuit depends upon the values of 'S_1' & 'S_2'.

If $\frac{1}{T_p^2} > \frac{4}{T^2}$ (S_1 & S_2 are real)

If $\frac{1}{T_p^2} < \frac{4}{T^2}$ (S_1 & S_2 are complex).

These conditions can be expressed in terms of 'η'

where $\eta = R/2\sigma$

If $\frac{1}{T_p^2} > \frac{4}{T^2}$ then $\eta < 1/2$

If $\frac{1}{T_p^2} < \frac{4}{T^2}$, then $\eta > 1/2$

When the roots are complex, $\eta > 1/2$

$$S_1 = \frac{-1}{2T_p}\left[1 - j(4\eta^2 - 1)^{1/2}\right]$$

$$S_2 = \frac{-1}{2T_p}\left[1 + j(4\eta^2 - 1)^{1/2}\right]$$

Now

$$I_L(t) = \frac{V_c(0)}{L} \frac{2T_p\, e^{-1/2T_p}}{(4\eta^2 - 1)^{1/2}} \sin(4\eta^2 - 1)^{1/2} \cdot t/2T_p.$$

When the roots are real, $\eta < 1/2$.

$$S_1 = \frac{-1}{2T_P}\left(1 - (1-4\eta^2)^{1/2}\right)$$

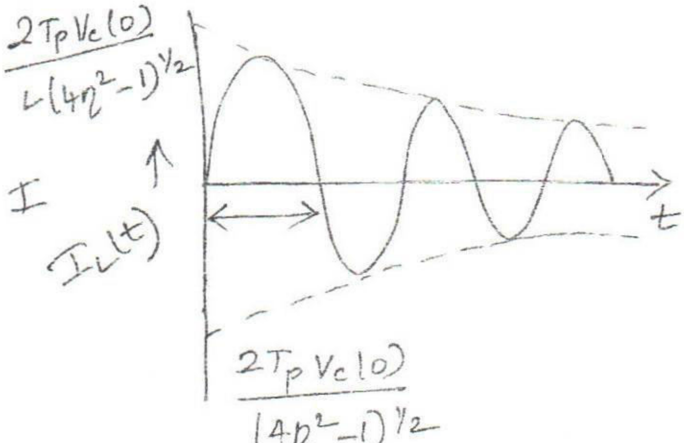
$$S_2 = \frac{-1}{2T_P}\left[1 + (1-4\eta^2)^{1/2}\right]$$

Now $$I_L(t) = \frac{V_c(0)}{L} \cdot \frac{2T_P \cdot e^{-t/2T_P}}{(1-4\eta^2)^{1/2}} \sinh (1-4\eta^2)^{1/2} \cdot t/2T_p$$

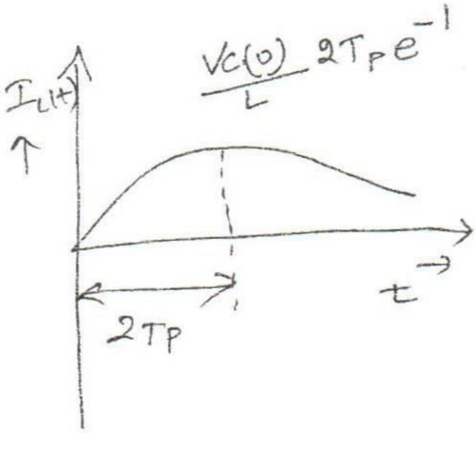
When $\eta = 1/2$

$$I_L(t) = \frac{V_c(0)}{L}\, t \cdot e^{-t/2T_P}.$$

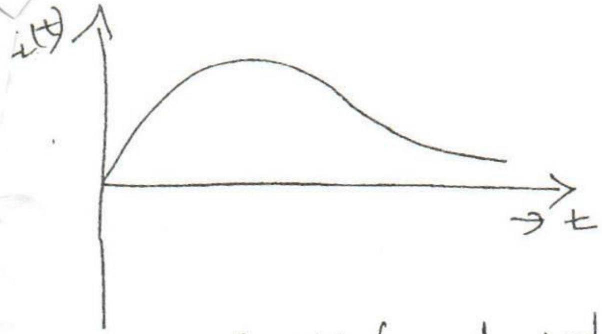
For diff values of "η", the inductor current is plotted from this curves we can extract the solutions at any instant.



$\eta > 1/2$ (underdamped)



$\eta = 1/2$ (Critically damped)



$\zeta < 1/2$ (overdamped).

b) Series RLC Circuit :-

Suppose a battery of voltage is connected to series RLC circuit, then

$$IR + L \cdot \frac{dI}{dt} + \frac{1}{C}\int I\,dt = V.$$

Diff and Rearranging

$$\frac{d^2I}{dt^2} + \frac{R}{L} \cdot \frac{dI}{dt} + \frac{1}{LC} = 0$$

$$\frac{L}{R} = T_S$$

Then $\frac{d^2I}{dt^2} + \frac{1}{T_s} \frac{dI}{dt} + \frac{1}{T^2} = 0$

L.T of the above eqn

$$\left(S^2 + \frac{S}{T_S} + \frac{1}{T^2}\right) i_S(S) = \left(S + \frac{1}{T_S}\right) I(0) + I'(0)$$

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If the current starts from zero, then $I(0)=0$

$$I'(0) = \frac{V}{L}$$

$$i(s) = \frac{V}{L} \frac{1}{\left(s^2 + \frac{s}{T_s} + \frac{1}{T^2}\right)}$$

Then

for oscillatory condition if $\lambda > 1/2$

Where $\lambda = \frac{1}{2} = \frac{Z_0}{R}$

At oscillatory condn

$$I(t) = \frac{V}{L} \cdot \frac{2T_s}{(4\lambda^2-1)^{1/2}} e^{-1/2T_s} \cdot \sin(4\lambda^2-1) \cdot \frac{t}{2T_s}$$

When $\lambda = 1/2$, Critical damping

$$I(t) = \frac{V}{L} t e^{-t/2T_s}$$

When $\lambda < 1/2$, (over damped condition)

$$I(t) = \frac{V}{L} \frac{2T_s}{(1-4\lambda^2)^{1/2}} e^{-1/2T_s} \sinh(1-4\lambda^2)^{1/2} \cdot \frac{t}{2T_s}$$

For this current equations curves are plotted for various values of "λ". From this curves the solutions can be extracted at any instant. (1)

Mrs. LAVANYA DHANESH.

UNIT-II SWITCHING TRANSIENTS.

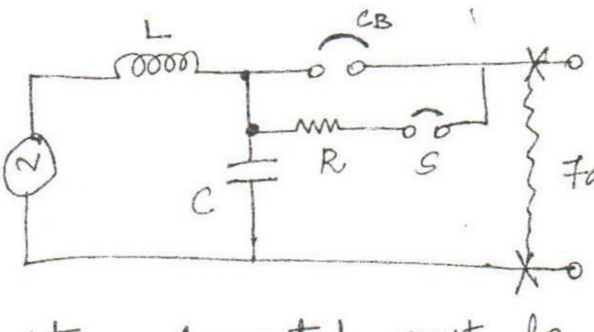
Resistance Switching:-

A delibrate connection of a resistance in parallel with the contact space (arc) is made to overcome the effect of transient recovery voltage. This is known as Resistance Switching.

→ The shunt resistors connected across the circuit breaker have '2' functions

→ To distribute the transient recovery voltage more uniformly across several breaks.

→ To reduce the severity of transient recovery voltage at the time of interruption by introducing damping into oscillation.



The resistance connected must be low compared with the reactance of capacitance shunting the breaks at the frequency of recovery transient

→ The lower value of resistor is only required to reduce the transient recovery voltage.

L ⇒ System Inductance

C ⇒ Stray capacitance which shunts the breaker

R ⇒ Resistor used to modify the recovery transient

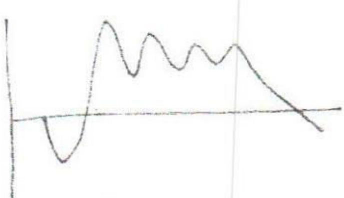
When fault current has been switched a residual current will flow through resistor 'R'. This must be interrupted by opening the auxillary interrupter 'S'

From this we know that if the value of Resistance 'R' is equal to or less than $\frac{1}{2}\sqrt{L/C}$, the oscillatory nature of transient will not be there

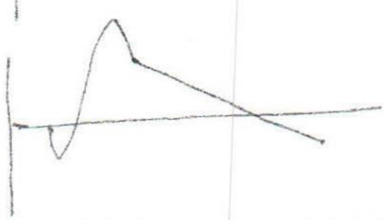
→ The rate of rise of restriking voltage will be within permissible limits of circuit breaker.

for critical damping

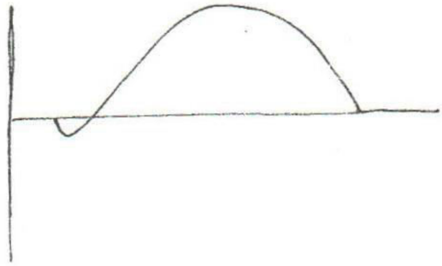
$$R = \frac{1}{2}\sqrt{L/C}$$



When $R = \infty$



When $R > \frac{1}{2}\sqrt{L/C}$



When $R < \frac{1}{2}\sqrt{L/C}$

We know

$$\frac{V}{S} = \left(RLCS^2 + LS + R\right) I_2(s)$$

Now

$$I_2(s) = \frac{V}{S\left[RLCS^2 + LS + R\right]}$$

$$I_2(s) = \frac{V}{S\,(RLC)\left[S^2 + \frac{1}{RC} + 1/LC\right]}$$

$$I_2(s) = \frac{V/RLC}{S\left[S^2 + \frac{1}{RC} + 1/LC\right]}$$

On resolving $I_2(s)$ we get

$$I_2(s) = \frac{V}{R}\left[\frac{1}{S} - \frac{S+x}{(S+x)^2 + (\sqrt{y})^2} - \frac{x}{(S+x)^2 + (\sqrt{y})^2}\right]$$

Here $x = \frac{1}{2RC}$

$$y = \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2$$

Taking L-T of $i_2(s)$

$$i_2(t) = \frac{V}{R}\left[1 - e^{-xt}\left[\cos\sqrt{y}\,t + \frac{x}{y}\sin\sqrt{y}\,t\right]\right]$$

The natural freq of oscillation is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4C^2R^2}}.$$

In air blast circuit breaker, it is observed that the rate at which dielectric strength of the gap increases is lower than the coil CB

This is because, the air has much lower dielectric strength than the gases at same temp and pressure in the oil C.B.

Dielectric strength of gas increases with pressure.

