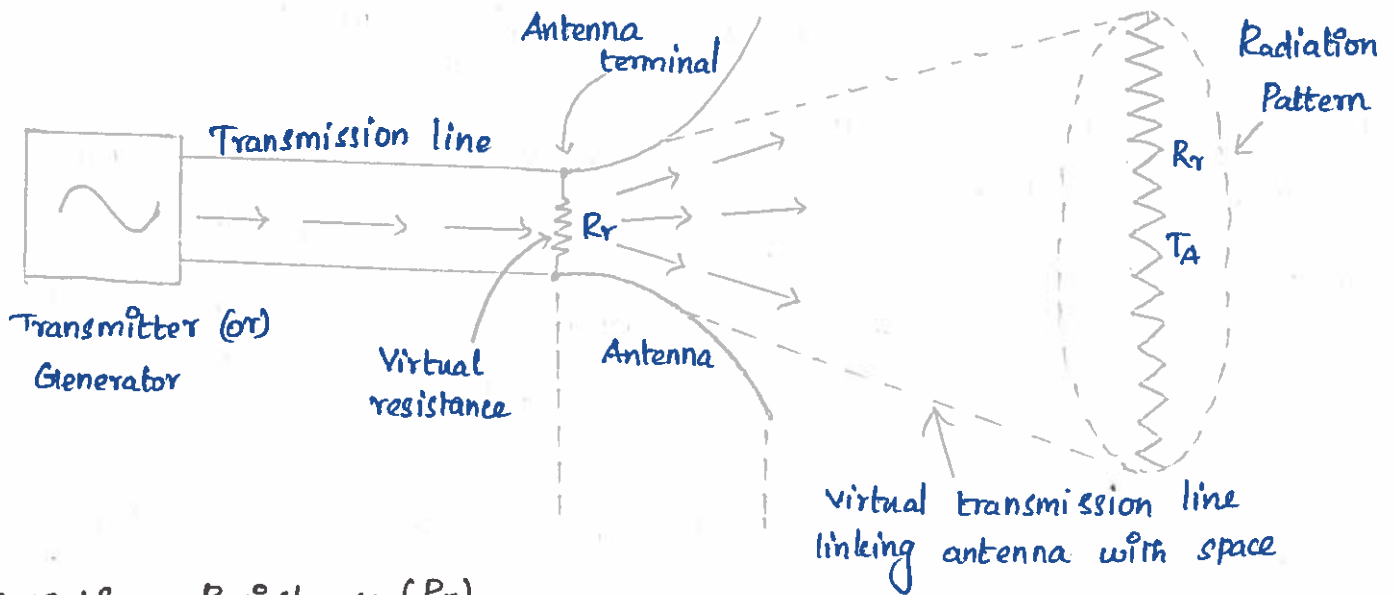


PHYSICAL CONCEPT OF RADIATION

An antenna is an important basic component in the communication system. Basically antennas are the metallic structures designed for radiating and receiving an Electromagnetic (EM) energy in an effective manner which is used for conveying the information.



(i) Radiation Resistance ( $R_r$ )

The antenna appears as a terminal resistance to the transmission line which is commonly called as radiation resistance and denoted by ' $R_r$ '. It is a resistance coupled from space to the antenna terminals.

(ii) Antenna Temperature ( $T_A$ )

The receiving antenna receives two types of radiations namely, passive radiations which are the reflections from any obstacle distant objects, while active radiations from other antennas. These radiation increases the apparent temperature of the radiation resistance, which is related to the temperature of the distant objects.

The basic equation of the radiation can be simply expressed as,

$$\mathcal{I}l = Qv (A - m/s)$$

Where,  $\mathcal{I}$  - Time changing current ( $As^{-1}$ )

$l$  - Length of the current element (m)

$Q$  - charge (c)

$v$  - time change of velocity which equals the acceleration of the charge ( $ms^{-2}$ ).

Radiation Pattern: Antenna Pattern.

\* Any antenna is characterized by its radiation pattern which is a mathematical or graphical representation of the radiation properties of an antenna as a function of space coordinates in a desired direction. This is called as the radiation pattern.

\* The total radiation field strength is expressed as,

$$E = \sqrt{E_{\theta}^2 + E_{\phi}^2}$$

where,  $E_{\theta}$  - amplitude of  $\theta$  component

$E_{\phi}$  - amplitude of  $\phi$  component

\* There are two basic types of radiation pattern:

(i) If the radiation of an antenna is expressed in terms of field strength ( $E$ ) in  $V/m$ , then the graphical representation is called field strength pattern or field radiation pattern.

(ii) Similarly, if the radiation of an antenna is expressed in terms of the power per unit solid angle, then the graphical representation is called Power radiation pattern or simply power pattern.

(2)

\* Field pattern typically represents a plot of the magnitude of an electric or magnetic field as a function of the angular space.

\* Power pattern (in linear scale) typically represents a plot of the square of the magnitude of an electric or magnetic field as a function of the angular space.

\* Power pattern (in dB) represents the magnitude of an electric or magnetic field in decibels, as a function of the angular space.

Normalized Field Pattern:

It is obtained, when dividing a field component of radiation pattern by its maximum value. It is a dimensionless number with a maximum value of unity.

$$E_{\theta}(\theta, \phi)_n = \frac{E_{\theta}(\theta, \phi)}{E_{\theta}(\theta, \phi)_{\max}} \text{ (dimensionless)}$$

Normalized Power pattern:

It is obtained, when dividing a power component of radiation pattern by its maximum value as a function of angle. It is a dimensionless number with the maximum value of unity.

$$P_n(\theta, \phi)_n = \frac{S_{\theta}(\theta, \phi)}{S_{\theta}(\theta, \phi)_{\max}} \text{ (dimensionless)}$$

where

$$\begin{aligned} S(\theta, \phi) &= \text{Poynting vector} \\ &= \frac{E_{\theta}^2(\theta, \phi) + E_{\phi}^2(\theta, \phi)}{Z_0} \text{ Wm}^{-2} \end{aligned}$$

$$Z_0 = \text{Intrinsic impedance of space} = 376.7 \Omega$$

## Antenna Beamwidth.

\* It is the measure of the directivity of an antenna and it is defined as, "the angular separation, that is, angular width in degrees between the two identical points on the opposite side of the main radiation pattern".

\* In an antenna pattern, there are a number of beam widths possible, but two of the most widely used beam-widths are:

(i) Half - Power Beam width (HPBW)

(ii) First - Null Beam Width (FNBW)

### (1) Half - Power Beam Width (HPBW)

\* HPBW is an angular width in degrees, measured on the major lobe radiation pattern between points where the radiated power has fallen to half of its maximum value, which is called half power points.

\* HPBW is also known as "3-dB beamwidth" because at half power points, the power is 3-dB down the maximum power value of the major lobe.

### (2) First - Null Beam - Width (FNBW)

\* FNBW is defined as "the angular width between the first nulls or first side lobes, which has a beamwidth of 10 dB down from the power maximum of the main lobe".

\* FNBW is also known as 10-dB beamwidth and it is usually used to approximate the HPBW as,

$$\text{HPBW} = \frac{\text{FNBW}}{2}$$



## Radiation Pattern Lobes:-

\* Different parts of radiation pattern are referred to as "lobes". A radiation lobe is a 3 dimensional portion of strong fields which is surrounded by a weak field. It is the portion of significant field strength in a particular direction.

\* Depends on the field strength, the radiation lobes of an antenna may be classified into four types:

- (i) Major lobe
- (ii) Minor Lobe
- (iii) Side Lobe
- (iv) Back Lobe

### (i) Major Lobe :-

\* This is the radiation lobe containing the maximum radiation in a desired direction, which is also referred to as main lobe or main beam.

\* The maximum power is transmitted to the free space from an antenna only by the major lobes.

### (2) Minor Lobe :-

\* All the lobes other than the main lobe are called minor lobes. It represents the radiation in an undesired direction. The level of a minor lobe is usually expressed as "a ratio of power density in that lobe to that of the major lobe".

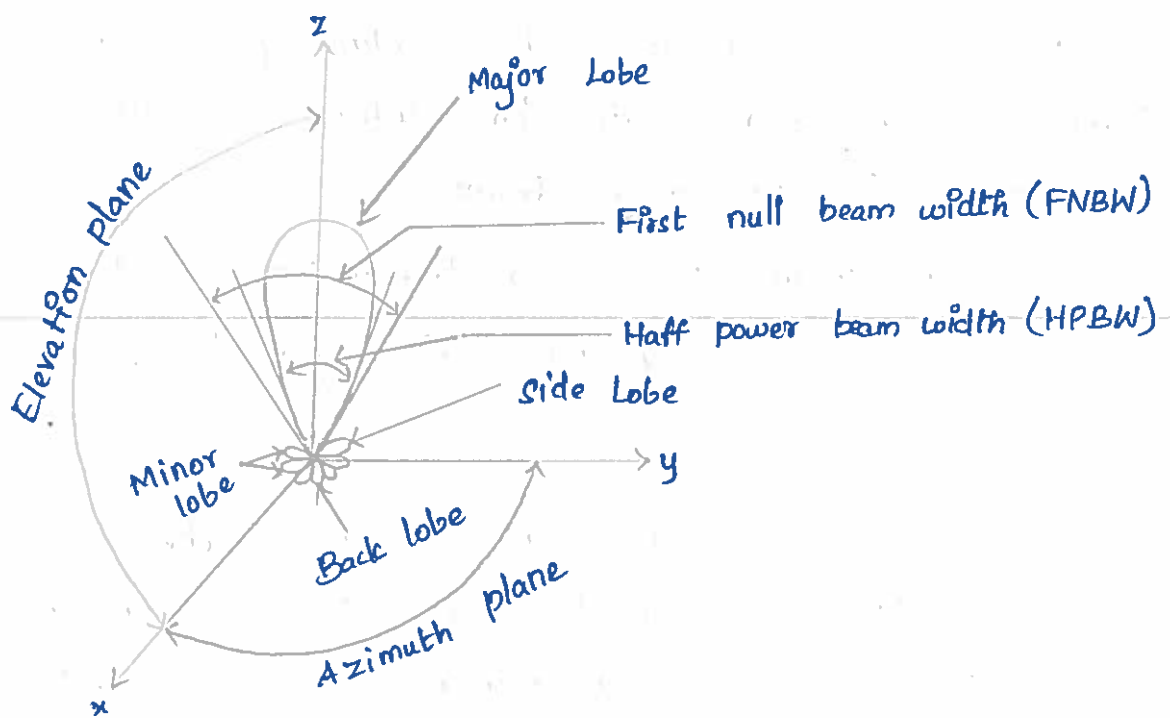
\* In most of the wireless systems, minor lobes are undesired. Hence, a good antenna design should minimize the minor lobes.

(3) Side lobe :

These are the minor lobes adjacent to the main lobe and which are separated by the various nulls. The side lobes are the largest among the minor lobes.

(4) Back Lobe :

This is the minor lobe diametrically opposite to the main lobe. It is the radiation lobe whose axis makes an angle of approximately  $180^\circ$  with the major lobe direction.



Lobes and beamwidths of an antenna radiation pattern.

## NEAR AND FAR-FIELD REGIONS: ANTENNA FIELD ZONES.

The space surrounding an antenna is usually subdivided into three regions.

- (i) Reactive near-field region (or) Antenna region.
- (ii) Radiating near-field (or) Near field (or) Fresnel region.
- (iii) Far-field regions (or) Fraunhofer region.

### Near-Field Region:

\* Near-field region (or) Fresnel region is defined as, "region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna."

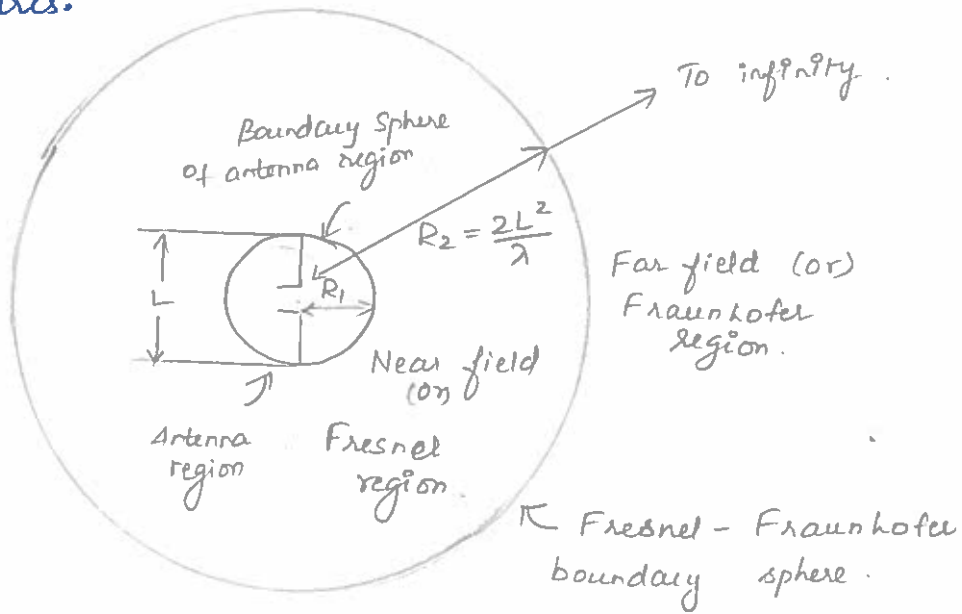
\* The inner boundary is taken to be the distance  $R_1 \geq 0.62 \sqrt{L^3/\lambda}$  (m) and the outer boundary distance is  $R_2 \geq 2L^2/\lambda$  (m).

### Far-field Region:

\* The far field region is a region which is commonly taken to exist at a distance greater than

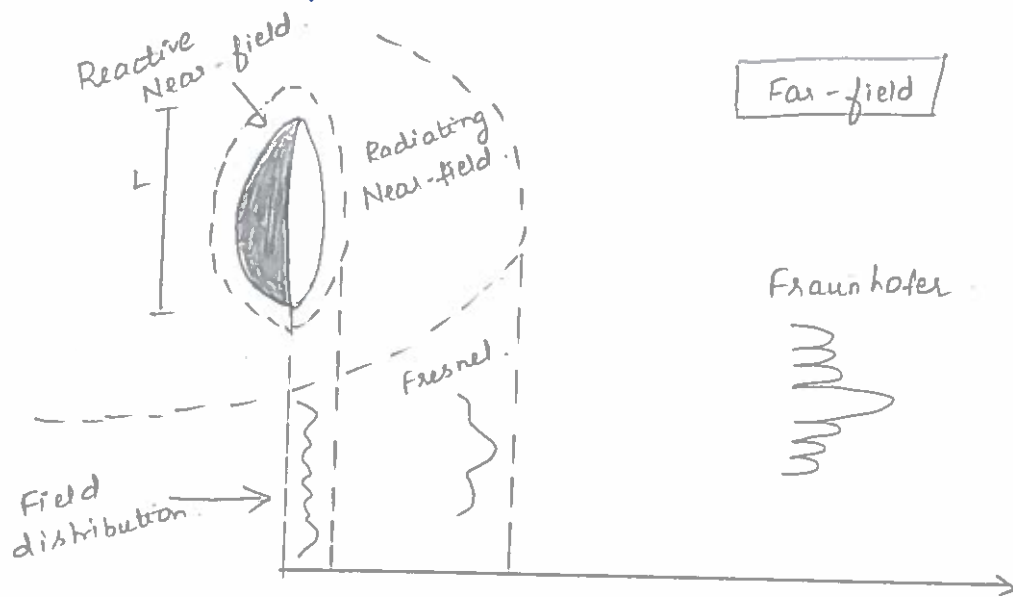
$$R_2 = \frac{2L^2}{\lambda} \text{ (m) from an antenna.}$$

\* The far-field patterns of certain antennas such as multibeam reflector antennas, are sensitive to the variations in phase over their apertures.



### Radiation Pattern:

The radiation pattern of an antenna, as the observation distance is varied from the reactive near field to the far field, changes in shape because of variations of the fields in both magnitude and phase.



5

\* A typical shape changes of radiation pattern from reactive near field toward the far field, with the largest dimension  $L$  as shown in figure.

\* It is apparent that in the reactive near field region the pattern is more spread out and nearly uniform, with slight variations. In the radiating near-field region, the pattern begins to smooth and form lobes.

\* In the far-field region, the pattern is well formed, usually consisting of few minor lobes and one, or more, major lobes.

## FIELD AND POWER RADIATED BY AN ANTENNA:

### Radiated Electric Field:

Consider an antenna located at the origin of a spherical coordinate system. At large distances where the localized near-zone fields are negligible, then the radiated electric field of an arbitrary antenna can then be expressed as,

$$\vec{E}(r, \theta, \phi) = [\hat{\theta} F_{\theta}(\theta, \phi) + \hat{\phi} F_{\phi}(\theta, \phi)] e^{-jk_0 r} \frac{1}{r} \text{ V/m} \rightarrow \textcircled{1}$$

where  $\vec{E}$  - Electric field vector.