Thus the air blast CB is more sensitive to the restriking voltage transient.

In low (or) medium voltage air blast CB, the rate of rise of restriking volt is higher.

The Shunt resistors are used for low & Mediu Voltage air blast CB.

The rate of the rise of restriking volt is higi Shunt Resistors are used for low & Medium voltag blast CB.

In case of oil CB, the resistance switching is not employed as it is not sensitive to RRRV.

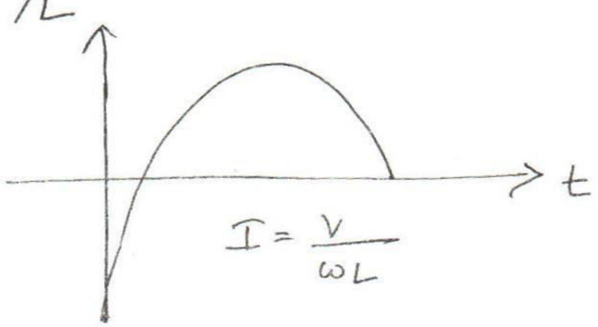
If the injected current is treated as a ramp with slope $\frac{V}{L}$

$$I = \frac{V}{L} t$$

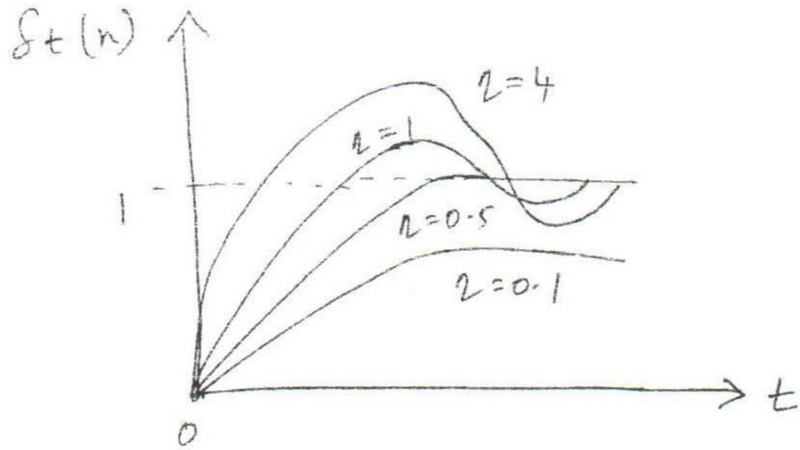
Where: V' – Instantaneous Voltage at time of interruption

$$V(s) = \frac{1}{s\left[s^2 + \frac{s}{T_p} + \frac{1}{T^2}\right]} \frac{I'}{C}$$

Where $I' = V/L$



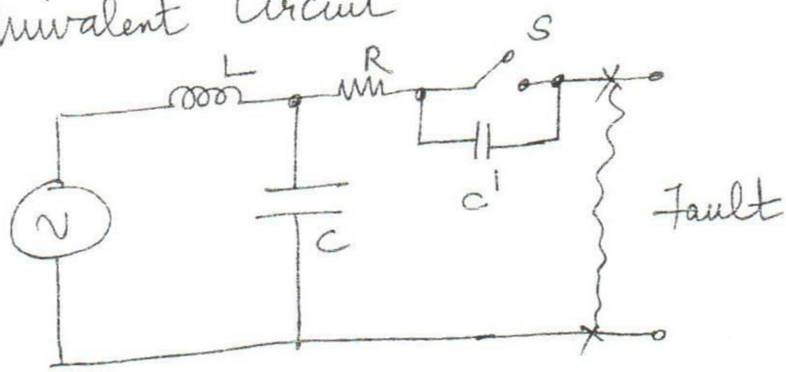
Different values of $\eta = R/2$



For Change in "R", the 'η' is modified, hence the transient voltage produced is also Changed.

When the resistor current is subsequently interrupted, a second transient will be initiated. To study it is necessary to introduce the Capacitance C', shunting the resistor break.

Equivalent Circuit

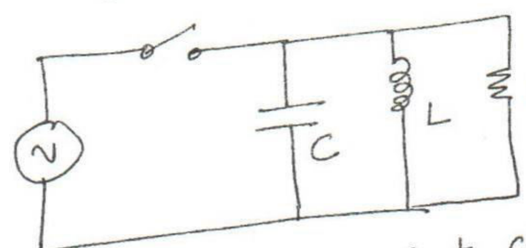


Load Switching :-

The frequent function performed by switching devices are to Switch "ON" and "OFF" the load (ie) load switching which is represented by parallel RL Circuit

Low P.f loads are Inductive
High P.f loads are resistive

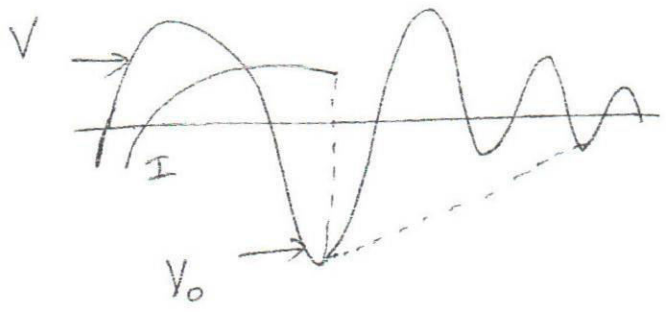
When high P.f load is switched off. the effective capacitance of load becomes important in determining the form of transient produced.



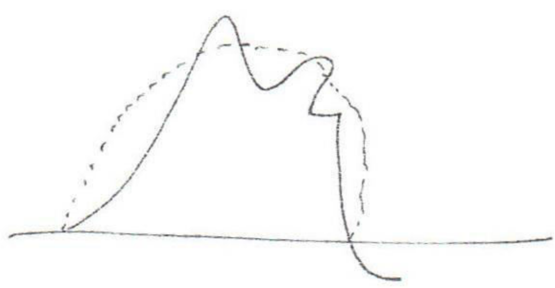
Simple equivalent circuit

When the Current extinguishes, the voltage access load is "V_0". Now 'C' is changed upto voltage 'V_0' and it is discharged through 'L' & "R".

As the P.f improves, the current comes more & more into phase with voltage. Thus "V_0" decreases



{Damped oscillatory discharge
Transient voltage across load.}



Transient voltage across switch.

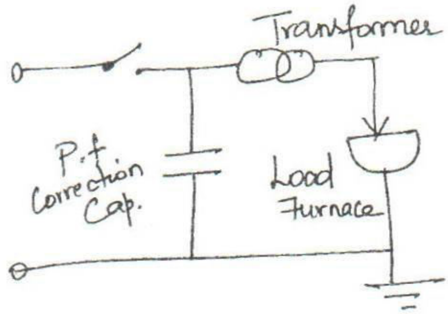
At UPF, voltage is zero, when current is zero. Thus No transient at all.

Thus the P.f plays a major role in the production of switching transient & magnitude of switching transient depends on the P.f.

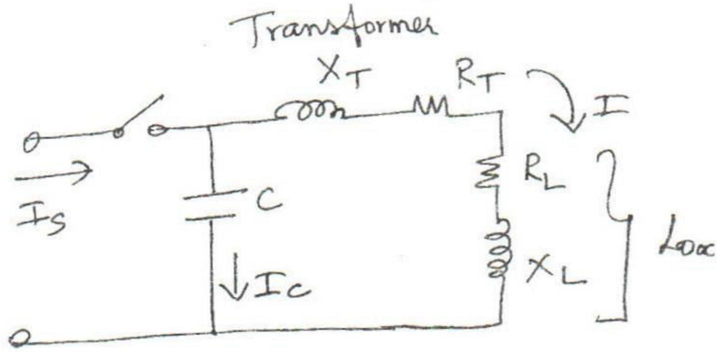
Arc furnace in industries operate at a low voltage and high current & consequently feed by a step down furnace transformer. They are characterised by low P.f and frequency.

Switching capacitors are frequently connected to high voltage bus to improve the P.f.

They are switched with the transformer & furnace



Schematic representation of arc furnace



(b) Equivalent circuit

In Fig (a), the transformer rating is 60 Hz, 13.8 kV 20 MVA star/star connected & solidly grounded at fully load P.f = 0.6.

To investigate the transient evoked by switching off the fully loaded transformer,

We have to determine load current,

$$\text{Load Current} = \frac{KVA}{\sqrt{3} \times KV} = 836.8 \text{ A}$$

$$\text{Total } Z = \frac{kV}{(\sqrt{3})(\text{Load Current})} = 9.52\,\Omega$$

$$\varphi = \cos^{-1}(0.6) = 53.13.$$

$$\text{Total "R"} = Z \cos\varphi = 5.7\,\Omega$$

$$\text{Total 'X'} = Z \sin\varphi$$

$$= 7.6\,\Omega$$

$$\boxed{L = 20.2\,mH}$$

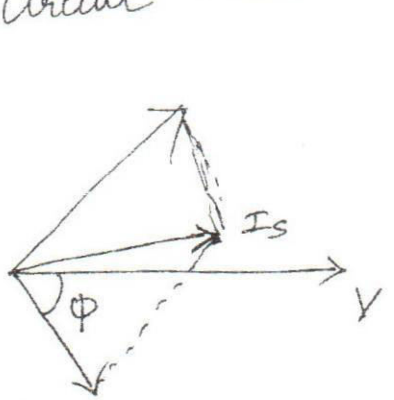
When the current $I(s)$ is interrupted at current zero, the currents 'I_c' and 'I' are equal & opp.

$$I_c(0) = -I(0) = I \sin\varphi$$

$$= 669.4\,A$$

$$V_c(0) = 0$$

The post interruption transient can be computed from the circuit with initial condition & as in eq.



Phasor diagram of loaded arc furnace.

Now the Current is given by

$$\frac{d^2 I}{dt^2} + \frac{1}{TS}\frac{dI}{dt} + \frac{1}{T^2} = 0$$

$$(s)\left[s^2 + \frac{s}{Ts} + \frac{1}{T^2}\right] = \left(s + \frac{1}{Ts}\right) I(0) + I'(0). \rightarrow (2)$$

We Know.

$$L.\frac{dI}{dt} + IR = V_c$$

$$I'(0) + I(0).R = V_c(0) = 0$$

$$I'(0) = -I(0)\cdot\frac{R}{L}$$

$$\boxed{I'(0) = \frac{-I(0)}{TS}} \rightarrow (3)$$

ub (3) in (1) we get.

$$i(s) = \frac{s}{\left(s^2 + \frac{s}{TS} + \frac{1}{T^2}\right)} I(0) \rightarrow (4)$$

o compute transformer terminal voltage

$$\frac{d^2 V_c}{dt^2} + \frac{1}{TS}\cdot\frac{dV_c}{dt} + \frac{V_c}{T^2} = 0.$$

$$V_c(s)\left[s^2+\frac{s}{Ts}+\frac{1}{T^2}\right]=\left[s+\frac{1}{Ts}\right]V_c(0)+V_c'(0). \rightarrow (6)$$

$$C.\frac{dV_c}{dt}=-I\;;\quad V_c'(0)=\frac{-I(0)}{C}\rightarrow (7)$$

Sub (7) in (6) we get

$$V_c(s)=\frac{1}{\left(s^2+\frac{s}{Ts}+\frac{1}{T^2}\right)}\frac{I(0)}{C}$$

Here peak voltage reaches about 72% of undamped Value

The first voltage after current zero.

$$=0.72\times 669.4\times 11.4 = 5.49\,kV$$

Since $I(0) = -669.4\,A$

Normal & Abnormal Switching transients :-

When a switch opens in 1φ circuit, it is possible for the recovery voltage to reach a value twice as high as normal peak voltage of the system.

When the switch closes, the current can reach a value twice that of the eventual steady state current.

But in practical circuits, this theoretical magnitudes Current & Voltages are not achieved, because of circuit damping.

Due to some other circumstance like transients voltage & current magnitude may rise high.

The transients occur due to the trapping of energy & its subsequent release somewhere in the circuits. Such transients are referred as Abnormal current voltage transients.

It is also caused due to charge on a capacitor line & current in an inductor. If a circuit is completely quiescent when a transient is initiated, the transient will be a normal transient.

The transient stores energy in the system, so that subsequently when a second transient is initiated it would be abnormal.

When the initial current $I_1(0)$, voltage across the capacitor $V_2(0)$ & current through the capacitor $I_2'(0)$ is finite, then an abnormal transient will develop in the system.

Capacitance Switching :-

The shunt Capacitor bank plays an important role in power systems.

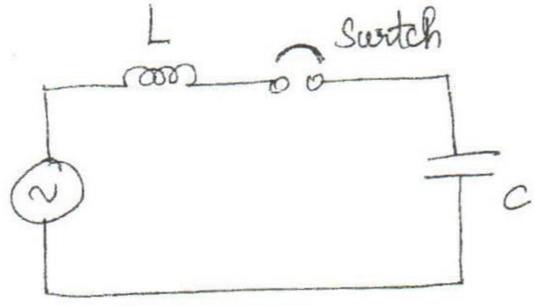
The shunt Capacitors are connected between the line & neutral (or) line & ground.

The shunt Capacitors are employed to correct a lagging p.f, to provide voltage support for the system. In some applications, they are switched in & out quite frequently as the system load varies & the system voltage fluctuates.

The switching operations are non-trivial & should be carefully considered when designing capacitor banks and their associated switching equipment.

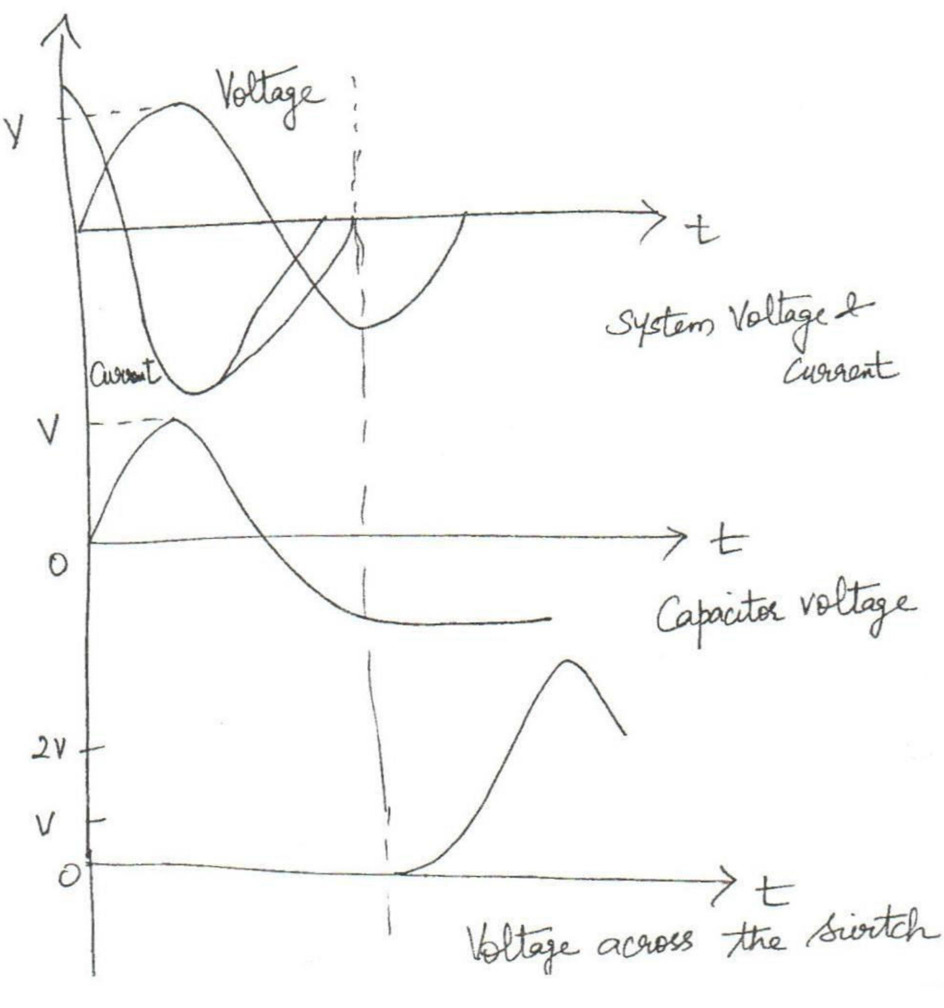
This is called as Capacitance switching.

The switching operation may be unsuccessful due to the recognition (or) restriking of the switch during opening.



From the figure, it is clear that there is a relative phase of current & voltage, since the current leads the voltage by 90°. When the switch interrupts, the capacitor is fully charged to maximum voltage.

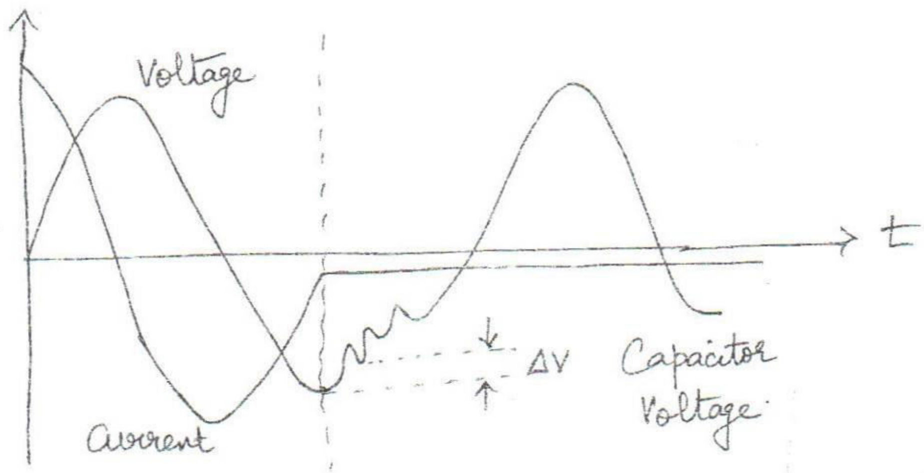
When switch opens, the capacitance is now isolated from the source, retains its charge. When the capacitor tries to retain its charge, the voltage across the switch reaches a peak value of 2v, which is potentially dangerous.



When a Capacitor is connected to a system, the leading Current that it draws flowing through the inductance of the System, causes the capacitor Voltage to be higher than the Open circuit System Voltage.

This condition is called as Ferranti Rise (OR) Negative Regulation

Capacitance Switching showing the effect of source Regulation:-



⇒ This is the event where the capacitor is disconnected from the source.

→ The potential of the source side of the circuit breaker will return to the lower value after some oscillations if the capacitor has been disconnected.

The oscillations are produced due to the presence of source inductance and stray Capacitance adjacent to the breaker on the source side.

Capacitance Switching with a restrike :-

Consider a restrike that takes place when the voltage reaches its peak. During this instant, the capacitor voltage subjected to oscillation due to this sudden disturbance.

$$\text{freq of such oscillation } "f_0" = \frac{\omega_0}{2\pi} = \frac{1}{2\pi (LC)^{1/2}}$$

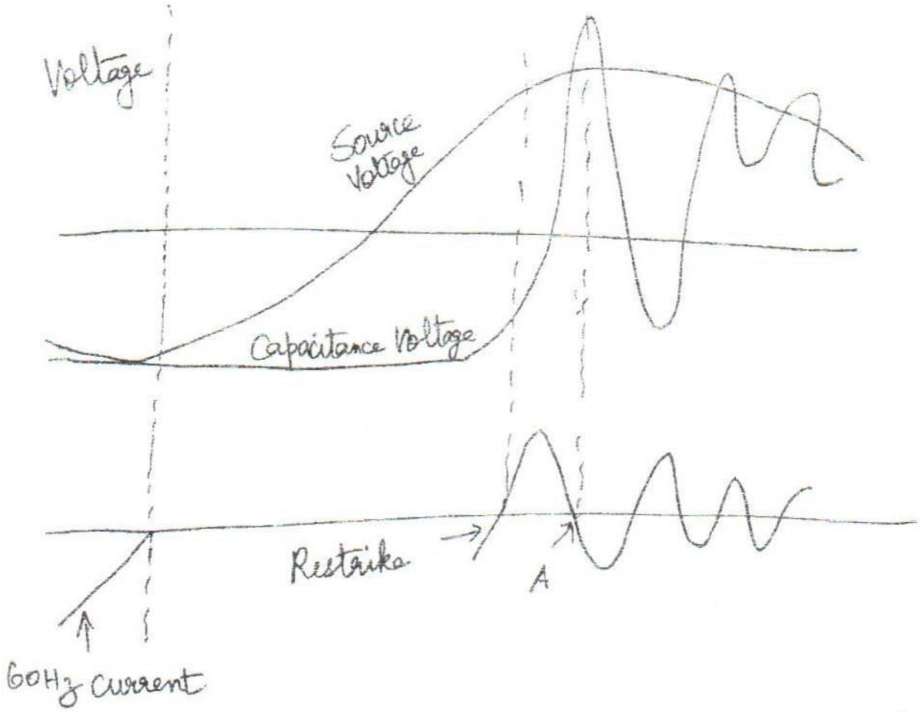
Where 'L' ⇒ Inductance of the supply
"C" ⇒ Capacitance of the bank.