$\hat{\theta}$  and  $\hat{\phi}$  - Unit vectors in the spherical coordinate system.

$r$  - Radial distance from an origin.

$k_0 = \frac{2\pi}{\lambda}$  - Free space propagation constant  
with wavelength  $\lambda = c/f$  and

$F_\theta(\theta, \phi)$  and  $F_\phi(\theta, \phi)$  - Antenna pattern function.

\* Equation (1) represents that this electric field propagates in the radial direction with a phase variation of  $e^{-jk_0 r}$  and an amplitude variation with distance of  $1/r$ . This is a TEM wave, so the electric field may be polarized in either the  $\hat{\theta}$  and  $\hat{\phi}$  direction but are not in the radial direction.

### Radiated Magnetic Field:

\* The magnetic fields associated with an electric field of equation (1) can now be expressed as

$$H_\phi = \frac{E_\theta}{\eta_0} \longrightarrow (2a)$$

$$H_\theta = -\frac{E_\phi}{\eta_0} \longrightarrow (2b)$$

where wave impedance of free space,  $\eta_0 = 377\Omega$ .

\* The magnetic field vector also polarized only in the transverse directions and the Poynting Vector for this wave is expressed as,

$$\vec{S} = \vec{E} \times \vec{H}^* \text{ W/m}^2 \longrightarrow (3)$$

\* And the time-averaged Poynting vector is

$$\vec{S}_{avg} = \frac{1}{2} \text{Re} \{ \vec{S} \} = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \} \text{ W/m}^2 \longrightarrow (4)$$

## Far-Field Distance

\* The far-field distance is the distance where the spherical wave front radiated by an antenna becomes a close approximation to an ideal planar phase front of a plane wave.

\* This approximation applies over the radiating aperture of an antenna ~~of an~~ which depends on the maximum dimension ( $L$ ) of the antenna. Then the far-field distance is defined as

$$R_{ff} = \frac{2L^2}{\lambda} \text{ m} \rightarrow (5)$$

\* The above result is derived from the condition that the actual spherical wave front radiated by an antenna departs less than  $\pi/8 = 22.5^\circ$  from a true plane wave front over the maximum extent of an antenna.

\* For electrically small antennas, such as short dipoles and small loops, this result (equation 5) may give a far-field distance that is too small; in the case, a minimum value of  $R_{ff} = 2\lambda$  should be used.



## Radiation Intensity:

\* The radiation intensity gives the variation in radiated power versus position around the antenna. The radiation intensity of the radiated electromagnetic field is expressed as,

$$U(\theta, \phi) = r^2 |\bar{S}_{avg}| = \frac{r^2}{2} \operatorname{Re} \{ E_\theta \hat{\theta} \times H_\phi^* \hat{\phi} + E_\phi \hat{\phi} \times H_\theta^* \hat{\theta} \} \rightarrow (6)$$

By using equations (1), (2) and (4) then equation (6) becomes.

$$U(\theta, \phi) = \frac{r^2}{2\eta_0} [ |E_\theta|^2 + |E_\phi|^2 ] = \frac{1}{2\eta_0} [ |F_\theta|^2 + |F_\phi|^2 ] W \rightarrow (7)$$

\* The units of the radiation intensity are Watts or Watts per unit solid angle, since the radial dependence has been removed.

\* We can find the total power radiated by an antenna by integrating the Poynting vector over the surface of a sphere of radius  $r$  that encloses the antenna which is equivalent to integrating the radiation intensity over a unit sphere:

$$\begin{aligned} P_{rad} &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \bar{S}_{avg} \cdot \hat{r} r^2 \sin\theta d\theta d\phi \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta, \phi) \sin\theta d\theta d\phi \rightarrow (8) \end{aligned}$$

## ANTENNA PATTERN CHARACTERISTICS:-

\* The radiation pattern (or) antenna pattern of an antenna is a plot of the magnitude of the far-zone field strength versus position around the antenna, at a fixed distance from an antenna.

### (i) Lobes:-

\* The pattern may exhibit several distinct lobes, with different maxima in different directions that is different part of the radiation pattern.

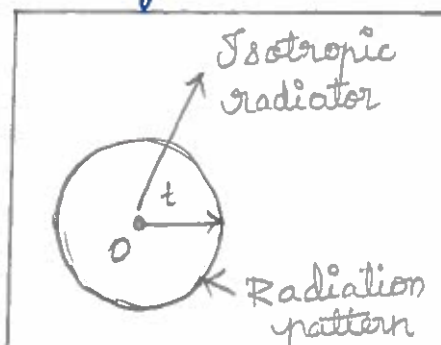
\* The lobe having the maximum value in the desired direction is called main beam, which those lobes at lower levels other than main lobe are called the side lobes.

\* A fundamental property of an antenna is its ability to focus power in a desired direction, not for the other directions. Thus an antenna with a broad main beam can transmit (or) receive power over a wide angular region, while it has a narrow main beam over a small angular region.

### (ii) Isotropic radiator:-

\* An isotropic radiator is a radiator which radiates uniformly in all the directions. It is also called as isotropic source or omni directional radiator or simply unipole.

\* Basically it is a lossless ideal radiator or antenna. Generally, all the practical antennas are compared with the characteristics of isotropic radiator. So it is also called as reference antenna.



## (b) Pencil Beam:

Patterns that have relatively narrow main beams in both planes are known as pencil beam antennas and are useful in applications such as radar and point-to-point radio links.

### (ii) Directivity (D):

\* Another measure of the focussing ability of an antenna is the directivity which is defined as, "the ratio of the maximum radiation intensity in the main beam to average radiation intensity over all space".

$$D = \frac{\text{Maximum Radiation Intensity in the main beam (or) test antenna}}{\text{Radiation Intensity of an Isotropic antenna.}}$$

$$D = \frac{U_{\max}}{U_{\text{avg}}}$$

\* The average radiation intensity is equal to the total power radiated ( $P_{\text{rad}}$ ) by an antenna divided by  $4\pi$ .

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi U_{\max}}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi} \rightarrow \textcircled{1}$$

\* Directivity is a dimensionless ratio of power and it is usually expressed in dB as  $D(\text{dB}) = 10 \log(D)$  and an isotropic element  $D=1$ , or 0 dB.

\* Typical directivities for some common antennas are 2.2 dB for a wire dipole, 7.0 dB for a microstrip patch antenna, 23 dB for a waveguide horn antenna and 35 dB for a parabolic reflector antenna.

Relationship between Directivity and Beamwidth:

$$D \approx \frac{32400}{\theta_1 \theta_2} \rightarrow (2)$$

- \* This approximation works well for antennas with antenna beam patterns.
- \* Here  $\theta_1$  and  $\theta_2$  are the beamwidth in two orthogonal planes of the main beam, in degrees.
- \* This approximation does not work well for the omnidirectional patterns because there is a well-defined main beam in only one plane for such patterns.

(iii) Antenna gain and Efficiency:

\* The gain is an useful measure describes the performance of an antenna which acts as the figure of merit for an antenna. It is closely related to the directivity which is a measure that takes into account an antenna efficiency as well as its directional capabilities.

\* The gain of the transmitting antenna is defined as, "the ability of an antenna to concentrate the radiated power in a given direction", where as for the receiving antenna, "it is an ability of absorbing incident power effectively from the particular radiation direction".

Radiation Efficiency (or) Antenna Efficiency:

The radiation efficiency of an antenna is defined as, "the ratio of the desired output power to the supplied input power".

$$\eta_{\text{rad}} = \frac{P_{\text{rad}}}{P_{\text{in}}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} \quad [ \because P_{\text{in}} = P_{\text{rad}} + P_{\text{loss}} ]$$

$$= \frac{P_{\text{in}} - P_{\text{loss}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}} \rightarrow (1)$$

where  $P_{\text{rad}}$  = Power radiated by the antenna  
 $P_{\text{in}}$  = Power supplied to the input of the antenna.

$P_{\text{loss}}$  = Power lost in the antenna.

\* Other factors that can contribute to an effective loss of transmit power are the impedance mismatch at the input to the antenna and polarization mismatch with the receive antenna.

\* These losses are external to an antenna and could be eliminated by the proper use of matching networks or the proper choice and positioning of the receive antenna.

\* Antenna directivity is a function only of the shape of the radiation pattern which is not affected by losses in an antenna itself. An antenna having a radiation efficiency less than unity will not radiate all of its input power.

\* The relation between antenna gain ( $G$ ) and directivity ( $D$ ) is expressed in terms of antenna efficiency ( $\eta$ ) as,

$$G = \eta_{\text{rad}} D \quad 0 \leq \eta_{\text{rad}} \leq 1 \rightarrow (2)$$

\* In most well designed antennas,  $\eta_{\text{rad}}$  may be close to the unity (100%). In practice,  $G$  is always less than  $D$  ( $G < D$ ) due to Ohmic losses in the antenna.

\* When an antenna efficiency is 100% ( $\eta_{rad}=1$ ), the gain (G) and directivity (D) are used interchangeably. Gain of an antenna is expressed in decibels as,

$$G(\text{dB}) = 10 \log_{10} G \rightarrow (3)$$

#### (iv) Aperture Efficiency and effective area:

\* Aperture antenna means that the antenna has a well-defined aperture area from which radiation occurs. Example: reflector antennas, horn antennas, lens antennas and array antennas.

\* The maximum directivity that can be obtained from an electrically large aperture of area A and it is given as

$$D_{\text{max}} = \frac{4\pi A}{\lambda^2} \rightarrow (1)$$

#### Aperture Efficiency:

\* An aperture efficiency is defined as, "the ratio of the actual directivity of an aperture antenna to the maximum directivity value possible to that of the antenna". Then the directivity of an aperture can be written as.

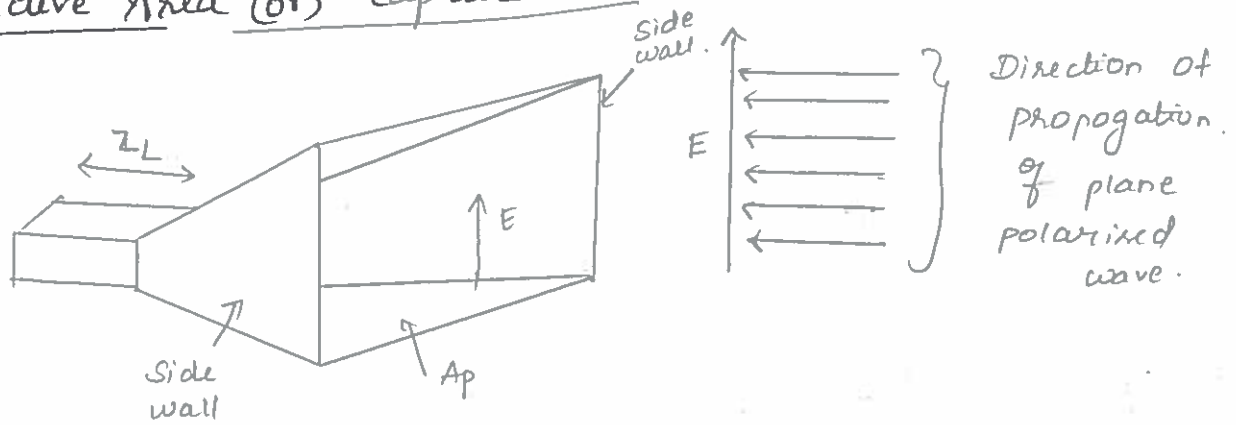
$$D = \eta_{ap} \frac{4\pi A}{\lambda^2} \quad 0 \leq \eta_{ap} \leq 1 \rightarrow (2)$$

\* An antenna efficiency is simply defined for an aperture antenna as, "the ratio of effective area (or) capture area to physical aperture of that antenna".

$$\eta_{ap} = \frac{A_e}{A_p} \quad (\text{dimensionless}) \rightarrow (3)$$



## Effective Area (or) Capture Area:



\* Effective area ( $A_e$ ) is an area over which an antenna extracts power from the incident radio waves. It may be defined as, "the ratio of power received at an antenna load terminal to the Poynting vector (power density) in  $W/m^2$  of an incident wave."

$$A_e = \frac{\text{Power received by the antenna}}{\text{Poynting vector (or) power density of the incident wave}}$$

$$A_e = \frac{P_r}{S_{avg}} \rightarrow (4)$$

where  $P_r$  - Power received in watts.

$S_{avg}$  - Power density [Power flow per sq. meter] or Poynting vector of an incident wave in  $W/m^2$  and

$A_e$  - Effective area in  $m^2$

$$P_r = A_e S_{avg} \rightarrow (5)$$

\* The maximum effective aperture area of an antenna can be related to the directivity of an antenna as,

$$A_e = \frac{D \lambda^2}{4\pi} \rightarrow (6)$$



where  $\lambda$  is the operating wavelength of the antenna and above expression does not include the effect of losses in the antenna. For electrically large aperture antennas, the effective aperture area is often close to the actual physical aperture area.

\* For many other types of antennas, such as dipoles and loops, there is no simple relation between the physical cross-sectional area of an antenna and its effective aperture area.

### (V) Antenna Noise Temperature:

The antenna temperature or antenna noise temperature for a lossless antenna is defined as, "the temperature of a far field region of space and near surroundings which are coupled to the antenna through radiation resistance".

### Radiation Efficiency ( $\eta_{rad}$ ):

It is the ratio of output power to input power of an antenna.

$$\eta_{rad} = \frac{P_o}{P_i} \rightarrow \textcircled{1}$$

\* If a receiving antenna has dissipative loss and its radiation efficiency  $\eta_{rad}$  is less than unity ( $\eta_{rad} < 1$ ). The power available at the terminals of the antenna is reduced by the factor  $\eta_{rad}$  from that intercepted by the antenna.

\* This reduction applies to received noise power as well as received signal power, so that the noise temperature ( $T_A$ ) of an antenna will be reduced from the brightness temperature ( $T_b$ ) by the factor  $\eta_{rad}$ .

\* The thermal noise will be generated internally by resistive losses in the antenna which will increase the noise temperature of an antenna.

\* In terms of noise power, a lossy antenna can be modeled as a lossless antenna and an attenuator having a power loss factor of  $L = 1/\eta_{rad}$ . Then, equivalent noise temperature of an attenuator, we can find the resulting noise temperature seen at the antenna terminals as,

$$T_A = \frac{T_b}{L} + \frac{(L-1)}{L} T_p = \eta_{rad} T_b + (1 - \eta_{rad}) T_p \rightarrow (1)$$

where

$T_A$  - Antenna noise temperature (K).

$T_b$  - Brightness temperature (K) and

$T_p$  - Antenna physical temperature (K).

\* The antenna noise temperature ( $T_A$ ) is a combination of the external brightness temperature seen by the antenna and the thermal noise generated by the antenna.

\* For lossless antenna,  $\eta_{rad} = 1$ , then eqn (1) reduces to  $T_A = T_b$ . If the radiation efficiency is zero ( $\eta_{rad} = 0$ ), it means that the antenna appears as a matched load and does not see any external background noise, then eqn (1) reduces to  $T_A = T_p$ , due to the thermal noise generated by the losses.

## Thermal Noise:

Assuming no losses or other contributions between the antenna and the receiver then the noise power transferred to the receiver is given by

$$P_r = k T_A \Delta f \rightarrow (2)$$

where  $P_r$  - Antenna noise Power (W)

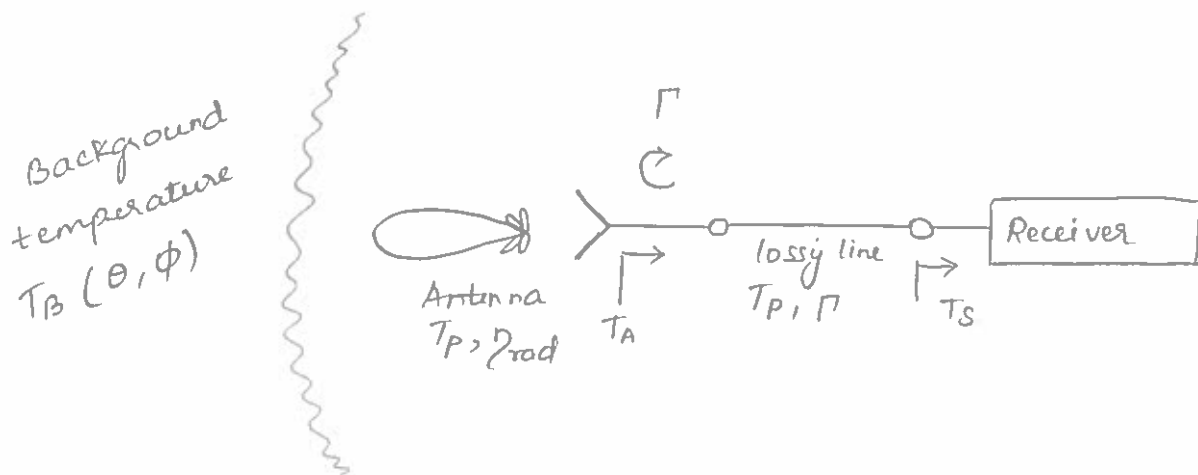
$k$  - Boltzmann's Constant ( $1.38 \times 10^{-23}$  J/K)

$T_A$  - Antenna temperature (K)

$\Delta f$  - Bandwidth (Hz).