The restrike current will be the instantaneous voltage across the switch divided by the circuit surge Impedence.

$$\text{Restrike Current} = \frac{2V_p}{\left(\frac{L}{C}\right)^{1/2}} \sin\omega t$$

$$= 2V_p \left(\frac{C}{L}\right)^{1/2} \sin\omega t.$$

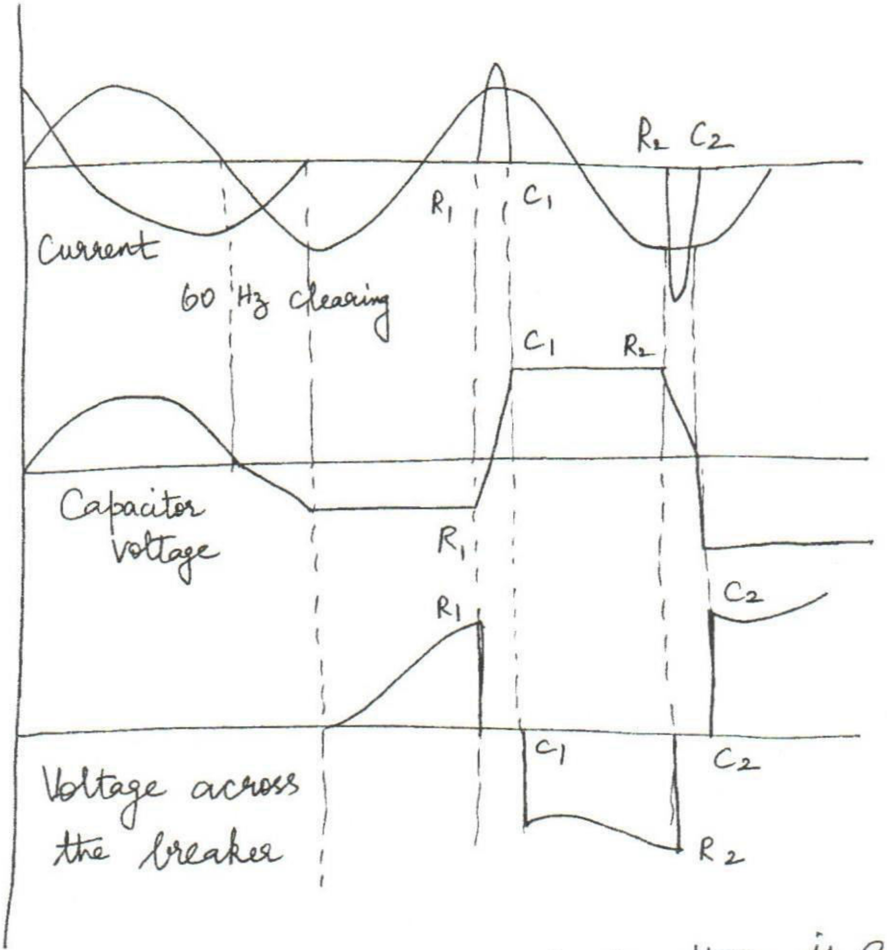
The figure shows the initial clearing, trapping of charge on the capacitor & the subsequent restrike.



When the circuit breaker interrupts, the current at point 'A', the voltage across the capacitor is high (Appro 3 times the peak value V_P).

The transient voltage excursion to $3V_P$ is an abnormal overvoltage by the definition and is the consequence of energy stored in the capacitor at the time of restrike.

Capacitance Switching with multiple restrikes:



During Capacitance switching, practically there is a chance of sequential restrikes. The fig represents the sequential restrikes and `C2' represents subsequent clearings. The sequence is idealised & to some extent oversimplified.

For eg: In practice restrikes will not occur precisely at the voltage peak, so that the voltage if it escalates, does more slowly. Again the Circuit becomes more Complicated.

Some Cap will exist on the source side of the breaker, which will inproduce higher freq of disturbances.

When the multiple restriking occurs, it is possible for a voltage of 4/unit to be developed across the switch, a point which is often overlooked.

A re ignition may occur at this time rather than half a cycle later which will propably result in the switch Conducting current for another half cycle.

Current suppression and other problems can arise during Capacitance switching Operations which are the examples of overvoltages Caused by the release of stored energy in the system.

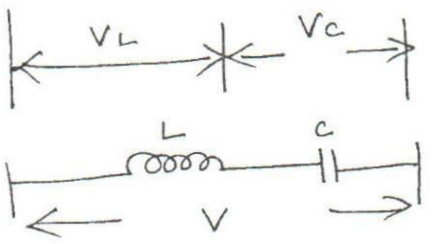
FERRO RESONANCE

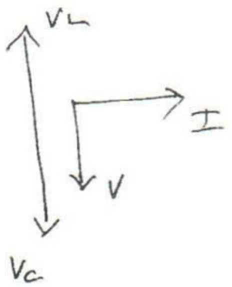
In the series resonance Circuits, a very high voltage Can appear across the elements of a series 'LC' circuit when it is excited at (or) near its natural frequency.

'V_L' & 'V_c' add to give the applied voltage 'V'.

But the voltage across the inductor leads the (5)

urrent in phase by 90°, the capacitor voltage
ags the current by the same amount.





Simple series resonance

If it is seen that both V_L and V_C can far exceed
, voltage conditions of this kind can be sustained
nd therefore called as <u>Dynamic</u> <u>Overvoltages</u> rather than
ransients.

Such resonant conditions are to be avoided in
ower circuits. This phenomenon such that both 'V_L' & 'V_C'
far exceed 'V' is called ferroresonance. This condition
across in the power circuit since the inductance involved
is usually icon cored and more often than not a
transformer. The non-linear character of an iron
lad inductance also inproduces some peculiar effects.

The voltage across the inductance will depend upon the
frequency "ω" and the current through a function (f(I)).
Thus voltage can be written as $V_L = \omega f(I)$.

'V_L' is plotted as a function of current. This Voltage will lead the current by 90°.

The voltage across the inductance will depend upon the frequency "ω" and the current through a function f(I). Thus voltage can be written as $V_L = \omega f(I)$.

'V_L' is plotted as a function of current. The voltage will lead the current by 90°. The voltage across the capacitor is given by

$$V_c = -\frac{1}{\omega C}$$

The '-ve' sign indicates that it is antiphase with 'V_L' and lags the current by 90°.

The total voltage will be

$$V = V_L + V_c = \omega f(I) - \frac{1}{\omega C}$$

(or)

$$\boxed{V_L = V + \frac{1}{\omega C}}$$

From the eq above, it is clear that 'V_L' has a fixed consistent 'V' and that is proportional to "I".

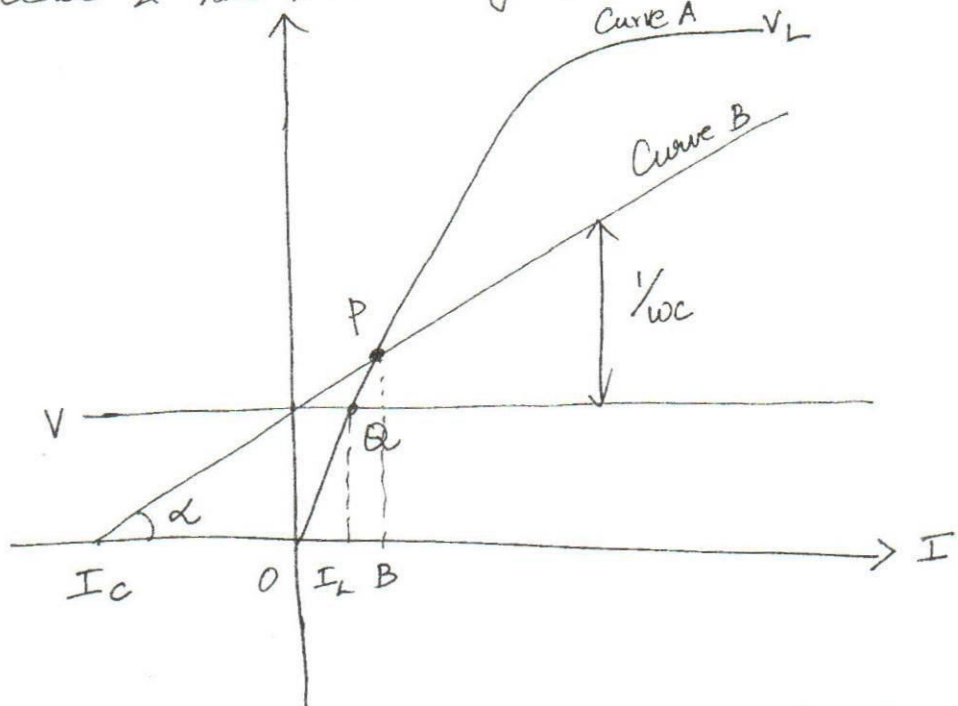
Since both the curves 'A' and 'B' represent the V_L, the operation point must be where the '2' lines cross at 'P'. The capacitor voltage in this (4) nistance 'PQ' and the inductor voltage PB which exceeds V, whereas the current is given by OB.

It is noted that the voltage 'V' applied to the pacitor alone, it would take a much larger current c, but if applied to the inductor alone, the current ould be smaller current I_L.

The slope of the inclined line is given by

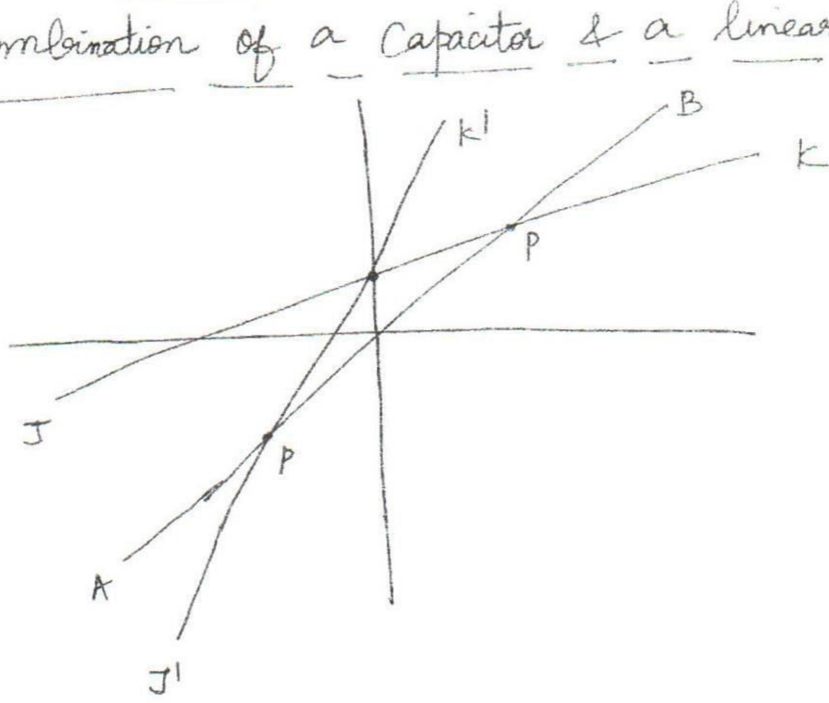
$$\tan \alpha = \frac{1}{\omega C}.$$

If the value of "ω" (or) 'C' is reduced, the slope will ncrease & the intersecting point 'P' will progress up the curv



Voltage & current relationship in Ferroresonant Circuit.

Ferro Resonance in a power circuit with series combination of a capacitor & a linear inductor



The fig illustrates the graphical representation of the operating conditions with a series connected capacitor & linear inductor.

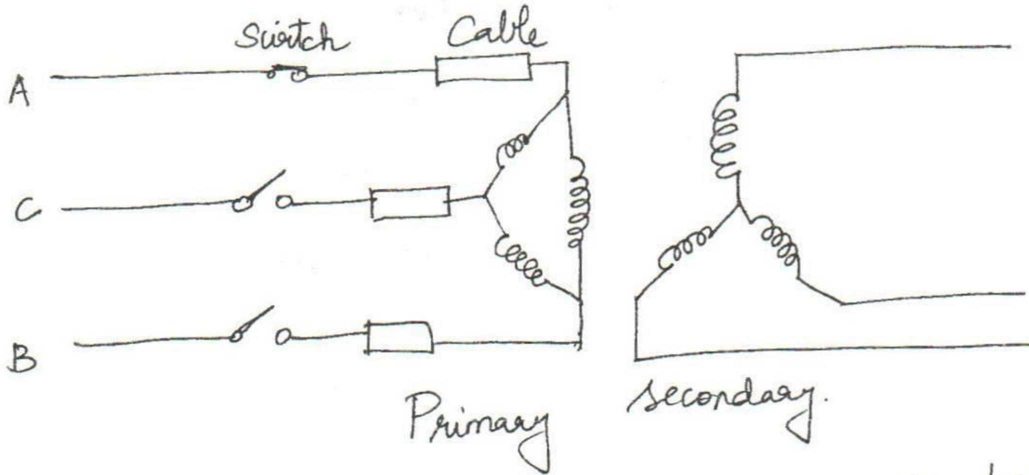
The characteristic of the capacitor is given by JK (or) JK' according to the value of capacitor. The operating point point will be P' (or) P'.

If the values $\omega_L > \frac{1}{\omega_C}$, the operating point is at "P" and if the values $\omega_L < \frac{1}{\omega_C}$, the operating pt is P'.

If 'C' is reduced, JK becomes stepper and J'K' becomes less steep.

Ferroresonance Situation :-

A practical Example.



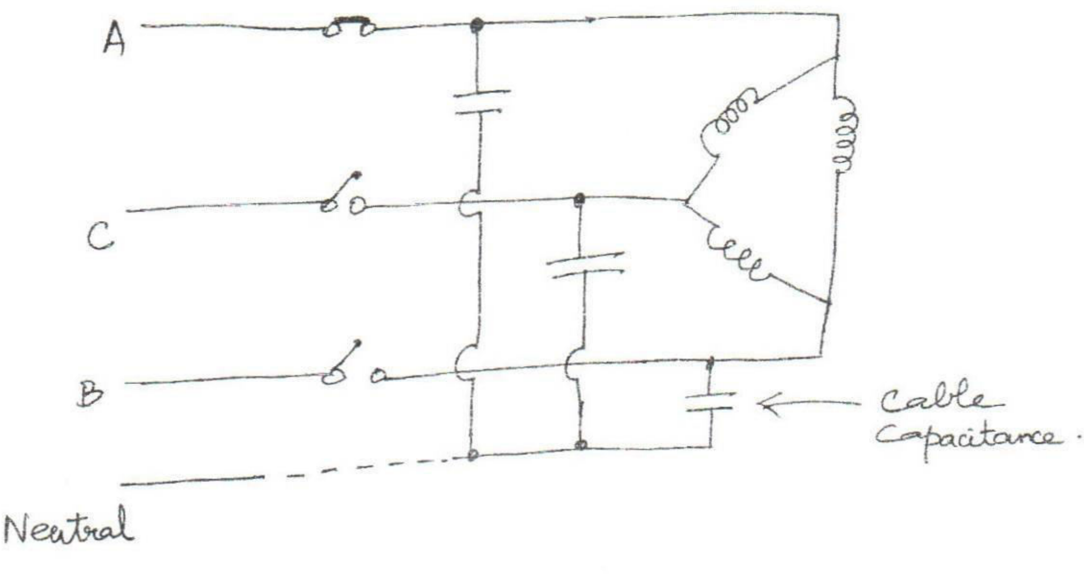
A Switch used to energise and Deenergise the primary Of the transformer. The Switch is interconnected to the primary by a length of cable. The Switching device may be mounted at the top of a pole & a transformer on a near pad at ground level.

Consider only one pole of the Switch is closed, then the transformer is not energised. Thus there is a path for the flow of current through '2' of the phase windings and the cable capacitance is obtained.

This current flowing in the Specified path Can produce resonance & impress excessive voltage across the transformer and the Cables on the unenergised phases.

It can cause lightning arrestors connected to B & C bushings of the transformer to operate. If the condition is sustained, repeated operation can destroy the arresters.

There is a possibility of ferroresonant overvoltages on Y - Δ transformer banks during the single phase switching as a function of transformer size and length of the cable.



Power System Transients

UNIT-III

Mrs. LAVANYA DHANESH

LIGHTNING TRANSIENTS

The voltage waves having magnitude more than its normal value and which remains for a very short duration are called Over voltage surges or transient Overvoltages.

→ The overvoltages occur due to lightning surges are called lightning transients.

→ There is a high rate of rise and high peak value in transient over voltages which is dangerous for insulators & hence protection is required against the over voltages.

Causes of over voltages:-

Overvoltages are caused due to Voltage Stress.

- External Over voltages
  ⇓
  Over voltages originate from atmospheric disturbance mainly due to lightning.
- Internal Over voltages

① Direct lightning strokes

→ Electromagnetically induced voltages due to lightning discharge taking place near the line

→ Voltage induced due to Changing atmospheric Condition along the line length.

→ Electrostatically induced over voltages due to the presence of Charge Clouds

→ Electrostatically induced over voltages due to the frictional effects of Small particles such as dust or dry snow in the atmosphere or due to change in the altitude of the line

Internal overvoltage
- → Switching overvoltages (or) Transient overvoltages of high freq).
- → Temporary Over voltages (Steady state overvoltages of power freq)

Switching overvoltages

Egs:
① Switching ON & OFF the equipments
② Switching of a transformer at no load.
③ Opening of a CB in order to clear a fault

Over voltage Factor (or) Amplitude factor

$$\text{Over voltage Factor} = \frac{\text{Peak over voltage}}{\text{Rated peak system frequency phase voltage}}$$

Comparison

Lightning overvoltages	Switching overvoltages
(1) It is a natural phenomenon	(1) They originate in the system itself by the connection and disconnection of CB contacts or due to initiation of faults
(2) The magnitude of lightning voltages appearing on transmission lines doesn't depend on line design	(2) Switching overvoltages are proportional to operating Voltage.
(3) If the system operating voltage is less than 500kV, lightning overvoltages have to be considered	(3) If the system operating voltage is in the range of 300 kV to 765kV both switching overvoltages & lightning overvoltages have to be considered.

Charge Formation in clouds:-

During thunderstorms, positive and negative charges become seperated by the heavy air currents with ice crystals in the upper part and rain in the lower parts of the cloud.

This charge seperation depends on the height of clouds which range from 0.2 to 10km with their charge centers probably at a distance of 0.25 to 2km.

→ The charge inside the cloud may be as high as 1 to 100°c.

→ clouds may have a high potential as 10^7 to 10^8 V. with field gradients ranging from 100 V/cm to 10 kV/cm

→ The energies associated with the cloud dischages can be very high

→ It is beleived that the upper regions of the cloud are usually positively charged whereas the lower region & the base are predominantly negative except the local region near the base & the head is positive.

Simpson's Theory

→ '3' essential regions in the cloud to be Conside for charge formation :

→ Below region "A", air current travel above 800cm and no raindrops fall through.

→ In region "A", air Velocity is high that is enough to break the falling raindrops Causing a positive Charge spray in the Cloud and negative Charge in the air.

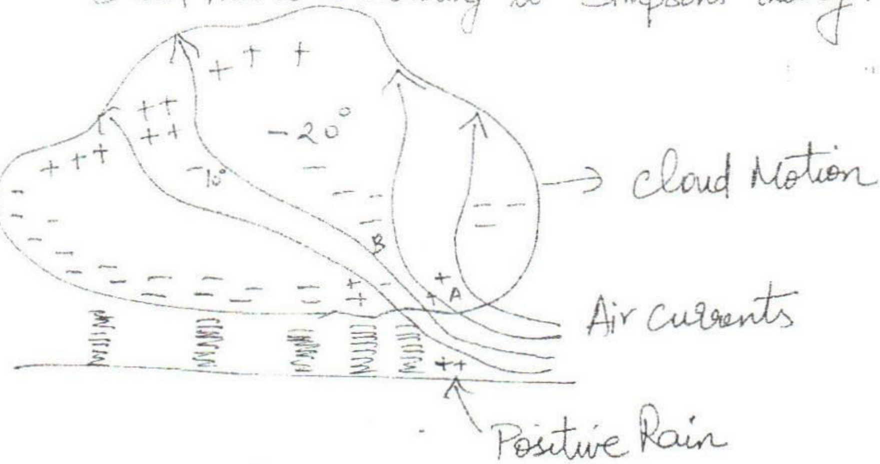
→ The Spray is blown upwards, but as the velocity of air decreases, the positively Charged water drops recombi with the larger drops and fall again.

→ Thus region 'A' becomes predominantly positively Charged while region "B" above it becomes negatively charged by air currents.

→ In the upper regions in the cloud, the temperature is low (Below freezing point) and only ice Crystals exist

→ The impact of air on these Crystals make them negatively charged, thus the distribution Of the Charge within the Cloud is shown in the figure below.

Cloud model according to Simpson's theory.



Reynolds and Mason Theory:-

→ According to this theory, the thunder clouds are developed at heights of 1 to 2km above the ground level.

→ They may go up to 14km above the ground.