## Overall System Noise Temperature:

\* The more general problem of a receiver is when it is connected through a lossy transmission line to an antenna and viewing as a background noise temperature distribution  $T_B$  when an impedance mismatch exists between the antenna and the line which can be represented by the system shown below.



\* Here, the antenna is assumed to have a radiation efficiency  $\eta_{rad}$  and the connecting transmission line has a power loss factor of  $L \geq 1$ , with both at physical temperature  $T_p$ .

\* The effect of an impedance mismatch between an antenna and the transmission line is represented by the reflection coefficient  $\Gamma$ .

\* The equivalent noise temperature seen at the output terminals of the transmission line consists of three contributions:

(i) Noise power from an antenna due to internal noise and the background brightness temperature.

(ii) Noise power generated from the lossy line in the forward direction.

(iii) Noise power generated by the lossy line in the backward direction and reflected from an antenna mismatch toward the receiver.

\* Due to above noise contributions, the noise due to the antenna is given by equation (1) is reduced by the loss factor of the line,  $1/L$  and the reflection mismatch factor,  $(1 - |\Gamma|^2)$ .

\* The forward noise power from the lossy line is reduced by the loss factor,  $1/L$ . The contribution from the lossy line reflected from the mismatched antenna is reduced by the power reflection coefficient,  $|\Gamma|^2$  and the loss factor  $1/L^2$ .

\* Therefore, the overall system noise temperature seen at an input to the receiver is given by

$$T_s = \frac{T_A}{L} (1 - |\Gamma|^2) + (L-1) \frac{T_P}{L} + (L-1) \frac{T_P}{L^2} |\Gamma|^2 \rightarrow (3)$$

By substituting eqn (1) in eqn (3) we get

$$T_s = \frac{(1 - |\Gamma|^2)}{L} [\eta_{\text{rad}} T_b + (1 - \eta_{\text{rad}}) T_P] + \frac{(L-1)}{L} \left(1 + \frac{|\Gamma|^2}{L}\right) T_P \rightarrow (4)$$

\* For a lossless line ( $L=1$ ), the effect of an antenna mismatch is to reduce the system noise temperature by the factor  $(1 - |\Gamma|^2)$  and the received signal power will be reduced by the same amount. For a matched antenna ( $\Gamma=0$ ) equation (4) reduces to

$$T_s = \frac{1}{L} [\eta_{\text{rad}} T_b + (1 - \eta_{\text{rad}}) T_P] + \frac{L-1}{L} T_P \rightarrow (5)$$

\* Radiation efficiency accounts for the resistive losses, and thus involves the generation of thermal noise but aperture efficiency does not.

\* Aperture efficiency applies to the loss of directivity in aperture antennas and by itself does not lead to any additional effect on noise temperature that would not be included through the pattern of an antenna.

(vi) Gain - Antenna Temperature Ratio:  $G/T$ :

\* Another useful figure of merit for receive antennas is the  $G/T$  ratio and it is defined as,

$$G/T \text{ (dB)} = 10 \log \frac{G}{T_A} \text{ dB/K} \rightarrow \textcircled{6}$$

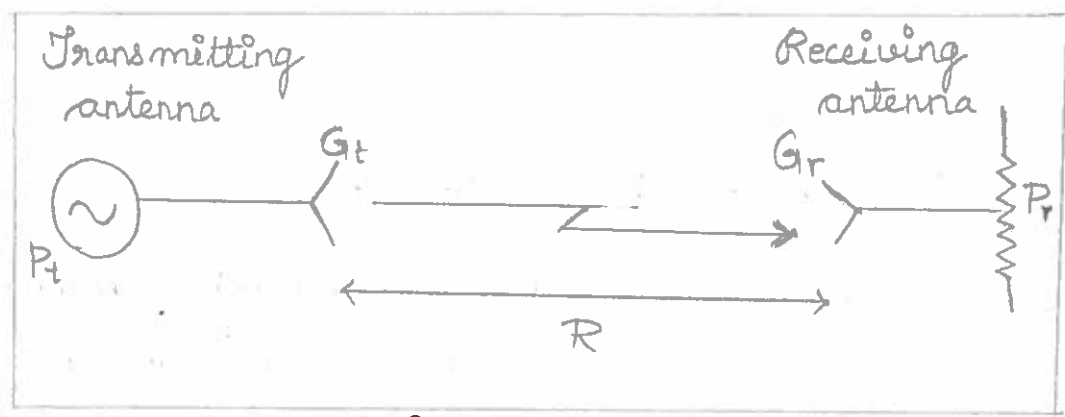
\* where  $G$  is the gain of the antenna and  $T_A$  is the antenna noise temperature.

\* This quantity is important because, the Signal-to-Noise Ratio (SNR) at an input to a receiver is proportional to  $G/T_A$ . The ratio  $G/T$  can often be maximized by increasing the gain of an antenna and usually minimizes the reception of noise from hot sources at the low elevation angles.

\* Higher gain requires a larger and more expensive antenna, and high gain may not be desirable for applications requiring omnidirectional coverage (eg. cellular telephones or mobile data networks) so often a compromise must be made.



# FRIIS TRANSMISSION EQUATION:



A Basic radio system

Where,

$P_t$  - Transmit power

$G_t$  - Transmit antenna gain

$G_r$  - Receive antenna gain

$P_r$  - Received Power which is delivered to a matched load

$R$  - Distance between transmit and receive antennas.

\* The power density radiated by an isotropic antenna at a distance  $R$  is given by

$$S_{avg} = \frac{P_t}{4\pi R^2} \text{ W/m}^2 \longrightarrow \textcircled{1}$$

\* The power is distributed isotropically, and the area of sphere is  $4\pi R^2$ . The general expression for the power density radiated by an arbitrary transmit antenna is,

$$S_{avg} = \frac{G_t P_t}{4\pi R^2} \text{ W/m}^2 \longrightarrow \textcircled{2}$$

\* If this power density is incident on the receive antenna of effective aperture  $A_{er}$  is

$$P_r = A_{er} S_{avg} = \frac{G_t P_t A_{er}}{4\pi R^2} \text{ W} \longrightarrow \textcircled{3}$$

\* The gain of the transmitting antenna can then be expressed as,

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \longrightarrow \textcircled{4}$$



Sub ④ in ③

$$P_r = \frac{P_t A_{er}}{4\pi R^2} \times \frac{4\pi A_{et}}{\lambda^2}$$
$$= \frac{P_t A_{er} A_{et}}{R^2 \lambda^2}$$

$$\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{R^2 \lambda^2} \text{ (Dimensionless)} \longrightarrow \textcircled{5}$$

where,

$A_{et}$  = Effective aperture of transmitting antenna,  $m^2$

$A_{er}$  = Effective aperture of receiving antenna,  $m^2$

From ④,

$$A_{er} = \frac{G_r \lambda^2}{4\pi} \longrightarrow \textcircled{6}$$

Sub ⑥ in ③

$$P_r = \frac{P_t G_t}{4\pi R^2} \times \frac{G_r \lambda^2}{4\pi}$$

$$\frac{P_r}{P_t} = \frac{G_t G_r \lambda^2}{4\pi R^2} W \longrightarrow \textcircled{7}$$

Eqn ⑤ and ⑦ are called as Friis transmission formula (or) Friis radio link formula.

\* These equations include impedance mismatch at either antenna, polarization mismatch between the antennas, propagation effects leading to attenuation or depolarization and multipath effects that may cause partial cancellation of the received field.

\* It is observed in eqn ⑦, that the received power decreases as  $1/R^2$  as the separation between the transmitter and receiver increases. For long distance communications, radio links will perform better than the wired links.

\* The received power is proportional to the product of  $P_t G_t$ , which characterizes the transmitter.

(14)

\* In the main beam of an antenna, the product  $P_t G_t$  can be interpreted equivalently as the power radiated by an isotropic antenna with input power  $G_t P_t$ . Thus this product is defined as the Effective Isotropic Radiated Power (EIRP).

$$EIRP = P_t G_t \text{ W} \rightarrow \textcircled{8}$$

\* For a given frequency, range and receiver antenna gain, the received power is proportional to the EIRP of the transmitter and can only be increased by increasing EIRP.

### LINK BUDGET

\* In a link budget, the various terms in the Friis transmission formula are often tabulated separately and each of the factors can be individually considered in terms of its net effect on the received power.

\* In the link budget, the additional loss factors, such as line losses, or impedance mismatch at the antennas, atmospheric attenuation and polarization mismatch can also be added.

Path loss :-

\* Path loss is one of the terms in a link budget which accounting for the free-space reduction in signal strength with distance between the transmitter and the receiver. It is defined as (in dB),

$$L_0(\text{dB}) = 20 \log \left( \frac{4\pi R}{\lambda} \right) > 0 \rightarrow \textcircled{1}$$

\* Path loss depends on wavelength and it provides a normalization for the units of distance.

Other Terms:-

\* The remaining terms of the Friis formula as shown in the following link budget:

Transmit power	$P_t$
Transmit antenna line loss	$(-)L_t$
Transmit antenna gain	$G_t$
Path loss	$(-)L_0$
Atmospheric attenuation	$(-)L_A$
Receive antenna gain	$G_r$
Receive antenna line loss	$(-)L_r$
Receive power	$P_r$

\* Atmospheric attenuation and line attenuation loss terms also included in link budget, if all of the above quantities are expressed in dB (or) dBm, then the receive power is

$$P_r \text{ (dBm)} = P_t - L_t + G_t - L_0 - L_A + G_r - L_r \rightarrow (2)$$

Impedance Mismatch Loss:

\* If the transmit or receive antenna is not impedance matched to the transmitter or receiver or to their connecting lines, then an impedance mismatch will reduce the received power by the factor  $(1 - |\Gamma|^2)$ , where  $\Gamma$  is the appropriate reflection coefficient.

The resulting impedance mismatch loss can be included in the link budget to account for the reduction in received power will be expressed as,

$$L_{imp} \text{ (dB)} = -10 \log (1 - |\Gamma|^2) \geq 0 \rightarrow (3)$$

## Polarization Matching:-

\* Maximum power transmission between transmitter and receiver requires both antennas to be polarized in the same manner. Therefore, the polarization matching of the transmit and receive antennas is an important entry in the link budget.

\* For example, if a transmit antenna is vertically polarized, maximum power will only be delivered to a vertically polarized receiving antenna, while zero power would be delivered to a horizontally polarized receive antenna, and half the available power would be delivered to a circularly polarized antenna.

## LINK MARGIN

\* In practical communications systems, generally it is desired to have the received power level greater than the threshold level required for the minimum acceptable quality of service which is usually expressed as the minimum carrier to noise ratio (CNR) or minimum SNR.

\* This design allowance for the received power is referred to as the link margin and it can be expressed as the difference between the design value of received power and the minimum threshold value of the receive power.

$$\text{Link Margin (dB)} = \text{LM} = P_r - P_r(\text{min}) > 0 \rightarrow \textcircled{1}$$

\* In equation  $\textcircled{1}$  all the quantities are in dB, Link margin should be a positive number and its typical values may range from 3 to 20 dB.

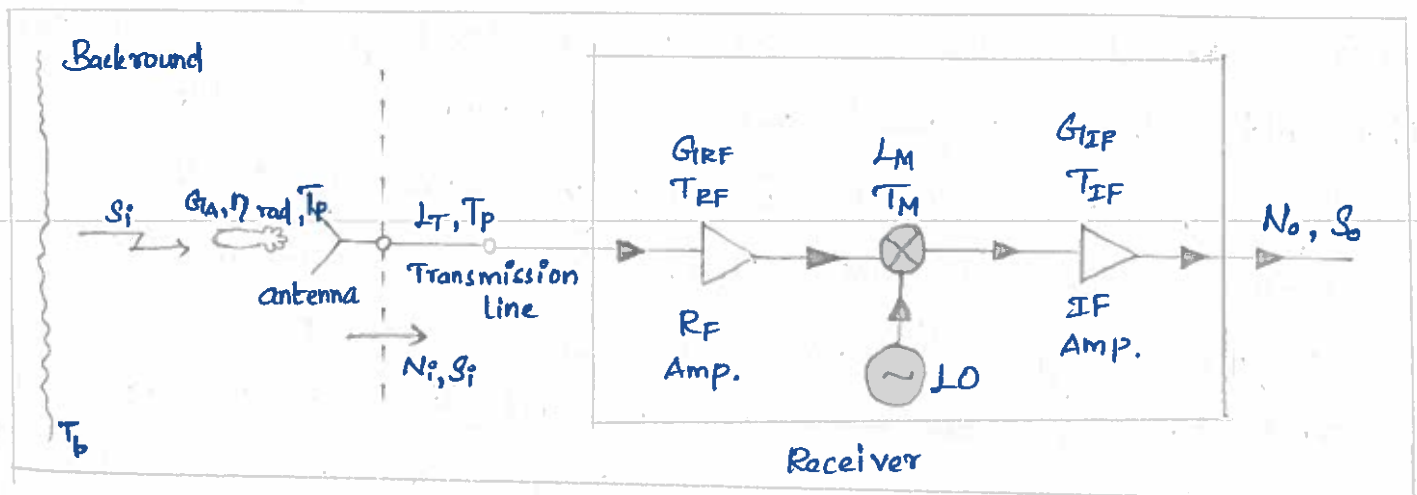
\* Link Margin is used to account for fading effects and it is sometimes referred to as fade margin. For example, satellite links operating at frequencies above 10 GHz, often require fade margins of 20dB or more to account for attenuation during heavy rain.

\* Link Margin for a given communication system can be improved by,

(i) Increasing the received power by increasing the transmit power or antenna gains.

(ii) Reducing the minimum threshold power by improving the design of the receiver or changing the modulation method.

### NOISE CHARACTERIZATION OF A MICROWAVE RECEIVER.



Noise analysis of a microwave receiver.

\* In this system, the total noise power at the output of the receiver is  $N_o$ , it will be due to the contributions from the antenna pattern, the loss in the antenna, the loss in the transmission line, and the receiver components.

\* This noise power will determine the minimum detectable signal level for the receiver and for a given transmitter

Power, the maximum range of the communication link.

\* The receiver components consists of an RF amplifier with gain  $G_{RF}$  and noise temperature  $T_{RF}$ , a mixer with an RF-to-IF conversion loss factor  $L_M$  and noise temperature  $T_M$  and an IF amplifier with gain  $G_{IF}$  and noise temperature  $T_{IF}$ .

\* The noise effects of later signals in the microwave receiver can usually be ignored since the overall noise figure (F) is dominated by the characteristics of the first few stages.

\* The component noise temperatures can be related to noise figures as  $T = (F-1)T_0$ . The noise temperature of the receiver can be expressed as,

$$T_{REC} = T_{RF} + \frac{T_M}{G_{RF}} + \frac{T_{IF} L_M}{G_{RF}} \longrightarrow \textcircled{1}$$

\* The transmission line connecting the antenna to the receiver has a loss  $L_T$ , and it is at a physical temperature  $T_P$ . Then its equivalent noise temperature is expressed as,

$$T_{TL} = (L_T - 1) T_P \longrightarrow \textcircled{2}$$

\* If the transmission line (TL) and receiver (REC) cascade, then the noise temperature at the antenna terminals that is the input to the transmission line is

$$\begin{aligned} T_{TL + REC} &= T_{TL} + L_T T_{REC} \\ &= (L_T - 1) T_P + L_T T_{REC} \longrightarrow \textcircled{3} \end{aligned}$$

\* The entire antenna pattern can collect noise power. If antenna has a reasonably high gain with relatively low sidelobes, we can assume that all noise power comes via the main beam, so that the noise temperature of the antenna is given as,

$$T_A = \eta_{rad} T_b + (1 - \eta_{rad}) T_b \longrightarrow \textcircled{4}$$



where  $\eta_{\text{rad}}$  - Efficiency of the antenna.

$T_p$  - Physical temperature of the antenna

$T_b$  - Equivalent brightness temperature of the back ground seen by the main beam.

\* The noise power at the antenna terminals that is, the noise power delivered to the transmission line is

$$N_i = kBT_A = kB[\eta_{\text{rad}}T_b + (1-\eta_{\text{rad}})T_p] \rightarrow (5)$$

where  $B$  - system bandwidth.