→ For the charge formation air currents in the clouds moisture and specific temperature range are required

→ The air current controlled by the temperature gradient move upwards carrying moisture & water droplets.

→ The temperature is 0°c at about 4km from the ground and may reach -50°c at about 12 km height.

→ The water droplets don't freeze as soon as temp is 0°c. They freeze only if the temperature is below -40°c.

→ They form solid particles on which crystalline ice patterns develop and grow.

→ In clouds the effective freezing temperature is $-33°c$ to $-40°c$.

→ The water droplets in the thunder cloud are — up by air currents and gets super cooled.

→ When such freezing occurs, the crystals grow : large masses and due to their weight & gravitation force start moving downwards.

→ Thus a thunder cloud consists of super cooled water droplets moving upwards and large hail stones moving downwards.

→ When the upward moving super cooled water droplets act on cooler hail stone, it freezes partially. Due to this the outer layer of the water droplets freezes forming a shell with water inside.

When the process of cooling extends to inside warmer water in the core, it expands thereby splintering and spraying the frozen ice shell.

The splinters being fine in size are moved up by the air currents and carry a net positive charge in the upper region of the cloud.

→ The hail stones that travel downwards carry on equivalent negative charge to the lower regions of the cloud and thus negative charge builds up in the bottom side of the cloud.

→ According to Mason, the ice splinters should carry only positive charge upwards.

→ water being ionic in nature, has the concentration of H^+ and OH^- ions

→ The ion density depends on the temperature

→ Thus in an ice slab with upper & lower surfaces at temperature T_1 and T_2 ($T_1 < T_2$), there will be a higher concentration of ions in the lower region.

→ H^+ ions are much lighter and they diffuse much faster all over the volume

→ The lower portion (warmer) has a net negative charge density (OH^-) and hence the upper portion (cooler) has a net positive charge density (H^+)

→ The outer shells of the frozen water droplets coming into contact with hail stones will be relatively cooler and acquire a positive charge.

When the shell splinters, the charge carried by them in the upward direction is positive.

According to the Reynold's theory, the hail packets get negatively charged when impinged upon by the warmer ice crystals.

When the temperature conditions are reversed the charging polarity reverses. This type of phenomenon also occurs in thunder clouds.

Rate of charging of thunder clouds:-

Mason considered thunder clouds to consist of a uniform mixture of positive & negative charges. Due to hail stones and air currents, the charges seperate vertically.

If 'λ' is a factor which depends on the conductivity of the medium, there will be a resistive leakage of charge from the electric field built up and this should be taken into account for cloud charging.

Let

E ⇒ Electric field intensity
V ⇒ Velocity of seperation of Charges.
ρ ⇒ Charge density in the cloud.

Then

Electric field intensity 'E' is given by

$$\frac{dE}{dt} + \lambda E = \rho V \rightarrow (1)$$

$$\frac{dE}{dt} = \rho V - \lambda E \rightarrow (2)$$

$$E = \frac{\rho V}{\lambda}\left[1 - \exp(-\lambda t)\right] \rightarrow (3)$$

At t = 0, E = 0, (There is no seperation initially)

Let

Q ⇒ Seperated charge, A ⇒ Area of the cloud
Q_g ⇒ Generated charge, h ⇒ Height of the Charged Region.
M ⇒ Electric moment of the thunder storm.

$$\rho = \frac{Q_g}{Ah} \rightarrow (4)$$

$$E = \frac{Q_s}{A\varepsilon_0} \rightarrow (5)$$

Lightning stroke

Sub: (4) in (5) we get (6)

$$\frac{Q_s}{A\varepsilon_0} = \frac{Q_g V}{Ah\lambda}\left[1-\exp(-\lambda t)\right]$$

$$Q_g = \frac{Q_s h \lambda}{V\left[1-\exp(-\lambda t)\right]\varepsilon_0} = \frac{M}{V(1-\exp(-\lambda t))} \rightarrow (6)$$

Where

$M = Q_s h$

The average values observed for thunder-clouds are

Time constant $= 1/\lambda = 20s$

Electric moment$(M) = 110$ C-km.

Time for the first lightning flash to appear, $t = 20s$

Velocity of seperation of charges, $V = 10$ to 20 m/s

Sub the values in eq (6) we get

$$Q_g = \frac{20{,}000}{V} C \Rightarrow \frac{20{,}000}{20} = 1000C$$

$Q_g = 1000C$ for $V = 20$ m/s.

Mechanism of Lightning strokes :-

The Mechanism of lightning strokes to ground involve the following

→ The breakdown of virgin air column between the cloud & earth by the stepped leader as it progresses from the cloud to earth, lowering down the negative charge on the base of the cloud.

→ Subsequent passage of a large amount of positive charge from the earth to the cloud through the conducting channel produced by the stepped leader

The sequence of events in a lightning discharge is as follows

Propagation of stepped leader :-

→ Critical breakdown voltage = 10 kV/cm

(7)

Mechanism of Lightning Strokes :-

→ Breakdown of virgin air column between the Cloud & earth by the stepped leader as it progresses from the cloud to earth, lowering down the negative charge on the base of the cloud.

→ Subsequent passage of a large amount of positive charge from the earth to the cloud through the conducting channel produced by the stepped leader

Sequence of events in a lightning discharge

1. Propagation of stepped leader
2. Return stroke
3. Multiple strokes
4. Return stroke current.

Propagation of Stepped leader :-

→ Critical breakdown voltage is 10 kV/cm in a cloud region occupied by the water droplets. 30kV/cm in air without water droplets.

→ When the field at some point in a charge concentrated cloud exceeds 10 kV/cm, an electric streamer with plasma starts towards the ground with a velocity of about 1/10 times that of a light.

→ A downward streamer towards the ground is formed.

→ Negative charges on the base of the cloud are lowered down the streamer.

→ The streamer can progress only about 50m (or) so towards the ground before coming to halt and emitting a bright flash of light at its head.

→ This is due the fact that some of the positive ions produced in the streamer recombine with negative charges.

→ This tends to reduce the conductivity of the channel to such an extent that the electrons at the head of the streamer don't obtain the necessary push to proceed on their way after a time.

→ With the channel losing its conductivity, sufficient charges from the cloud cannot be lowered down to keep the electric field stress in front of the avalanche to a value, which ensures the progress of the streamer.

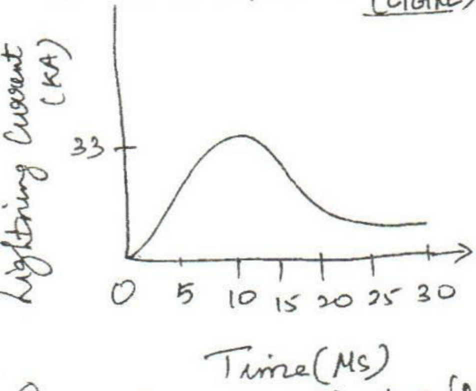
6

CHARACTERISTICS OF LIGHTNING STROKES

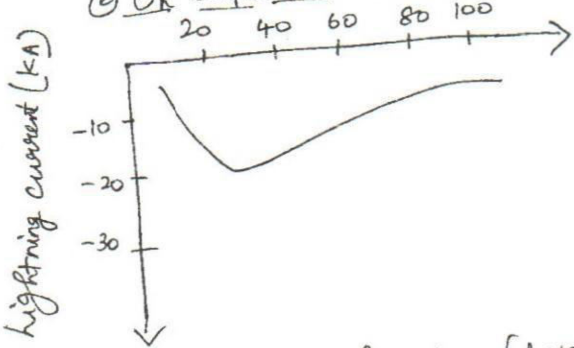
The parameters & Characteristics of lightning include

1. Amplitude of the currents
2. Rate of rise
3. Probability distribution of the Rate of rise
4. Waveshapes of the lightning voltage & current

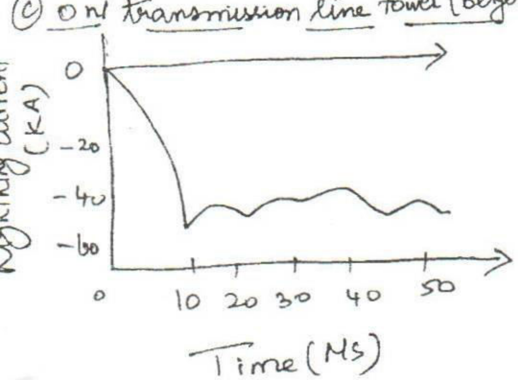
(a) To a Capacitive Balloon (CIGRE)



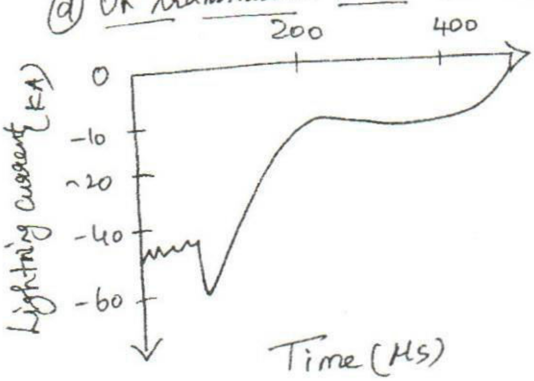
(b) On empire state building (Mc Eachron)



(c) on transmission line tower (Berger)



(d) On transmission line tower (Berger)



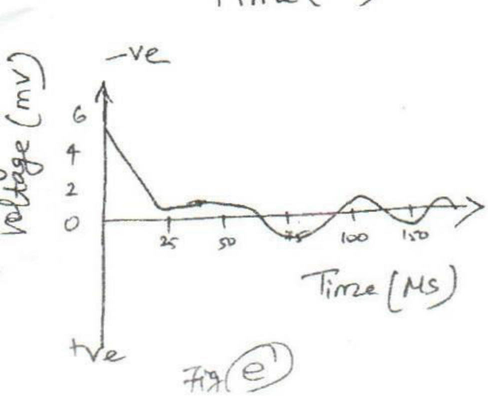


Fig (e)

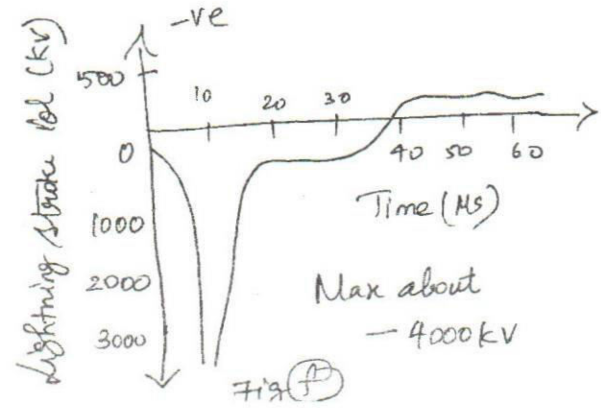


Fig (f)

The fig ⓐ, ⓑ ⓒ & ⓓ indicates the typical lightning current oscillograms.