\* If  $S_i$  is the received power at the antenna terminals, then the input SNR at the antenna terminals is  $S_i/N_i$ . The o/p signal power is,

$$S_o = \frac{\Phi_i G_{\text{RF}} G_{\text{IF}}}{LTL M} = S_i G_{\text{sys}} \rightarrow (6)$$

\*  $G_{\text{sys}}$  defines system power gain. The output noise power is

$$\begin{aligned} N_o &= (N_i + kBT_{\text{TL}} + R_{\text{REC}}) G_{\text{sys}} \\ &= kB(T_A + T_{\text{TL}} + R_{\text{REC}}) G_{\text{sys}} \rightarrow (7) \end{aligned}$$

By sub (3) and (4) in (7)

$$\begin{aligned} N_o &= kB[\eta_{\text{rad}}T_b + (1-\eta_{\text{rad}})T_p + (L_T - 1)T_p + L_T T_{\text{REC}}] G_{\text{sys}} \\ &= kBT_{\text{sys}} G_{\text{sys}} \rightarrow (8) \end{aligned}$$

\*  $T_{\text{sys}}$  is the overall system noise temperature. Then the output SNR is written as,

$$\frac{S_o}{N_o} = \frac{S_i}{kBT_{\text{sys}}}$$

By using (8)

$$\frac{S_o}{N_o} = \frac{S_i}{kB[\eta_{\text{rad}}T_b + (1-\eta_{\text{rad}})T_p + (L_T - 1)T_p + L_T T_{\text{REC}}]} \rightarrow (9)$$

\* This SNR is improved by various signal processing techniques. It is very convenient to use an overall system noise figure to calculate the degradation in SNR from an i/p to o/p of the above system.



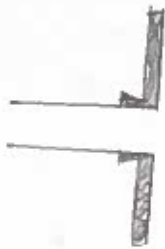
## Unit - 2

### Radiation Mechanisms and Design aspects.

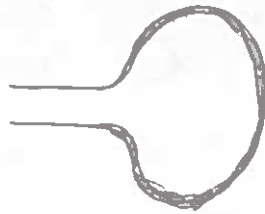
#### Types of Antennas:-

##### (1) Wire Antennas:-

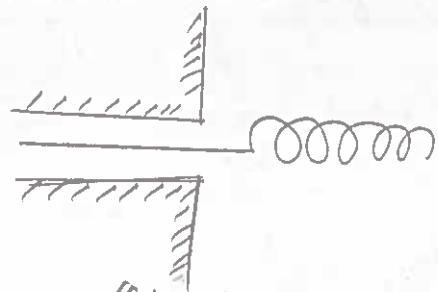
These antennas are used on automobile, buildings, ships, aircrafts, spacecraft and soon. These are various shapes of microantennas such as straight line wire (dipole), loop, helix as shown below.



(a) Dipole.



(b) circular loop.



(c) Helix.

Loop antennas need not only be circular. They may take the form of rectangular, square, ellipse etc.,

##### (2) Aperture Antennas:-

These antennas are very useful for aircraft and space craft applications.

##### Types:-

(i) Rectangular Horn

(ii) Conical Horn

(iii) Rectangular waveguide.

### (3) Microstrip Antennas:

\* It is very popular in 1970's for spaceborne applications. Today they are used for government and commercial applications.

\* These antennas are low profile, comfortable to planar and non-planar surfaces.

\* Simple and inexpensive to fabricate.

\* Can be mounted on the surface of high performance aircraft, spacecraft, satellites etc.

### (4) Reflector Antennas:

Larger dimensions are needed to achieve the high gain required to transmit (or) receive signals after millions of miles of travel.

### Linear wire Antennas:-

\* Antennas which is in the form of linear wire is called linear wire antenna.

### Types of linear wire antenna.

- 1) Infinitesimal dipole  $l < \lambda/50$
- 2) Short dipole  $\lambda/50 < l < \lambda/10$ .
- 3) Half wavelength dipole  $l = \lambda/2$ .

## APERTURE ANTENNA :

The term aperture refers to an opening in a closed surface. The aperture antenna represent a class of antennas which are analysed by considering the antenna as an opening in a surface, in which radiation is considered to occur from an aperture. EM waves are transmitted or received through this opening.

Example for ~~Aperture~~ Aperture antennas are

- 1) Slot antenna.
- 2) Horn antenna.
- 3) Reflector antenna.
- 4) Lens antenna.

### Horn antenna:

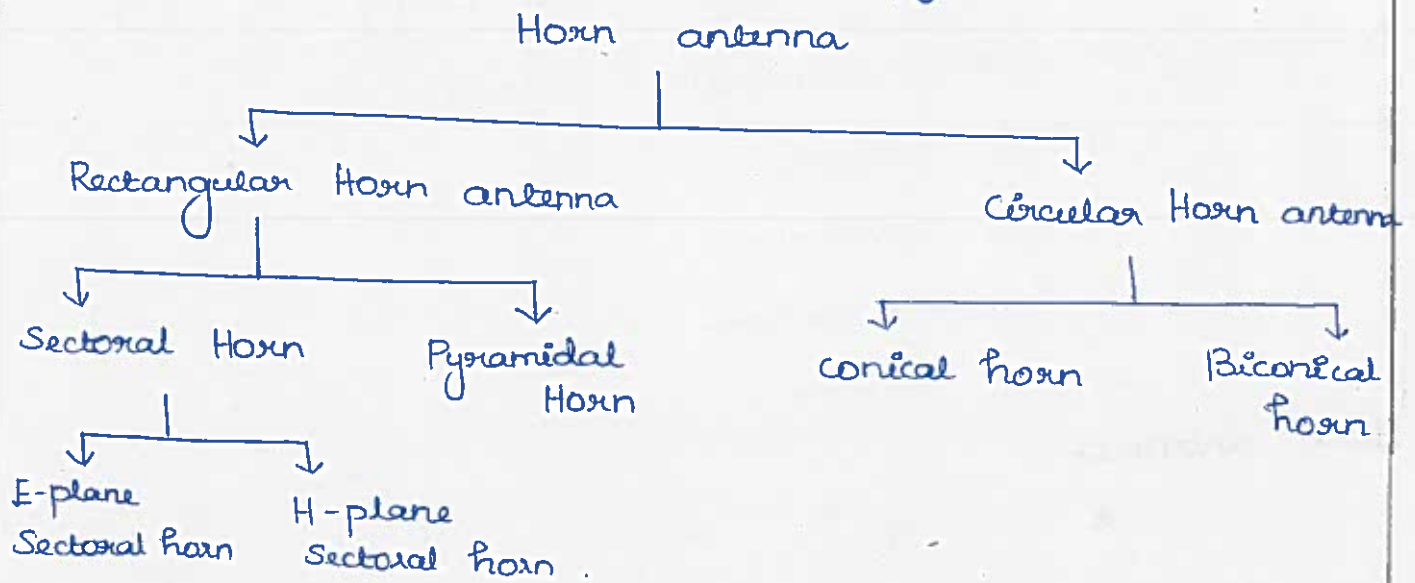
\* One of the simplest and probably the most widely used microwave antenna is the horn antenna. It serves as a feed element for large radio astronomy, communication dishes and satellite tracking throughout the world.

\* The function of horn is to produce a uniform phase front with a larger aperture than that of the magnitude and hence the greater directivity.

\* A horn antenna (or) microwave horn is an antenna that consists of a flaring metal waveguide shaped like a horn to direct radio waves in a beam, Horns are widely used as antennas of UHF & microwave frequencies above 300MHz.

## Types of horn antenna:

All the horn antennas are energized from either rectangular (or) circular waveguide. Basically horn antennas are classified as rectangular antennas and circular horn antennas. The rectangular horn antennas are fed with rectangular waveguide, while circular horn antennas are fed with circular waveguide.

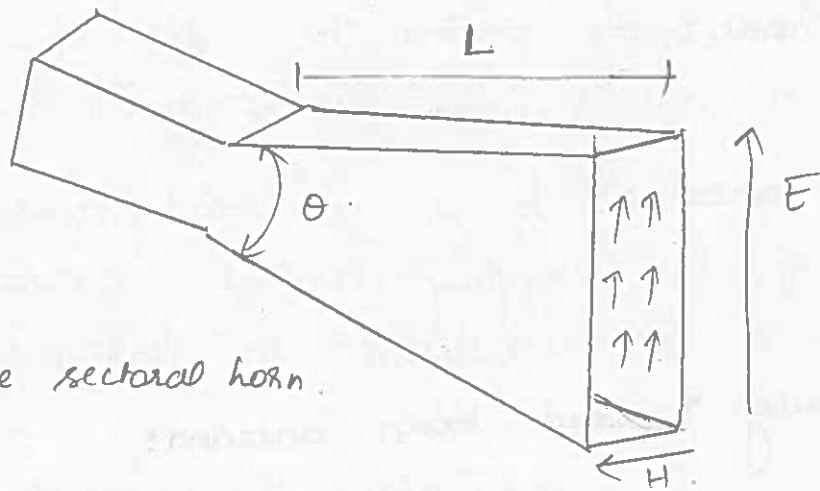


### 1) Rectangular Horn antenna:

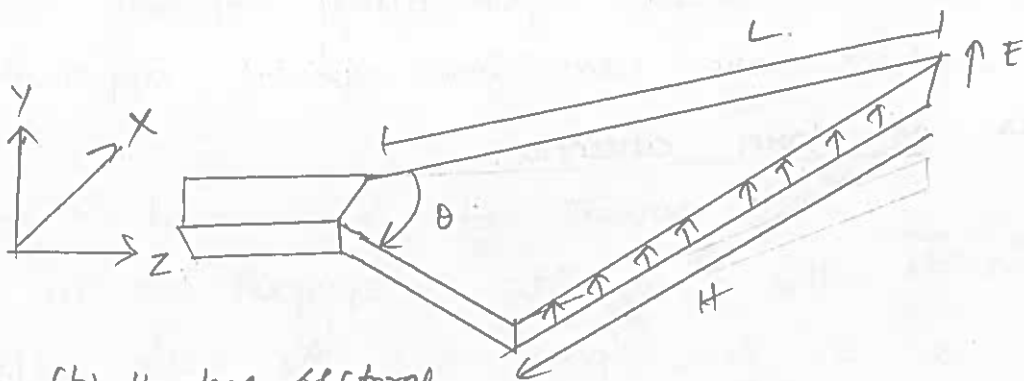
\* Depending upon the direction of flaring, the rectangular horns are classified as sectoral and pyramidal horn.

\* The sectoral horn is a rectangular type waveguide with a flaring is done in only one direction. It is along the single wall of a rectangular waveguide.





(a) E-plane sectoral horn.



(b) H-plane sectoral horn.

\* It is further classified as E-plane sectoral form and H-plane sectoral form.

E-plane sectoral form is obtained when the flaring is done in the direction of electric field vector.

H-plane sectoral horn is obtained if the flaring is done in the direction of magnetic field vector.

**Pyramidal Horn:** If the flaring is done along both the walls of the rectangular waveguide in the direction of both electric field and magnetic field vector, then horn obtained is called pyramidal horn and allow indicates E-plane and H-plane.

## 2) Circular horn:

**Conical horn:** By flaring the walls of a circular waveguide, a conical horn is formed.

**Biconical antenna:** It is a broad bandwidth antenna made up of two roughly conical conductive objects, which is non-directional in horizontal plane.

## 3) Exponentially Tapered Horn antenna:

To minimize the reflections of the guided wave, the transition region or horn between the waveguide at the throat and free space at the aperture could be given a gradual exponential taper. Such horns are called exponentially tapered horn antenna, which are used for special applications.

### Principles of Horn antenna:

In general, it is observed that the fields inside the waveguide propagate in the same manner as in free space. But the main difference is that the propagation of wavefield is constrained by the walls of the waveguide from being spherically spreading.

### Huygen's Principle:

\* It says that each point on a primary wave front can be considered to be a new source of a secondary spherical wave and the secondary wavefront can be constructed as the envelope of these secondary spherical waves.

\* At the end of the waveguide, the guided propagation changes to free space propagation, so this region is generally called as transition region.



\* If the flare angle is small, results in small aperture area. Beam width is decreased and directivity is increased.  $D$  is inversely proportional to aperture.

\* In a E plane of the Horn,  $\delta$  is usually held to  $0.25\lambda$  (or) less. However in the H-plane,  $\delta$  can be larger (or) about  $0.4\lambda$ .

For an optimum flare horn, the HPBW can be approximately.

$$\theta_H = \frac{67^\circ \lambda}{a_H} = \frac{67\lambda}{W} \text{ degree}$$

$$\theta_E = \frac{56^\circ \lambda}{a_E} = \frac{56\lambda}{a} \text{ degree}$$

Directivity  $D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \epsilon_{ap} A_p}{\lambda^2}$

where,

$A_e$  - effective aperture in  $m^2$ .

$A_p$  - physical aperture in  $m^2$  = Area of horn mouth opening.

$$\epsilon_{ap} = \frac{A_e}{A_p} = \text{Aperture efficiency}$$

For a pyramidal rectangular horn,

$$A_p = a_E a_H = a \times W$$

where,

$a$  - height of the aperture.

$w$  - height of the aperture.

$a_E$  - E plane aperture.

$a_H$  - H plane aperture.

similarly for a conical horn,  $A_p = \pi r^2$ .

For eg: if  $a_E = a_H = \lambda = 1\text{m}$  and  $\epsilon_{ap} = 0.6$ , then the directivity of rectangular horn is given by

$$D = \frac{4\pi(0.6)A_p}{\lambda^2} = \frac{7.5A_p}{\lambda^2}$$

$$D(\text{dB}) = 10 \log \frac{7.5A_p}{\lambda^2}$$

Advantages:

\* The directivity of the pyramidal horn and conical horn is highest as they have more than one flare angle.

\* It can be operated over a wide range of high frequency as there is no resonant element in the antenna.

Applications:

\* Used in microwave applications.

\* Used as feed element for large radio astronomy, satellite tracking, communication dishes in parabolic reflector.

\* Used in short range radar systems.

\* The waveguide impedance and free space impedance do not match with each other, the flaring (tapering) of the walls of the waveguide must be done so that the impedance matching is achieved along the concentrated radiation pattern with high directivity and narrow bandwidth.

### Design of Horn antenna:

\* Consider a pyramidal horn of length ' $L$ ' and aperture height ' $h$ ' with flaring along  $\theta$  as shown in figure 2.6.2. The function of the horn is to produce a uniform phase front with a larger aperture in comparison to the waveguide. Because of this, the directivity increases.

\* Consider an imaginary axis 'O' of horn as shown in figure 2.6.2. There is a path difference between a ray travelling along the side and along the axis of the horn.

Let ' $\delta$ ' be the difference in the path of travel.

$\theta$  - flare angle ( $\theta_E$  for E plane,  $\theta_H$  for H plane) in degree.

$h = a$  - Aperture ( $a_E$  for E plane,  $a_H$  for H plane)

$L$  - length of horn in (m) in m.

$$\sin \frac{\theta}{2} = \frac{h/2}{L+\delta}$$

From the geometry  $\triangle OBA$ ,  $\cos \frac{\theta}{2} = \frac{OB}{OA} = \frac{L}{L+\delta}$

$$\tan \frac{\theta}{2} = \frac{\sin \theta/2}{\cos \theta/2} = \frac{h/2}{L}$$

$$\theta = 2 \tan^{-1} \left( \frac{h}{2L} \right) = 2 \cos^{-1} \left( \frac{L}{L+\delta} \right) \longrightarrow \textcircled{1}$$

From figure,

$$(L+\delta)^2 = L^2 + (h/2)^2$$

$$L^2 + \delta^2 + 2\delta L = L^2 + \frac{h^2}{4} \quad [ \because \delta \text{ is small \& it can be neglected} ]$$

$$2\delta L = \frac{h^2}{4}$$

$$\boxed{L = \frac{h^2}{8\delta}} \longrightarrow \textcircled{2}$$

Equations  $\textcircled{1}$  &  $\textcircled{2}$  are the design equations of the horn antenna.

\* If the flare angle  $2\theta$  is very large, the wavefront of the mouth of horn will be curved rather than plane. This will result in non-uniform phase distribution over the aperture, resulting increased beam width and decreased directivity.

## Reflector Antenna

Reflector antennas (or) reflectors are widely used to modify the radiation pattern of a radiating element, for example backward radiation from an antenna may be eliminated by using a plane sheet reflector of large enough dimensions.

The antenna which is a radiating source in the reflector antenna is called primary antenna (or) feed, while the reflector antenna is called ~~primary~~ the secondary antenna. The most common feeds are dipole, horn and slot etc.

### Types of reflector Antennas

The reflector antennas are of several types and they are listed as :

- 1) plane reflector (or) flat sheet reflector
- 2) corner reflector
- 3) parabolic reflector
- 4) Hyperbolic reflector
- 5) Elliptical reflector
- 6) conical reflector.

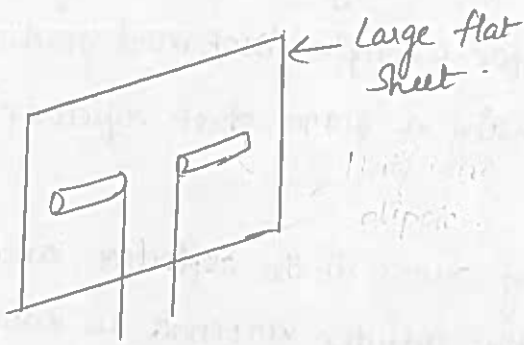
#### a) Flat sheet reflector (or) plane reflector

The plane reflector is the simplest form of the reflector antenna. The main advantage is that dipole (feed) backward directions are reduced and gain in the forward direction increases.

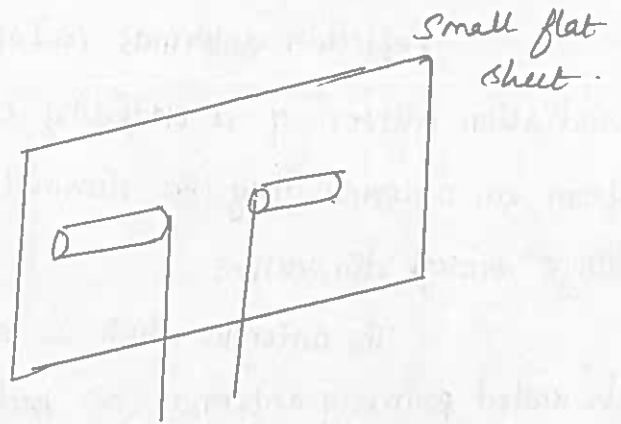
The gain can be increased further by reducing the spacing between the feed and sheet reflector, however bandwidth is narrow for small spacings.

The polarization of the radiating source and its position relative to the reflecting surface can be used to control the radiating (pattern, impedance, directivity) of the overall system.

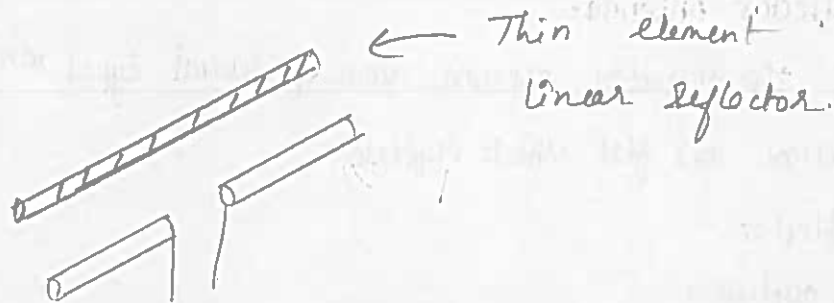




(a) Large sheet with half wave dipole feed.



(b) Small flat sheet with half wave dipole feed.



(c) Thin linear element with half wave dipole feed.

Fig 2.b.3 various shapes of reflectors

(a) large sheet with half wave dipole feed.

(b) small flat sheet with half wave dipole feed.



(c) ~~Thin plane reflector element with~~  
~~half wave dipole feed~~

~~Fig 2.6.4 various types of plane reflector.~~

A large flat sheet reflector can convert a bidirectional antenna array into a unidirectional system. Small spacing between the antenna and the sheet will improve the gain in the forward direction.

The desirable properties of the sheet reflector may be largely preserved with the reflector reduced in size as shown in figure 2.6.4 (b)

Thin reflector element is used which is highly sensitive to the frequency changes as shown in figure 2.6.4 (c) which can be used to increase directivity

D Analysis of plane reflector by method of images

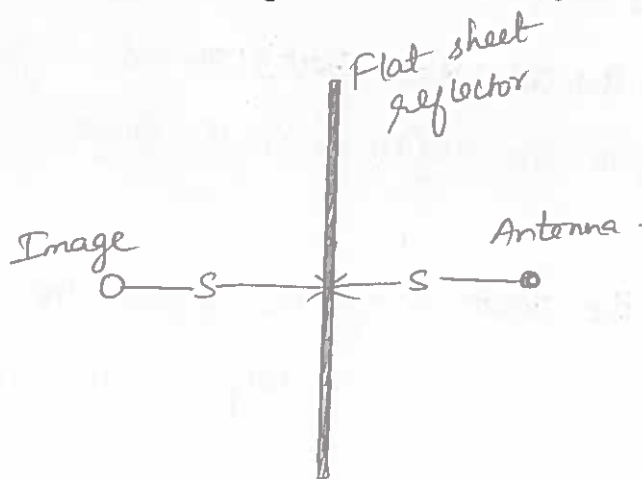


Fig 2.6.5 Antenna with flat sheet reflector & its image.

The problem of an antenna at a distance  $s$  from a perfectly conducting plane sheet reflector of infinite extent is readily handled by the method of images, here the reflector is replaced by an image of the antenna at a distance  $2s$  from the antenna as shown in figure 2.6.5

Assuming zero reflector losses, the gain in terms of field intensity of a  $\lambda/2$  dipole antenna at a distance  $s$  from an infinite plane reflector and it is expressed as

$$G_f(\theta) = 2 \sqrt{\frac{R_{11} + R_L}{R_{11} + R_L - R_{12}}} \left| \sin(sr \cos\theta) \right| \rightarrow \textcircled{1}$$

where  $s_r = \frac{2\pi s}{\lambda}$

$s$  - distance bet reflector & feed

$R_{11}$  - impedance of antenna

$R_{12}$  - Mutual impedance of the antenna & reflector

$R_L$  - Antenna loss resistance.

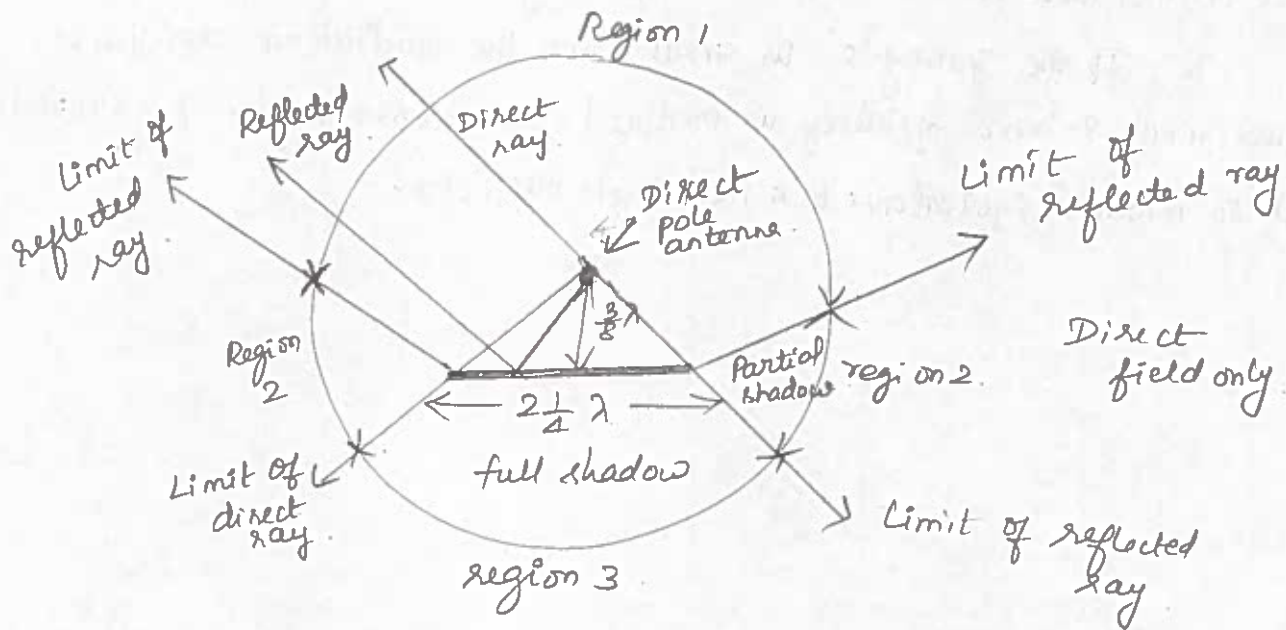
When the reflecting sheet is reduced in size, the analysis can be done by considering three principal angular regions as shown in figure 2.6.6  
 Region 1 (above or) in front of the sheet): In this region, the radiated field is given by the addition of the resultant of the direct field of the dipole and reflected field from the sheet.

Region 2 (above or) below at the sides of the sheet): In this region there is only the direct field from the dipole. This region is in the shadow of the reflected field.

Region 3 (below or) behind the sheet): In this region, the sheet acts as a shield producing a full shadow, that is only the diffracted fields & there is no direct or) reflected fields.

Fig: 2.6.6 Dipole antenna with  $2.25\lambda$  flat sheet reflector.

### 1) Corner reflectors:-



### 2) Corner reflectors :-

The disadvantage of plane reflector is that there may be a radiation in back & side directions. In order to overcome this limitation, the geometrical shape of the plane reflector is modified as two plane reflectors were joined to form a corner with some angle, which reflects EM waves back towards source. The reflector is known as corner reflector.

The angle between two plane reflectors were joined is called as included angle (or) corner angle ( $\alpha$ ). A corner included with a driven element is called active corner reflector antenna at an angle of ( $\alpha < 180^\circ$ ) that produces a sharper radiation pattern than a flat sheet reflector ( $\alpha = 180^\circ$ ).

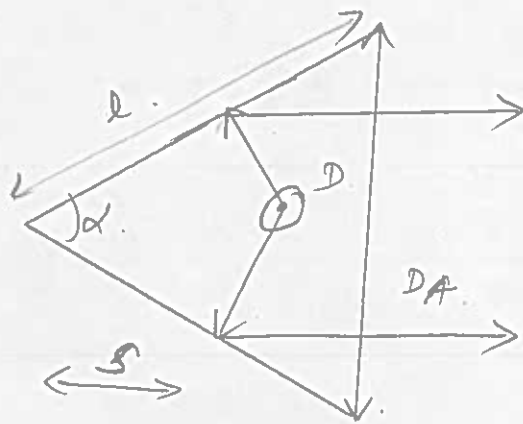
For most practical applications,  $\alpha = 90^\circ$  is used. A corner reflector without any driven element is called passive corner (or) retro (or) square corner reflector antenna.

For example, if the reflector is used as a passive target for radar (or) communication applications, it will return the signal exactly in the same direction as it is received when its include angle is  $90^\circ$ .

The system efficiency depends on the spacing between the vertex

of the corner reflector and the feed element is 's'. If 'α' decreases 's' must be increased to achieve desired efficiency.

If the spacing 's' is small, then the radiation resistance becomes small & hence efficiency is reduced. The corner angle of  $\alpha = \pi$  radian (or)  $180^\circ$  which is equivalent to a flat sheet reflector.



D → Driven element  
 DA → Aperture size  
 l → length  
 S → spacing between reflector and feed point location.  
 α — corner angle.

Fig 20b.7 Active corner reflector

Design equations of corner reflector :-

- 1) the aperture of the corner reflector  $DA$  is selected between one and two wavelength ( $\lambda < DA < 2\lambda$ )
- 2) The spacing (s) between the vertex of the reflector & the feed element is selected as a fraction of wavelength. ( $\lambda/\alpha < s < \frac{2\lambda}{3}$ )
- 3) the length of the reflectors is approximately given as  $l = \alpha s$  for  $\alpha = 90^\circ$  &  $\alpha < 90^\circ$  then  $l > \alpha s$ .
- 4) the height of the reflector (h) is generally selected as about 1.2 to 1.5 times greater than the total length of the feed element.
- 5) the radiation resistance is the function of 's'. If 's' is too large, the unwanted multiple lobes are produced & hence the directivity of antenna is lost.
- 6) If 's' is very small, radiation resistance decreases.



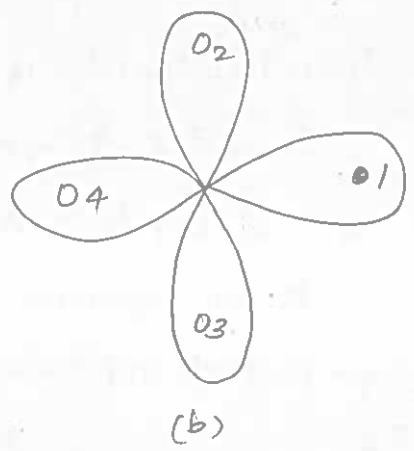
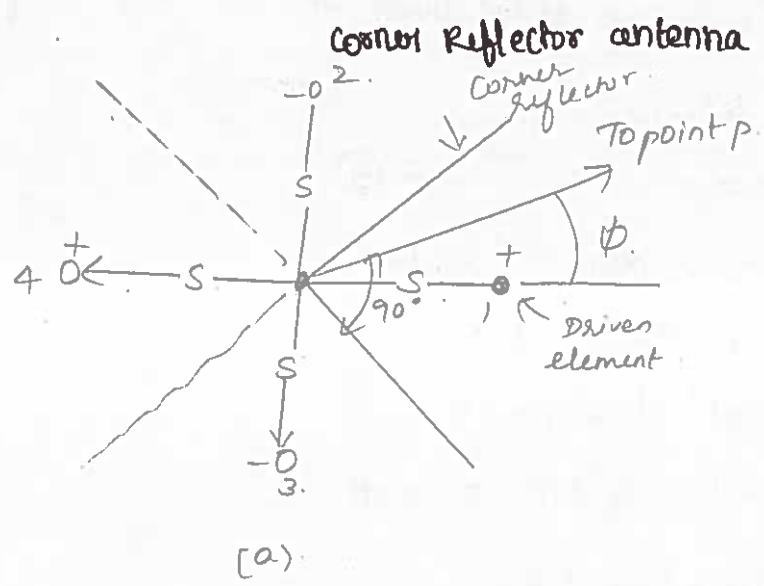
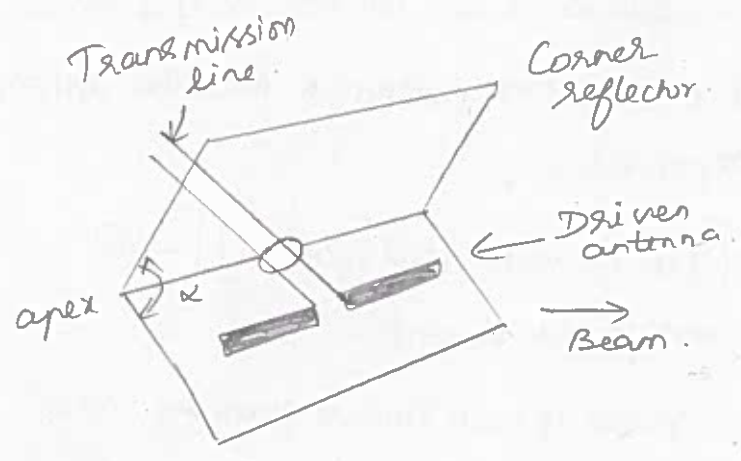


Fig. 2.6.8. Square corner reflector with images used in analysis (a) 4-lobed pattern of driven element and images.

I > general  $D_A = \sqrt{l^2 + l^2} = \sqrt{2l^2} = 1.414 l$

but  $l = 2.5 \rightarrow ①$

$D_A = 2.828 l \rightarrow ②$

The above equations are design equations of the corner reflector.

consider a square corner reflector with corner angle  $\alpha = 90^\circ$  which consists of one driven element D. Based on one driven element, three corresponding images are represented by points (2) (3) & (4) - the driven element along three images can carry current of same magnitude. The phase of current in 1 & 4 are

same as phase of currents in 3 & 2 but  $180^\circ$  out of phase.

when point 'p' is at large distance R from the antenna, then electric field intensity is expressed as,

$$E(\theta) = 2k I_1 \left| \left[ \cos(s_r \cos\theta) - \cos(s_r \sin\theta) \right] \right| \rightarrow (3)$$

where  $I_1$  = current in each element

$s_r = \frac{2\pi s}{\lambda}$   $\rightarrow$  spacing of each element from the corner

$k$  = propagation constant.

the terminal voltage  $V_1$  at the centre of the driver element is given by

$$V_1 = I_1 Z_{11} + I_1 Z_{12} + I_1 Z_{14} - 2I_1 Z_{12}$$

$$= I_1 (Z_{11} + Z_{12} + Z_{14} - 2Z_{12}) \longrightarrow (4)$$

where  $Z_{11}$  - self impedance of driver element ( $1/2$  dipole)

$Z_{12}$  - mutual impedance bet' element 1 & 2

$Z_{14}$  - mutual impedance bet' element 1 & 4

$Z_{12}$  - equation loss impedance of driver element.

in terms of  $R$ ,  $R = \frac{V_1}{I_1} = R_{11} + R_{12} + R_{14} - 2R_{12}$

$$I_1 = \sqrt{\frac{P}{R_{11} + R_{12} + R_{14} - 2R_{12}}}$$

Sub  $I_1$  value in eqn (3),

$$E(\theta) = 2k \sqrt{\frac{P}{R_{11} + R_{12} + R_{14} - 2R_{12}}} \left| \left[ \cos(s_r \cos\theta) - \cos(s_r \sin\theta) \right] \right|$$

Field intensity at point p at a distance D from  $1/2$  dipole with

reflector removed is  $E_{HW}(\phi) = k \sqrt{\frac{P}{R_{11} + R_{12}}}$

$$\text{Gain } G_f(\phi) = \frac{E_\phi}{E_{HW}(\phi)} = 2 \sqrt{\frac{R_{11} + R_{12}}{R_{11} + R_{12} + R_{14} - R_{12}}} \left| \begin{array}{l} \cos s_r \cos\phi \\ -\cos(s_r \sin\phi) \end{array} \right|$$

### 3) parabolic reflector

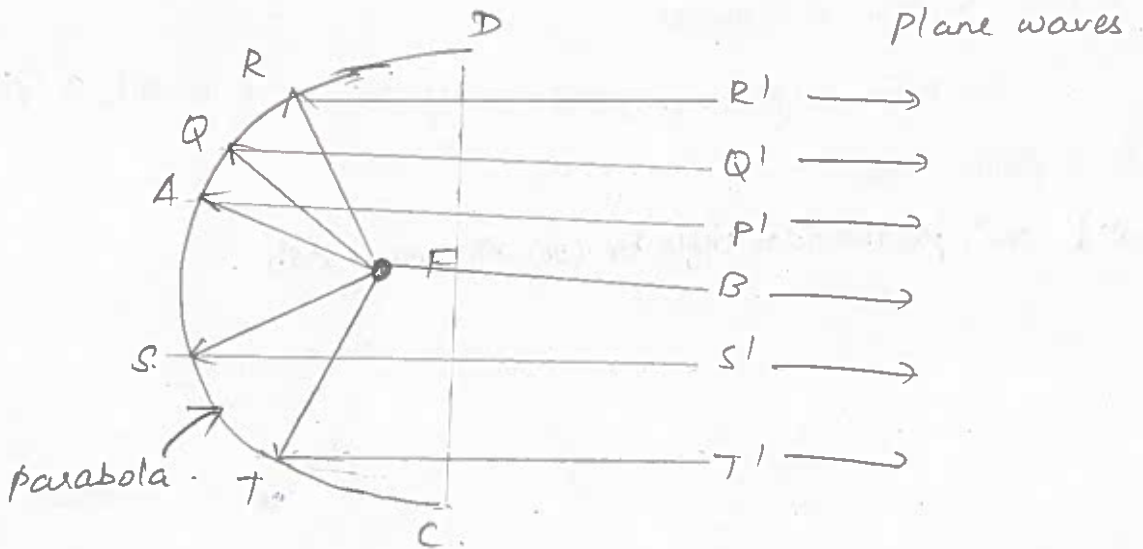


Fig 2.6.9 parabolic reflector.