Fig ⓔ & ⓕ indicates the typical lightning stroke Voltage on a transmission line without ground wire.

The lightning current oscillograms indicate

→ Initial high current portion has short front times upto 10 μs.

→ The high current peak lasts for some tens of μs followed by the duration low current position for several milliseconds.

Lightning currents are measured

→ Directly from high towers or buildings

→ From the transmission tower legs

Other Important Characteristics :-

→ Time (or) peak value

→ Its rate of rise

Specification Of lightning stroke currents :- (5)

→ Peak amplitude > 100 KA

→ Rate of rise 7.5 KA/ms to 25 KA/ms

→ Duration Of stroke Currents 30 μs.

Specification & of lightning stroke voltages (9)

→ Peak amplitude (max) - 5000KV in transmission line

→ Front time - 2 to 10 μs

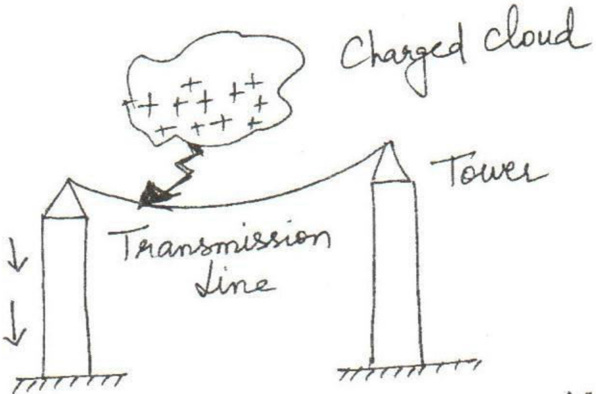
→ Tail time - 20 to 100 μs

→ Rate of rise of voltage - 1 MV/μs.

Classification of lightning strokes on transmission line

(1) Direct strokes (2) Induced strokes

Direct Lightning strokes



→ When the thunder cloud directly discharges onto a transmission line tower (or) line wires it is called direct stroke

→ Most severe form of the stroke

→ For bulk of transmission systems, the direct strokes are rare

Induced Lightning Stroke :-

When the thunderstorm generates, negative charge at its ground end, the earth objects (transmission line, towers) develop induced positive charge.

Normally the line are unaffected because they are insulated by string insulators

However because of high field gradients involved, the positive charges leave from the tower along the insulator surfaces of the line conductors. This process may take a quite long time (100s of sec).

The transmission line & the ground will act as a huge cap charged with a positive charge & hence over voltages occur due to induced charges. This would result in a stroke & hence named as "Induced lightning stroke".

Back Flashover

When a direct lightning stroke occurs on a tower, the tower has to carry huge impulse currents.

If the tower footing resistance is considerable, the potential of the tower rises to a large value, steeply wrt the line & consequently a flashover may take place along the insulator strings.

(4)

FACTORS CONTRIBUTING TO GOOD LINE DESIGN:-

In order to reduce the hazard that lightning poses to power system, Certain factors that determine the line performance must be understood.

→ The Objective of good line design is to reduce the number of Outages Caused by lightning.

→ First we try to minimize the effects of those strokes that do terminate on the system.

→ Before that the incidence of the strokes to the system to be minimum.

→ Minimize the effects of those strokes that do terminate on the system.

→ Lightning problems Can be eliminated if all transmission Was through tunnels atleast 20ft under the ground.

→ Tall towers are more Vulnerable than low goal post-like structures. In order to prevent the lightning, some adequate Clearances must be provided

→ High ground Impedence (OR) tower footing resistance are to be avoided.

→ High Surge impedence in ground wires, tower structures are to be avoided.

PROTECTION AGAINST LIGHTNING TRANSIENTS.

Protection offered by ground wires:-

Ground wire :-

It is a conductor run parallel to the main Conductor of the transmission line supported on the same tower & earthed at every equally & regularly spaced towers. It is run above the main conductor of the line.

The ground wire shields the transmission line conductor from induced charges from clouds as well as from a lightning discharge.

Function of ground wires:-

(1) Ground wire system can dramatically reduce the number of outages.

(2) First function of ground wires is to shield the phase conductors.

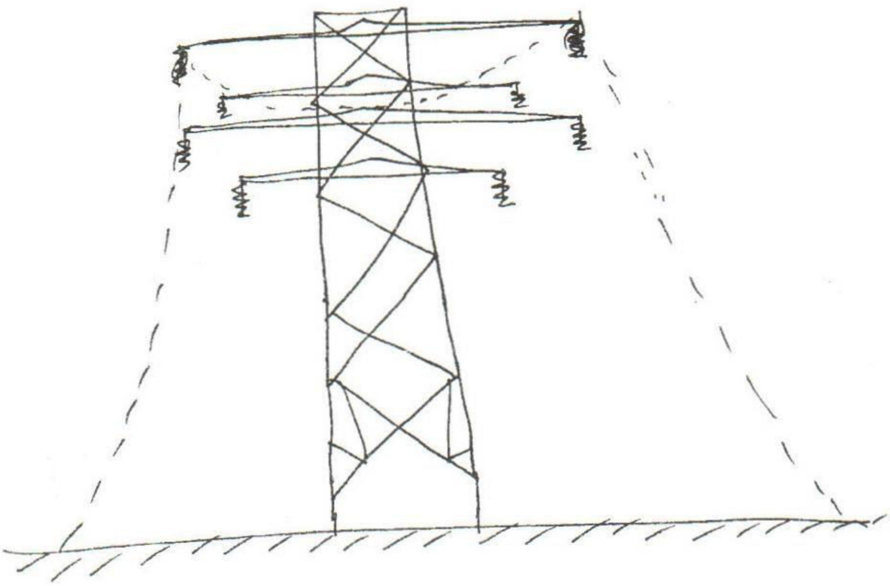
(3) Serve the line of those conductors as the termination of the lightning stroke.

(4) The degree of protection offered depends upon the disposition of ground wires wrt the conductors. (3)

(11)

Lacey states that a ground can be regarded as providing adequate protection to any conductor lying below a quarter circle drawn with its center at the height of the ground wire & within its radius equal to the height of the ground wire above the ground.

If '2' or more such wires are provided, the vulnerable area between two adjacent wires can be taken as a semicircle having as its diameter, a line connecting the '2' ground wires.



Protection afford by ground wires

Mechanism of lightning protection in the transmission line :-

If a positively charged cloud is assumed to be above the line, it induces a negative charge on the portion below it of the transmission line, with the ground wire present.

Both the ground wire & line conductor get the induced charge.

If the ground wire is earthed at regular intervals and as such the induced charge is drained to the earth, only the potential difference between the ground wire & the cloud

Between the ground wire and the transmission line will be the inverse ratio of their respective capacitance

As the ground wire is nearer to the line wire, the induced charge on it will be much less and hence the potential rise will be quite small.

The effective protection (or) shielding given by the ground wire depends on the

→ Height of the ground wire above the ground
→ Protection (or) shielding angle "θ" usually 30°. (2)

Mrs. Lavanya B.

UNIT-IV

TRAVELLING WAVES ON TRANSMISSION LINE COMPUTATION OF TRANSIENTS.

On an electrical transmission line, the voltages, current, power and energy flow from the source to a load located at a distance "L", propagating as electromagnetic waves with a finite velocity.

It takes a short time for the load to receive the power. This gives the concept of a wave travelling on the line which has distributed line parameters r, l, g, c per unit length.

The current flow is governed mainly by

→ Load Impedence

→ Line Charging current at power frequency

→ voltage.

If the Load Impedence doesn't match with the line Impedence, some of the energy transmitted by the source is not absorbed by the load and is reflected back to the source.

At every point on the line, there are '2' waves present and the resulting voltage (or) current is equal to the sum of the transmitted and reflected quantities. These waves are called travelling waves.

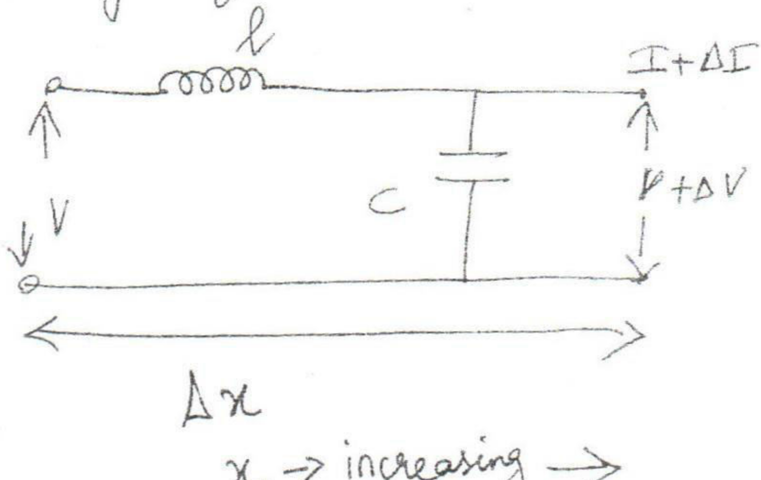
Any disturbance on a transmission line (or) system such as sudden opening (or) closing of line, a short circ or a fault results in the development of overvoltages or overcurrent at that point.

This disturbance propagates as a travelling wave to the ends of the line (or) to a termination, such as a substation.

TRANSIENT RESPONSE OF SYSTEMS WITH SERIES AND SHUNT DISTRIBUTED LINES:-

Consider a typical 2-wire transmission line along with the distributed electrical elements r, l, g and c.

The propagation of any travelling wave, say a voltage wave can be analysed by considering an elemental length of the line Δx



Let $l \Rightarrow$ Line Inductance H/m length

$c \Rightarrow$ Capacitance F/m length

$\Delta x \Rightarrow$ Elementary length of the line at a distanc "x" from the origin.