The parabolic structure is used to improve the overall radiation characteristics such as antenna pattern, antenna efficiency, polarization etc, of the reflector antenna. The geometry of parabolic reflector in transmitting mode is shown in figure 2.6.9.

Here  $AB \Rightarrow$  axis of parabola

$f \rightarrow$  focus

$CD \rightarrow$  direction

$CD$  - mouth diameter  $DA$

$A$  - vertex

$CAD$  - parabola

From the definition of a parabola, we have

$$FP + PP' = FQ + QQ' = fS + SS' = \text{constant } (k)$$

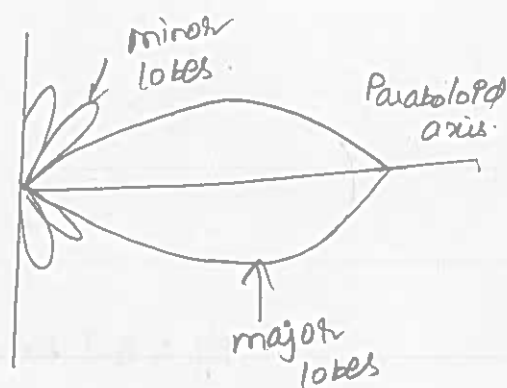
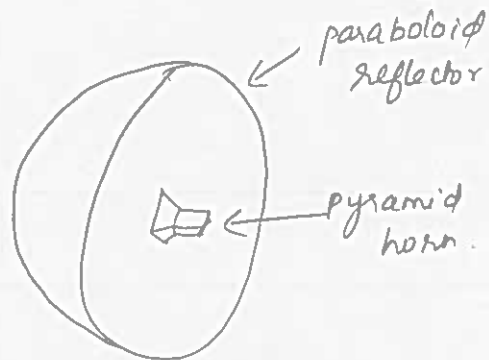
Operation:- If a feed antenna is placed at the focus, all the waves are incident on the reflector and they are reflected back, forming a plane wavefront. By the time the reflected waves reach the directrix, all of them will be in phase, irrespective of the point on the parabola from which they are reflected. Hence the radiation is very high & is concentrated along the



axis of the parabola. same time waves will be cancelled in other directions as result of path & phase differences.

The main purpose of parabolic reflector is to convert a spherical wave into a plane wave.

2) paraboloid (or) paraboloidal reflector (or) microwave dish.



(a) paraboloid

(b) Radiation pattern.

Fig : 2.6.10 paraboloid with pyramided form as feed.

The paraboloid is also called as microwave dish which produces sharp major lobe & smaller minor lobes.

If an isotropic source is placed at the focus point (F) of a parabolic reflector, here the radiation pattern B of source is intercepted & it is reflected as a plane wave of circular cross section.

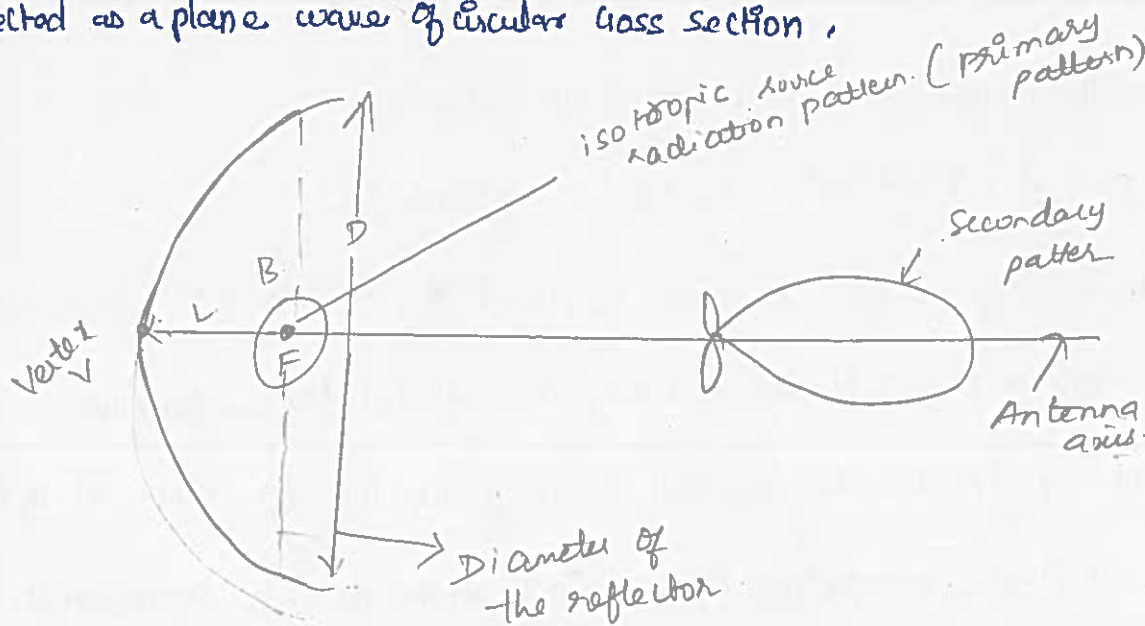


Fig 2.6.11 paraboloid & its radiation pattern.

The paraboloid and its radiation pattern is as shown in figure 26.11, the radiation pattern consists of very sharp major lobes & smaller minor lobes.

The distance bet<sup>n</sup> the focus (F) & vertex (V) is 'L' of the paraboloid. If an even number of  $\lambda/4$ , then direct radiation in the axial direction from the source will be in opposite phase & will tend to cancel the central region of reflected wave.

$$L = n\lambda/4 \text{ where } n = 2, 4, 6.$$

If n is odd (n=1, 3, 5, ...), then the direct radiation in the axial direction from the source will be in the same phase & will tend to support the central region of the reflected wave. When focal ratio is small we will get the tapered field distribution (or) illumination. To get more uniform aperture field distribution, it is necessary to make angle  $\theta$  small by increasing the focal length 'L' which keeping the reflector diameter D constant & hence the focal ratio will be larger.

Patterns of large circular apertures with uniform illumination

According to Huygen's principle, the normalized field pattern  $E(\theta)$  as a function of  $\theta$  & D is expressed as,

$$E(\theta) = \frac{2J_1}{\pi D} \frac{J_1 \left[ \frac{\pi D}{\lambda} \sin \theta \right]}{\sin \theta}$$

where, D  $\rightarrow$  Diameter of aperture in m

$\theta \rightarrow$  angle wrt the normal to the aperture

$J_1 \rightarrow$  first order Bessel function.

a) for circular aperture :-

$$\text{BWFN} = \alpha^\circ = \frac{140}{D\lambda} \text{ degrees.}$$

$$\text{HPBW} = \frac{58}{D\lambda}$$

The directivity (D) of the uniformly illuminated aperture is

Given by  $D = \frac{4\pi A_2}{\lambda^2}$

for circular Aperture  $A_2 = \frac{\pi D^2}{4}$

$$\therefore D = \frac{4\pi \frac{\pi D^2}{4}}{\lambda^2} = \pi^2 \left(\frac{D}{\lambda}\right)^2 \approx 9.9 D \lambda^2$$

Power gain  $G_p = \frac{4\pi A_0}{\lambda^2}$

$A_0 =$  capture area  $= k.A$

$k$  - constant dependent on feed antenna used.

$$\therefore G_p = \frac{4\pi k A}{\lambda^2} = \frac{4\pi \times 0.65 \times A}{\lambda^2}$$

for circular aperture  $A = \frac{\pi D^2}{4}$  &  $k = 0.65$  for dipole

$$G_p = \frac{4\pi \times 0.65}{\lambda^2} \frac{\pi D^2}{4}$$

$$\boxed{G_p = 64 \left(\frac{D}{\lambda}\right)^2 = 64 D \lambda^2}$$

b) Rectangular aperture :-

$$\text{BWFN} = \frac{115 \lambda}{L} = \frac{115}{L \lambda} \text{ degree}$$

$$L \lambda = \frac{L}{\lambda}$$

$L$  - length of rectangular aperture in terms of  $\lambda$  or  $m$

c) square aperture :-

$$D = 4\pi \frac{L^2}{\lambda^2} = 12.6 L \lambda^2$$

and power gain over a  $\lambda/2$  dipole is given as,

$$G_p = 7.7 L \lambda^2$$

length of side in term of  $\lambda = L \lambda = \frac{L}{\lambda}$ .

### 2.7 Feeding systems (or) structures:-

WKT parabolic reflector antenna consists of two basic parts

- namely (i) A source of radiation placed at the focus called primary radiator or feed.
- (ii) the reflector called secondary radiator.

The feed is said to be ideal feed, if it radiates entire energy towards the reflector. therefore, the entire surface of the reflector is illuminated & no energy is radiated in any unwanted direction.

practically there are number of possible feeds to the parabolic reflector antenna. the secondary radiator used is a parabolic most of the times.

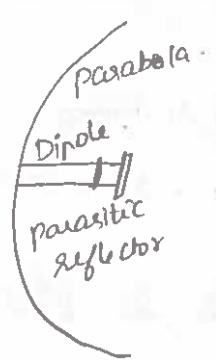
The various feeds used as:-

- (i) Dipole antenna.
- (ii) Horn antenna.
- (iii) End fire antenna
- (iv) Cassegrain feed
- (v) offset feed.

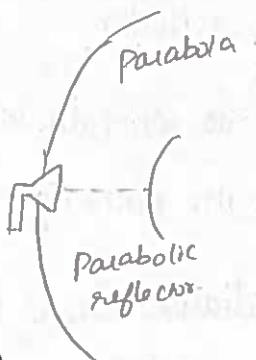
(i) Dipole feed:- The simplest type of the feed.

It is not suitable feed for the parabolic reflector antenna. Instead of only dipole, a feed consisting of dipole with parasitic reflectors (yagi-uda) can be used as a feed system.

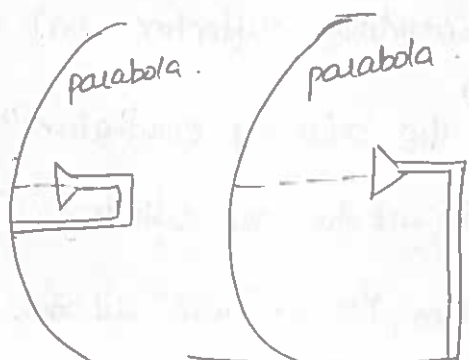
The spacing bet' the dipole as a driver element & parabolic parasitic reflector is  $0.125 \lambda$  & for plane reflector  $0.4 \lambda$



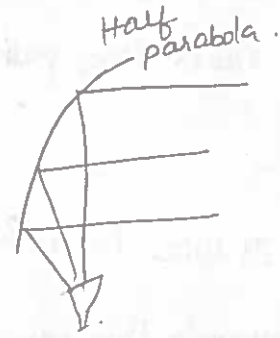
(a) Rear feed using half wave dipole



(b) Rear feed using horn

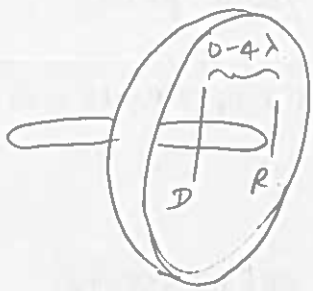


(c) Front feed using horn

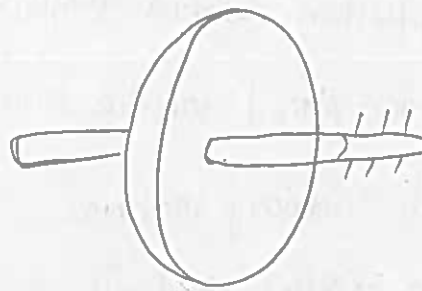


(d) offset feed using horn

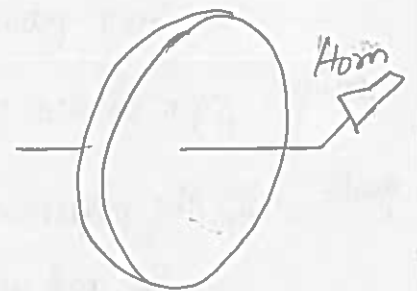




a) Dipole with plane reflector.



b) End fire array dipole



c) Horn with waveguide.

### 2.7.1 Different types of feed system.

- 1) End fire feed:- In some cases, an end fire array is used as feed radiator.
- 2) Horn Feed:- The most widely used feed system in the parabolic reflector antenna is waveguide turn antenna the form antenna is fed with a waveguide. In case, if circular polarization is required, then the rectangular form is replaced by a conical form. In all cases, the feed (or) primary radiator is placed at the focus to obtain maximum term pattern.

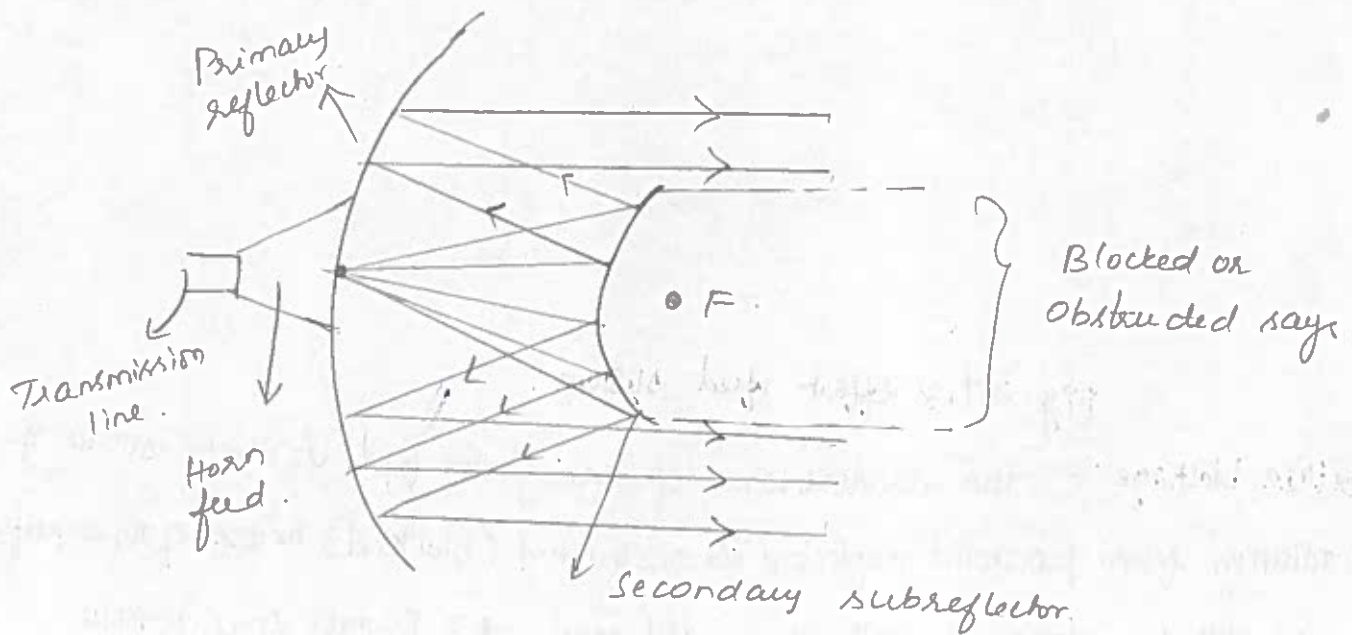
#### 4) Cassegrain feed

The system of feeding paraboloid reflector is named after a mathematician prof. Cassegrain. In this system, the feed is placed at the vertex of the parabolic reflector instead of placing it at the focus.

This system uses a hyperboloid reflector placed whose one of the foci coincides with the focus of the parabolic reflector, this hyperboloid reflector is called Cassegrain secondary reflector (or) sub-reflector.

The primary radiator used is generally a horn antenna, it aims its radiate at the sub-reflector, when the primary radiator radiates towards the Cassegrain, it radiates all the radiations. Due to this, the parabolic reflector gets illuminated similar to the feed radiator placed at the focus.

then the parabolic reflector collimates all the radiations as previous feed system.



### 2.7.2 Cassegrain feed system.

Advantages of Cassegrain feed system:-

- 1) It reduces the spill over & minor lobe radiations.
- 2) The system has ability to place a feed at convenient place.
- 3) Using this system, beam can be broadcast by adjusting one of the reflector surfaces.

offset feed system,

Due to aperture blockage effect, the minor lobes are increases.

Here the feed radiator is placed at the focus. With this system, all the rays are perfectly collimated without formation of the region of blocked rays.

5) offset feed system:-

Due to aperture blockage effect, the minor lobes increases.

Here the feed radiator is placed at the focus as shown in figure 2.7.3.

With this system all the rays are perfectly collimated without formation of the region of blocked rays.



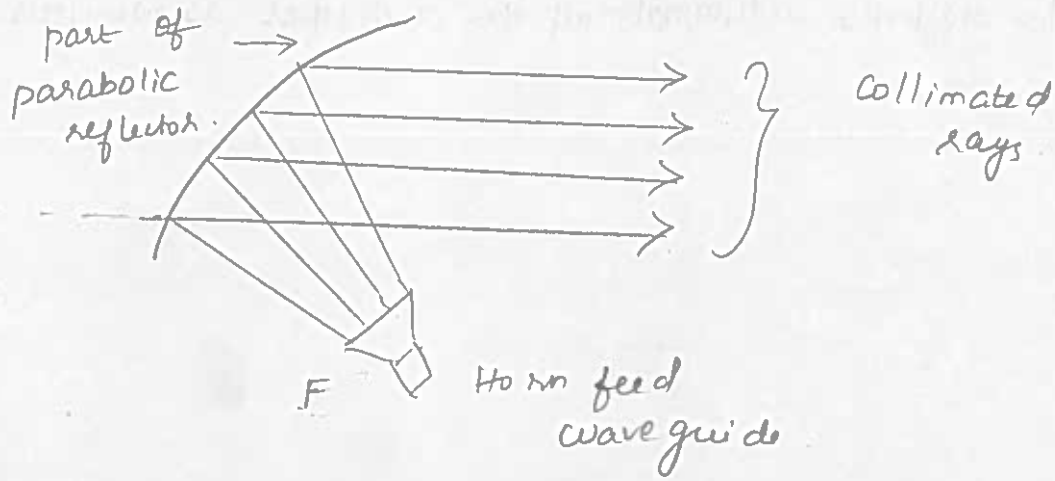


Fig: 8.7.3 offset feed system.

Aperture blockage:- the disadvantage of cassegrain feed is that some of the radiation from paraboloid reflector is obstructed (blocked) becoz of the presence of sub reflector along the path of parallel rays. this is not very serious problem in case of a parabolic reflector of larger dimensions. But for smaller dimension parabolic reflector, it is the main drawback of the cassegrain feed system.

the aperture blockage effect can be avoided by using an offset reflector which is applicable to focal point feed.

Some of the radiations from the parabolic reflector were blocked by the hyperbolic reflector creating region of blocked rays.

Applications of parabolic reflector:-

1) It is used in microwave communication

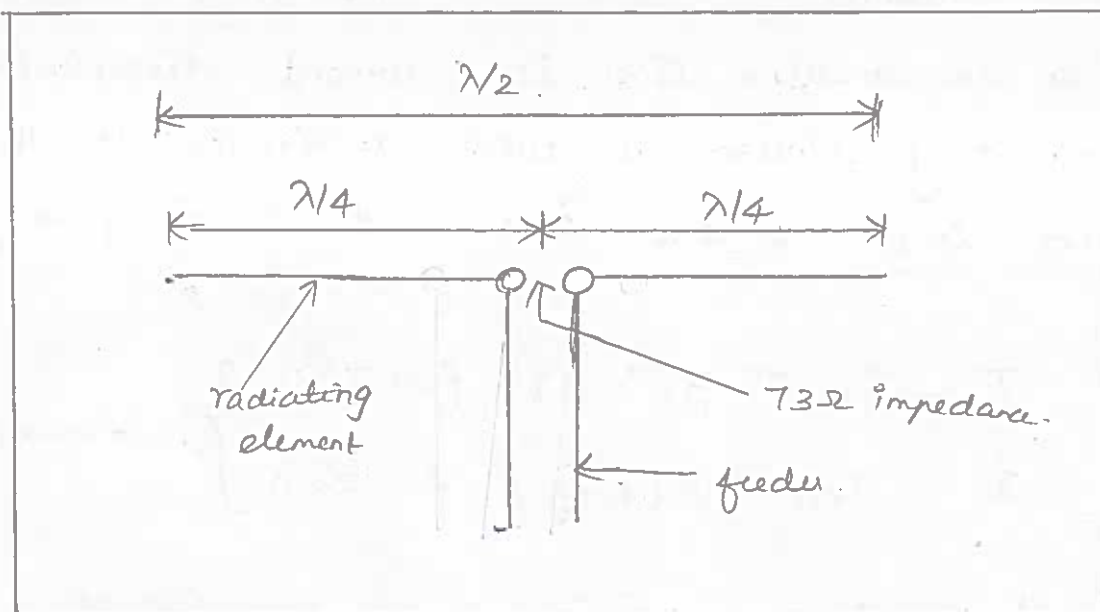
2) Radio astronomy

3) satellite transmission and reception.

## HALF WAVE DIPOLE:

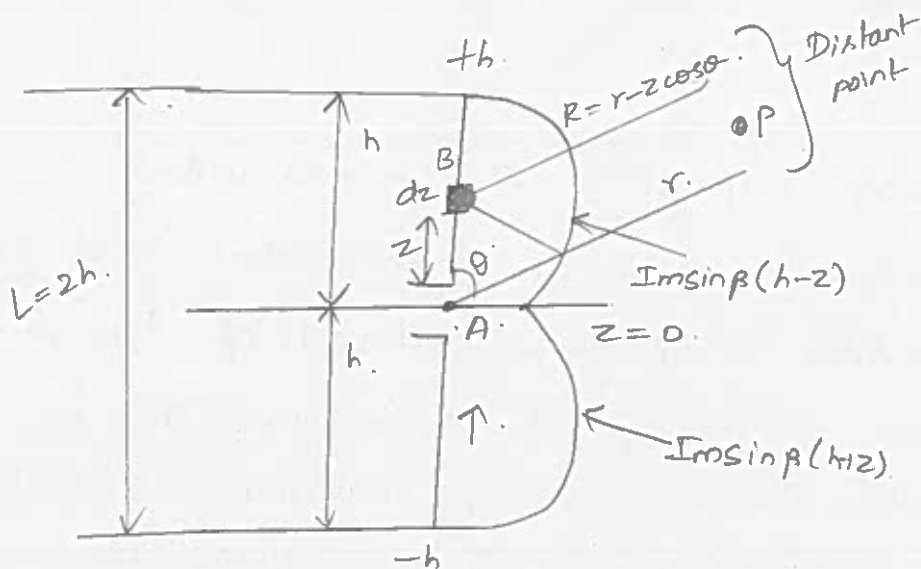
### Definitions:-

Half wave dipole is the fundamental radio antenna which is made up of a metal rod or thin wire and it has a physical length of  $\lambda/2$  in free space at the frequency of operation. This is usually fed at the centre having maximum current, that is, maximum radiation is in the plane normal to the axis.



### Power radiation from Half Wave dipole:

It is a symmetrical antenna in which the two ends are at an equal potential relative to the midpoint. The overall specified length is  $L=2h$  and vertical antenna height is  $h = \frac{L}{2}$ .



$$\text{In } \triangle ABC, AC = r - R$$

$$\frac{r - R}{z} = \cos \theta$$

$$r - R = z \cos \theta$$

$$R = r - z \cos \theta$$

This antenna is fed at the centre with the help of a transmission line, its current distribution is approximately sinusoidal with maximum at the centre and zero at the ends and it is given by,

$$\left. \begin{aligned} I &= I_m \sin \beta (h - z) \quad \text{for } z > 0 \\ I &= I_m \sin \beta (h + z) \quad \text{for } z < 0 \end{aligned} \right\} \longrightarrow \textcircled{1}$$

where  $I_m$  = current maximum at the current element  $I dz$ ,

$$\beta = \frac{2\pi}{\lambda} = \text{phase constant and}$$

$I dz$  - current element placed at a distance ' $z$ ' from  $z=0$  plane.

Consider a point 'P' located at a far distance from the current element. Then the Vector potential at point P due to current element  $I dz$  is given by,

$$dA_z = \frac{\mu}{4\pi R} I dz e^{-j\beta R} \longrightarrow \textcircled{2}$$

Where  $R =$  Distance between Idz to distant point P.

By integrating 2nd equation, we get over the total of the antenna and it is given by,

$$\int dA_z = \int_{-h}^0 \frac{\mu I dz e^{-j\beta R}}{4\pi R} + \int_0^h \frac{\mu I dz e^{-j\beta R}}{4\pi R}$$

$$A_z = \frac{\mu}{4\pi} \int_{-h}^0 \frac{I_m \sin\beta(h+z) e^{-j\beta R}}{R} dz + \frac{\mu}{4\pi} \int_0^h \frac{I_m \sin\beta(h-z) e^{-j\beta R}}{R} dz \quad \text{--- (3)}$$

Since the point P is at a large distance, the lines to the point P may be assumed to be parallel. We know that,

$$R = r - z \cos\theta$$

$R = r$ , When 'P' is at a large distance and replace  $R$  in the denominators of equation (3) only by  $r$ . But, in numerator, 'R' represents the phase factor and therefore the difference between  $R$  and  $r$  is very important.

$$A_z = \frac{\mu}{4\pi} \int_{-h}^0 \frac{I_m \sin\beta(h+z) e^{-j\beta(r-z\cos\theta)} dz}{r} + \frac{\mu}{4\pi} \int_0^h \frac{I_m \sin\beta(h-z) e^{-j\beta(r-z\cos\theta)} dz}{r}$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[ \int_{-h}^0 \sin\beta(h+z) e^{j\beta z \cos\theta} dz + \int_0^h \sin\beta(h-z) e^{j\beta z \cos\theta} dz \right]$$

For a  $\lambda/2$  antenna,  $L=2h=\lambda/2$  &  $z=h=\lambda/4 = \pi/2$  degrees

where

$$\sin\beta(h+z) = \sin\beta\left(\frac{\pi}{2}+z\right) = \cos\beta z$$

$$\sin\beta(h-z) = \sin\beta\left(\frac{\pi}{2}-z\right) = \cos\beta z$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[ \int_{-h}^0 \cos\beta z e^{j\beta z \cos\theta} dz + \int_0^h \cos\beta z e^{+j\beta z \cos\theta} dz \right]$$

Now  $\int_{-h}^0 e^{j\theta} d\theta = \int_0^h e^{-j\theta} d\theta$ . By using this property, change

limits of integration of first term.

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[ \int_0^h \cos\beta z e^{-j\beta z \cos\theta} dz + \int_0^h \cos\beta z e^{j\beta z \cos\theta} dz \right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[ \int_0^h \cos\beta z (e^{-j\beta z \cos\theta} + e^{j\beta z \cos\theta}) dz \right] \times \frac{2}{2}$$

WKT,

$$\frac{e^{-j\beta z \cos\theta} + e^{j\beta z \cos\theta}}{2} = \cos(\beta z \cos\theta)$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \int_0^h \cos\beta z \cdot 2 \cos(\beta z \cos\theta) dz \longrightarrow \textcircled{4}$$

$$2 \cos\alpha \cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \int_0^h \cos\beta z (1 + \cos\theta) + \cos\{\beta z (1 - \cos\theta)\} dz$$

↳ ⑤



Integrating eqn ⑤. &  $h = z = \lambda/4$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r} \left[ \frac{\sin(\beta z (1+\cos\theta))}{\beta(1+\cos\theta)} + \frac{\sin\beta z (1-\cos\theta)}{\beta(1-\cos\theta)} \right]_0^{\lambda/4}$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r} \left[ \frac{(1-\cos\theta) \sin\beta z (1+\cos\theta) + (1+\cos\theta) \sin\beta z (1-\cos\theta)}{1-\cos^2\theta} \right]_0^{\lambda/4}$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r} \left[ \frac{(1-\cos\theta) \sin\left(\frac{\pi}{2} + \frac{\pi}{2} \cos\theta\right) + (1+\cos\theta) \sin\frac{\pi}{2} (1-\cos\theta)}{1-\cos^2\theta} \right]$$

$\left[ \because \beta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \right]$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r} \left[ \frac{(1-\cos\theta) \cos(\pi/2 \cos\theta) + (1+\cos\theta) \cos(\pi/2 \cos\theta)}{\sin^2\theta} \right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r} \left[ \frac{\cos(\pi/2 \cos\theta) [1-\cos\theta + 1+\cos\theta]}{\sin^2\theta} \right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r} \left[ \frac{\cos(\pi/2 \cos\theta)}{\sin^2\theta} \right] \longrightarrow \textcircled{6}$$

The next step is to find the magnetic field using Maxwell's equation for spherical co-ordinate system.

The  $\phi$  components of H is given by,

$$H_\phi = \frac{1}{\mu} (\nabla \times A)_\phi = \frac{1}{\mu} \times \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} (A_r) \right] \longrightarrow \textcircled{7}$$

But now the current element is placed along z-axis, then  $A_r = -A_z \sin\theta$  and  $A_\theta = 0$  and by substituting in eqn (7) we get



$$H_{\phi} = \frac{1}{\mu} \times \frac{1}{r} \left[ \frac{d}{dr} (r A_z \cos \theta) \right] \longrightarrow \textcircled{8}$$

By substituting,  $A_z$  of eqn (6) in eqn (8)

$$= \frac{1}{\mu} \times \frac{1}{r} \left\{ \frac{d}{dr} \left[ \frac{-r \mu I_m e^{-j\beta r}}{2\pi\beta r} \left\{ \frac{\cos(\pi/2 \cos \theta)}{\sin^2 \theta} \right\} \sin \theta \right] \right\}$$

$$= \frac{-I_m}{2\pi\beta r} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] \frac{d}{dr} [e^{-j\beta r}]$$

$$= \frac{-I_m}{2\pi\beta r} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right] e^{-j\beta r} (-j\beta)$$

$$H_{\phi} = \frac{j I_m e^{-j\beta r}}{2\pi\beta r} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]$$

The magnitude of the magnetic field strength or magnetic field intensity for the radiation field of a half wave dipole is given by,

$$|H_{\phi}| = \frac{I_m}{2\pi r} \left\{ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right\} \text{ A/m} \longrightarrow \textcircled{9}$$

The electric field expression for the radiation field can be achieved from

$$\frac{E_{\theta}}{H_{\phi}} = \eta = 120\pi$$

$$|E_{\theta}| = 120\pi |H_{\phi}| \longrightarrow \textcircled{10}$$

By substituting  $H_{\phi}$  from eqn (9) in eqn (10), we get

$$|E_{\theta}| = 120\pi \times \frac{I_m}{2\pi r} \left\{ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right\}$$

$$|E_0| = \frac{60 I_m}{r} \left\{ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right\} \text{ V/m} \rightarrow (11)$$

This is the amplitude of electric field intensity of radiation field of a  $\lambda/2$  antenna (or) a  $\lambda/4$  antenna.

Power radiated by a half wave dipole and its radiation resistance.