$l\Delta x \Rightarrow$ Inductance

$c\Delta x \Rightarrow$ Capacitance

Voltage $\Delta V = -l \cdot \Delta x \cdot \frac{\delta I}{\delta t} \rightarrow (1)$

Current $\Delta I = -c \cdot \Delta x \cdot \frac{\delta V}{\delta t} \rightarrow (2)$

$(1) \Rightarrow \frac{\Delta V}{\Delta x} = -l \cdot \frac{\delta I}{\delta t}$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \lim_{\Delta x \to 0} -l \cdot \frac{\delta I}{\delta t}$$

$$\boxed{\frac{\delta V}{\delta x} = -l \cdot \frac{\delta I}{\delta t}} \rightarrow (3)$$

Also

$(2) \Rightarrow \frac{\Delta I}{\Delta x} = -c \cdot \frac{\delta V}{\delta t}$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = \lim_{\Delta x \to 0} -c \cdot \frac{\delta V}{\delta t}$$

$$\boxed{\frac{\delta I}{\delta x} = -c \cdot \frac{\delta V}{\delta t}} \rightarrow (4)$$

Diff eq (3) wrt 'x'

$$\frac{\delta^2 V}{\delta x^2} = -l \cdot \frac{\delta^2 I}{\delta x \, \delta t} \rightarrow (5)$$

Diff eq (4) wrt 't'

$$\frac{\delta^2 I}{\delta t \, \delta x} = -c \cdot \frac{\delta^2 V}{\delta t^2} \rightarrow (6)$$

from (5) & (6) (Sub (6) in (5))

$$\frac{\delta^2 V}{\delta x^2} = -l\left(-c \cdot \frac{\delta^2 V}{\delta t^2}\right)$$

$$\frac{\delta^2 V}{\delta x^2} = +lc \cdot \frac{\delta^2 V}{\delta t^2}$$

$$\frac{\delta^2 V}{\delta t^2} = \frac{1}{lc} \cdot \frac{\delta^2 V}{\delta x^2}$$

Here $V^2 = 1/lc$

$$V = \frac{1}{\sqrt{lc}}$$

$$\boxed{\frac{\delta^2 V}{\delta t^2} = V^2 \cdot \frac{\delta^2 V}{\delta x^2}} \rightarrow (7)$$

(5)

The general solution of the voltage wave eqn is given by

$$V(x,t) = f_1(x+vt) + f_2(x-vt) \rightarrow (9)$$

Diff eq (9) wrt "x" (2 times)

$$\frac{\delta V}{\delta x} = f_1'(x+vt) + f_2'(x-vt)$$

$$\frac{\delta^2 V}{\delta x^2} = f_1''(x+vt) + f_2''(x-vt) \rightarrow (10)$$

Diff eq (9) wrt "t"

$$\frac{\delta V}{\delta t} = f_1'(x+vt)(v) + f_2'(x-vt)(-v)$$

$$\frac{\delta^2 V}{\delta t^2} = f_1''(x+vt)(v^2) + f_2''(x-vt)(v^2)$$

$$\boxed{\frac{\delta^2 V}{\delta t^2} = v^2\left[\frac{\delta^2 V}{\delta x^2}\right]}$$

Which satisfies the eq (7)

3

Physical Significance of the Solution of equation

Any solution of the form $f(x \pm vt)$ represents a travelling wave, because for any value of "t" a Corresponding value of "x" can be found such that $f_1(x \pm vt)$ has a Constant value.

The Voltage distribution has moved intact a distance vt_1 in the direction of negative "x".

For $f_2(x_2 - vt)$, represents a travelling wave moving in the direction of positive "x"

We Know

$$\frac{\delta I}{\delta t} = -\frac{1}{l}\frac{\delta V}{\delta x}$$

$$\frac{\delta I}{\delta t} = -\frac{1}{l}\left[f_1'(x+vt) + f_2'(x-vt) \right]$$

Integrating wrt "t" we get

$$I = -\frac{1}{l}\left[f_1\frac{(x+vt)}{v} + f_2\frac{(x-vt)}{-v} \right]$$

$$= -\frac{1}{lv}\left[f_1(x+vt) - f_2(x-vt)\right]$$

$$= -\frac{1}{lv}\left[f_1(x+vt) - f_2(x-vt)\right]$$

$$= \frac{1}{lv}\left[f_2(x-vt) - f_1(x+vt)\right]$$

Here $v = 1/\sqrt{lc}$

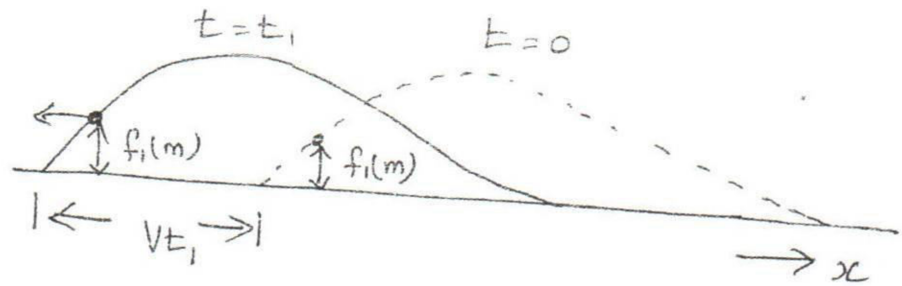
$$I(x,t) = \frac{1}{l \cdot \frac{1}{\sqrt{lc}}}\left[f_2(x-vt) - f_1(x+vt)\right]$$

$$= \frac{\sqrt{l} \cdot \sqrt{c}}{\sqrt{l} \cdot \sqrt{l}}\left[f_2(x-vt) - f_1(x+vt)\right]$$

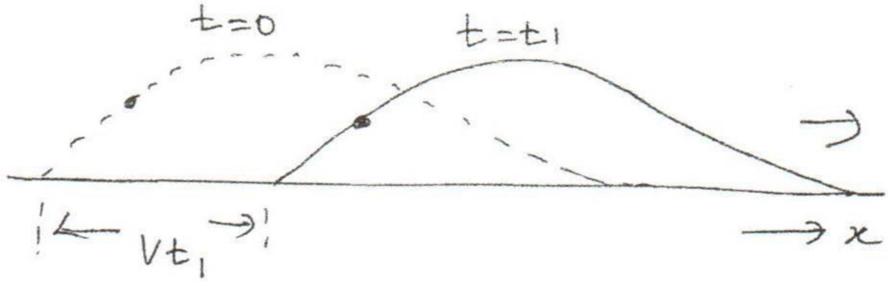
$$= \frac{1}{\sqrt{l/c}}\left[f_2(x-vt) - f_1(x+vt)\right]$$

$$\boxed{I(x,t) = \frac{1}{Z_0}\left[V_f - V_r\right]}$$

(4)



(a) $f_1(x + vt)$



(b) $f_2(x - vt)$

Propagation of wave.

Where $\sqrt{L/C}$ is Surge Impedence (or) Characteristic Impedence 'Z_0' of a lossless line and is a pure resistance.

$V_f \Rightarrow$ Forward voltage waves

$V_r \Rightarrow$ Backward voltage waves

Forward current wave $I_f = \frac{1}{Z_0}$ (Forward Voltage wave V_f)

Backward current wave $I_f = \frac{1}{Z_0}$ (Backward voltage wave V_r)

On loss-free transmission lines, current & voltage waves have the same shape being related by the characteristic impedence of the line and they travel undistorted.

The value of 'Zo' for overhead transmission line

⇒ 350-400 Ohms

Underground cables ⇒ 50-60 Ohms

If the losses are considered in the line the waves will suffer both attenuation & distortion while travelling along the line.

TRAVELLING WAVE PARAMETERS ON TRANSMISSION LINES:-

Consider a '2' wire transmission line along with the distributed electrical elements r, l, g, c. the voltage drop in the positive 'x' direction is given as

$$\frac{\delta V}{\delta x} = r + l \cdot \frac{\delta I}{\delta t}$$

Taking L.T w.r.t 'time' t.

$$\frac{\delta V}{\delta x} = (r + ls) I$$ (5)

$$\frac{\delta V}{\delta x} = Z I \rightarrow (1)$$

Where

$Z = r + ls$

$Y = g + cs$

and $$\frac{\delta I}{\delta x} = (g + cs) V$$

$$= Y V \rightarrow (2)$$

Diff eq (1) wrt "x"

$$\frac{\delta^2 V}{\delta x^2} = Z \cdot \frac{\delta I}{\delta x} \rightarrow (3)$$

Sub (2) in (3) we get

$$\frac{\delta^2 V}{\delta x^2} = Z \cdot (Y V) \Rightarrow P^2 \cdot V$$ (Where $P^2 = YZ$) Propagation Constant

Diff eq (2) wrt "x"

$$\frac{\delta^2 I}{\delta x^2} = Y \cdot \frac{\delta V}{\delta x}$$

$$= Y Z \cdot I \Rightarrow P^2 I$$ ($YZ = P^2$ Propagation Constant)

Here $P^2 = YZ = (g + cs)(r + ls)$

$$= rg + rcs + gls + cls^2$$

$$P^2 = (g+cs)(r+ls)$$

$$P = \sqrt{(g+cs)(r+ls)} = \left[(g+cs)(r+ls)\right]^{1/2}$$

$$P = \left[c\left(g/c+s\right)\ l\left(r/l+s\right)\right]^{1/2}$$

$$P = (lc)^{1/2}\left[\left(s+r/l\right)^{1/2}\left(s+g/c\right)^{1/2}\right]$$

$$V^2 = \frac{1}{lc} \Rightarrow lc = \frac{1}{V^2} \Rightarrow \boxed{\sqrt{lc} = 1/V}$$

$$\boxed{P = \frac{1}{V}\left[\left(s+r/l\right)^{1/2}\left(s+g/c\right)^{1/2}\right]}$$

Here $\alpha - \beta = g/c$, $\alpha + \beta = r/l$
↳ (4) ↳ (5)

Eq (4) + (5) ⇒

$$2\alpha = g/c + r/l \Rightarrow \alpha = \frac{1}{2}\left[g/c + r/l\right]$$

Eq (4) − (5)

$$P^2 = (g+cs)(r+ls)$$

$$P = \sqrt{(g+cs)(r+ls)} = \left[(g+cs)(r+ls)\right]^{1/2}$$

$$P = \left[c\left(\frac{g}{c}+s\right) l\left(\frac{r}{l}+s\right)\right]^{1/2}$$

$$P = (lc)^{1/2}\left[\left(s+\frac{r}{l}\right)^{1/2}\left(s+g/c\right)^{1/2}\right]$$

$$V^2 = \frac{1}{lc} \Rightarrow lc = \frac{1}{V^2} \Rightarrow \boxed{\sqrt{lc} = 1/V}$$

$$\boxed{P = \frac{1}{V}\left[\left(s+\frac{r}{l}\right)^{1/2}\left(s+g/c\right)^{1/2}\right]}$$

Here $\alpha - \beta = g/c$, → (4) $\quad \alpha + \beta = r/l$ → (5)

Eq (4) + (5) ⇒

$$2\alpha = g/c + r/l \Rightarrow \alpha = \frac{1}{2}\left[g/c + r/l\right]$$

Eq (4) − (5)

(6) $\alpha - \beta - \alpha - \beta = g/c - r/l$

$$-2\beta = g/c - r/l \Rightarrow 2\beta = \frac{r}{l} - g/c$$