The product of magnitude values of  $E_0$  and  $H_\phi$  and it is given as

$$P_{\max} = |E_0| |H_\phi|$$

$$= \left[ \frac{60 I_m}{r} \left\{ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right\} \right] \left[ \frac{I_m}{2\pi r} \left[ \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] \right]$$

$$P_{\max} = \frac{30 I_m^2}{\pi r^2} \left[ \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \right]^2 \rightarrow (12)$$

Average power in terms of effective or R.M.S current:

The average value of the power is half of the maximum power and it is expressed as,

$$P_{\text{avg}} = \frac{E_0}{\sqrt{2}} \cdot \frac{H_\phi}{\sqrt{2}} = \frac{1}{2} E_0 \cdot H_\phi = \frac{P_{\max}}{2}$$

$$P_{\text{avg}} = \frac{15 I_m^2}{\pi r^2} \left[ \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right]^2 \text{ W/m}^2 \rightarrow (13)$$

The effective or RMS current is related to the maximum current by the relation is given by,

$$I_{\text{r.m.s}} = \frac{I_m}{\sqrt{2}} \Rightarrow \frac{I_m}{1} = \sqrt{2} I_{\text{r.m.s}}$$

$$= \frac{15 (\sqrt{2} I_{\text{rms}})^2}{\pi r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2$$

$$P_{\text{avg}} = \frac{30 I_{\text{rms}}^2}{\pi r^2} \left[ \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \right] \text{ W/m}^2 \rightarrow (14)$$

This is expression for average power in terms of RMS current. This total radiated power is given by the surface integral of Poynting vector over any surrounding surface,

$ds =$  elemental area of spherical shell  $= 2\pi r^2 \sin\theta d\theta$

$$\text{Power radiated } (P_r) = \oint P_{\text{avg}} \cdot ds \rightarrow (15)$$

By substituting (14) in (15), we get

$$= \int_0^\pi \frac{30 I_{\text{rms}}^2}{\pi r^2} \left\{ \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \right\} 2\pi r^2 \sin\theta d\theta$$

$$= 60 I_{\text{rms}}^2 \int_0^\pi \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} d\theta$$

$$= 60 I_{\text{rms}}^2 \int_0^\pi \frac{1}{2} \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta$$

$$= 60 I_{\text{rms}}^2 \int_0^\pi \frac{1}{2} \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta$$

$$= 60 I_{\text{rms}}^2 \cdot I$$

$$\text{Where } I = \frac{1}{2} \int_0^\pi \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta \rightarrow (16)$$

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The value of 'I' after integration is

$$I = 1.219$$

$$\text{Power radiated} = 60 I_{\text{rms}}^2 \times 1.219$$

$$P_r = 73.140 I_{\text{rms}}^2 \longrightarrow (17)$$

The equation (17) is an expression for the total power radiated by a half wave dipole in free space.

Radiation Resistance:

$$W_r = R_r \cdot I_{\text{rms}}^2 \longrightarrow (18)$$

By comparing equation (17) with equation (18), we get

$$R_r = 73.14 \simeq 73 \Omega$$

The radiation resistance of a centre fed half wave dipole or simply dipole antenna is  $73.14 \Omega$  (or) approximately  $73 \Omega$ .

## FREQUENCY INDEPENDENT ANTENNAS.

\* A frequency - independent antenna is physically fixed size and operates on an instantaneous basis over a wide bandwidth (entire frequency band) with relatively constant impedance, pattern, polarization and gain.

\* These antennas are broadband antennas which are using 10 to 10000 MHz region for practical applications such as TV, point to point communication feeds for reflectors and lenses.

### Rumsey's Principle:

The condition of the frequency independent antenna was pointed out by V. H Rumsey. He stated that, "the performance that is, the impedance and pattern properties of a lossless antenna is independent of frequency if the dimensions of an antenna are specified in terms of angles such that they remain constant in terms of wavelength.



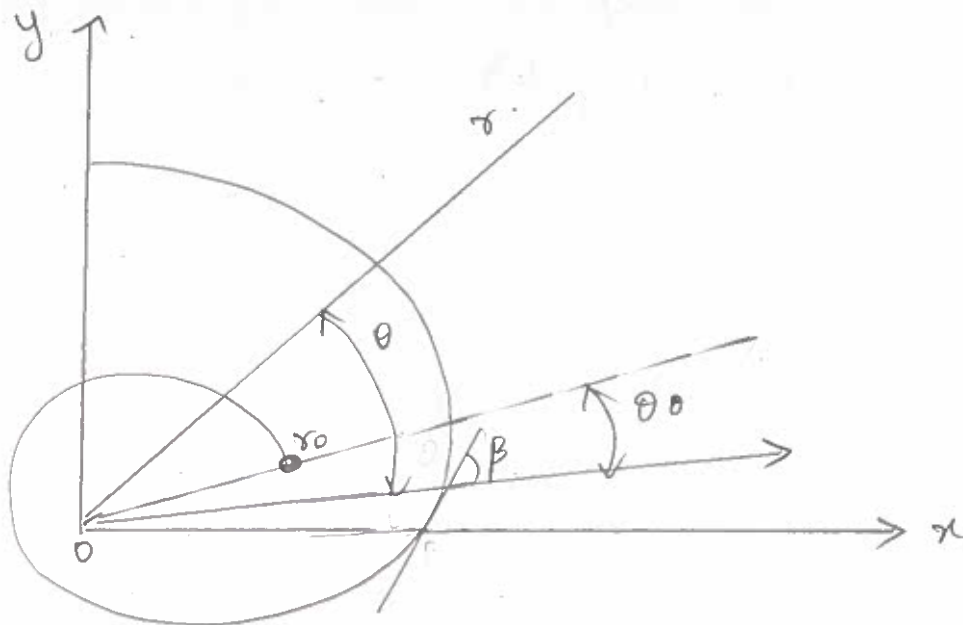
### SPIRAL ANTENNA:

\* Spiral Antennas are a frequency independent antenna. It radiates a bidirectional main lobe perpendicular to the plane of the antenna. It produces circularly polarised waves within the band of operation and the radiation is elliptically polarized outside the band of operation.

\* The surface of an equiangular spiral shape can be described completely by the angles. It fulfills all the necessary conditions that are employed to design the frequency independent antennas.

\* When the total arm length is comparable with the wavelength, the frequency of an operation will be the lowest cut-off frequency and for all other frequencies above this, the pattern and impedance characteristics are frequency independent.

### Planar Log-Spiral Antenna:





\* The equation of a logarithmic (or) log spiral is given by

$$r = a^\theta \longrightarrow (1a)$$

$$(or) \ln r = \theta \ln a \longrightarrow (1b)$$

where  $r \rightarrow$  radial distance to point P on spiral  
 $\theta \rightarrow$  Angle with respect to x axis  
 $a \rightarrow$  constant.

\* From equation (1a) the rate of change of radius with an angle is obtained as

$$\frac{dr}{d\theta} = a^\theta \ln a = r \ln a. \longrightarrow (2)$$

\* The constant 'a' in equation (2) is related to the angle  $\beta$  between the spiral and a radial line from the origin is given by,

$$\ln a = \frac{dr}{r d\theta} = \frac{1}{\tan \beta} \longrightarrow (3)$$

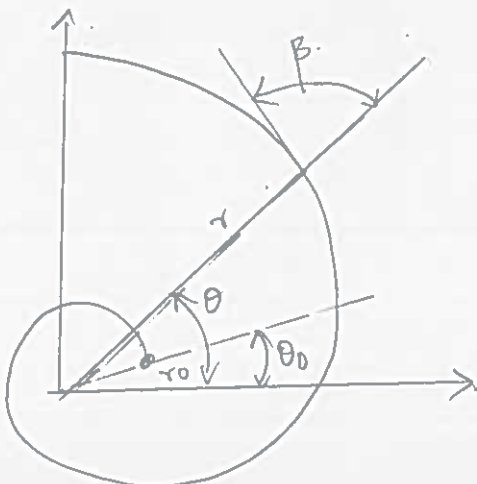
\* From equation (1b),

$$\ln r = \theta \ln a$$

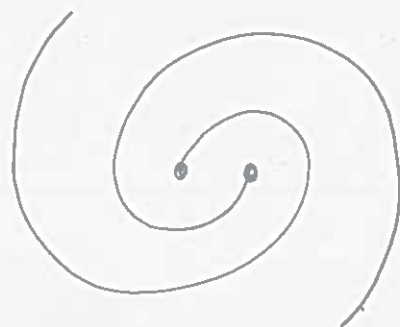
By substituting equation (3) in above expression we have

$$\ln r = \frac{\theta}{\tan \beta}$$

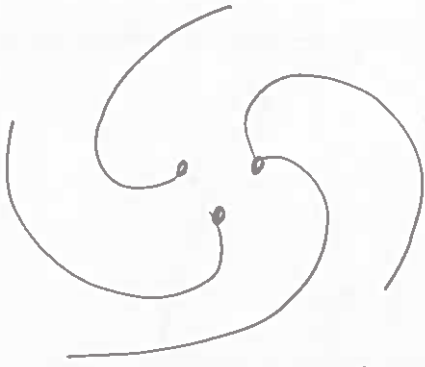
$$\theta = \tan \beta \ln r \longrightarrow (4)$$



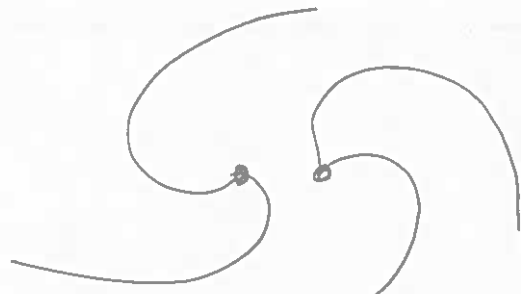
(a) Single spiral.



(b) Two spiral ( $\theta_0 = \theta, \pi$ )



(c) Multiple spiral  
( $\theta_0 = 0, \pi/2, \pi, 3\pi/2$ )



(d) Multiple spiral  
( $\theta_0 = 0, \pi/2, \pi, 3\pi/2$ )

\* The log spiral is constructed so as to make  $r=1$  and  $\theta=0$  and  $r=2$  at  $\theta=\pi$ . These conditions determine the value of constant 'a' and 'beta'. From equation (3) and (4)  $\beta = 77.6^\circ$  and  $a = 1.247$ .

\* The shape of the spiral is determined by the angle  $\beta$  which is the same for all points on the spiral.

\* For a second log spiral, which is identical in the form to the fig (a) and be generated by an angular rotation  $\delta$ , so that equation (1) becomes,

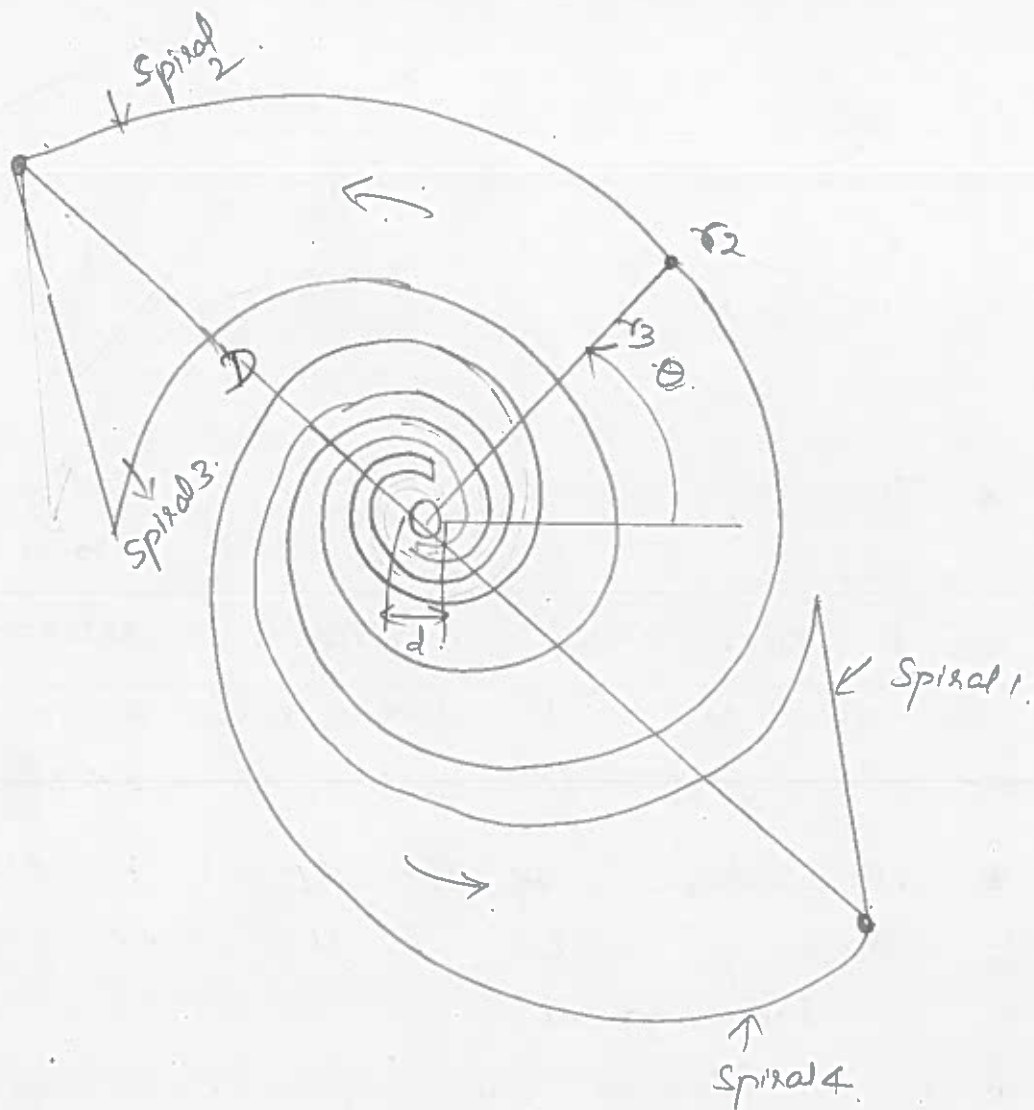
$$r_2 = a^{\theta - \delta} \rightarrow (5a)$$

and a third and fourth spiral is given by,

$$r_3 = a^{\theta - \pi} \rightarrow (5b)$$

and  $r_4 = a^{\theta - \pi - \delta} \rightarrow (5c)$

\* Then for a rotation  $\delta = \frac{\pi}{2}$  we have 4 spirals at  $90^\circ$  angles. Metalizing the areas between (spirals) 1 and 4 and 2 and 3, with the other areas open, self-complementary and the congruence conditions are satisfied.



\* From the above figure, the arrows indicate the direction of the outgoing waves travelling along the conductors resulting in right circularly polarized (RCP) radiation outward from the page and left-circularly polarized radiation in the pages.

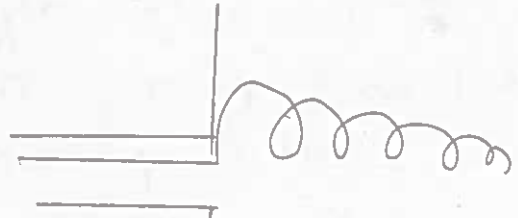
\* The high-frequency limit of operation is determined by the spacing 'd' of the input terminal and the low frequency limit by the overall diameter 'D'. The ratio  $D/d$  for the above figure is 25 to 1.

### Conical - Spiral (CP) Antenna:

\* A tapered helix is a conical spiral antenna and these were described and investigated extensively in the years following 1947.



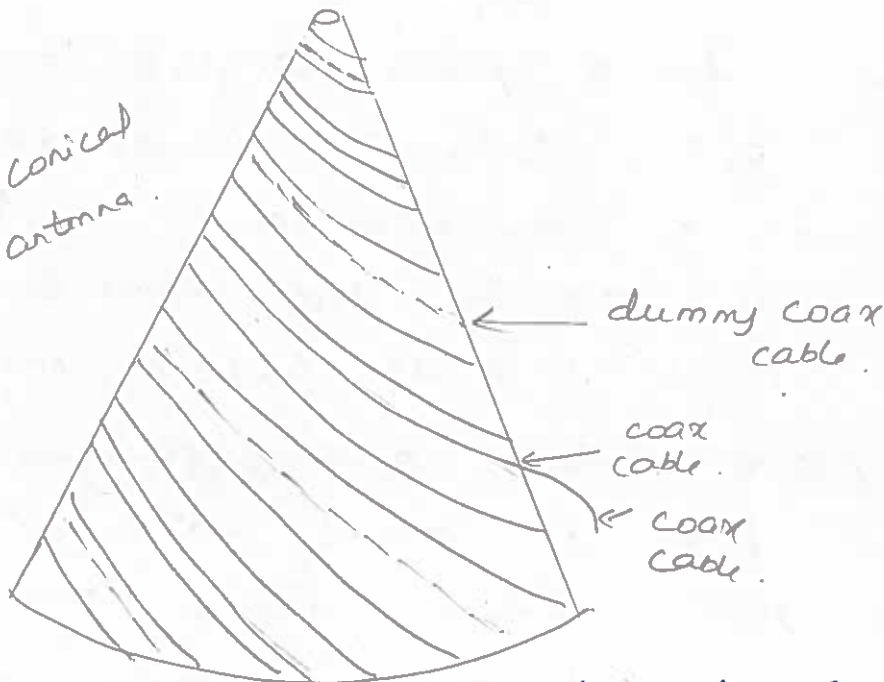
(a)  $\alpha$ -increasing



(b)  $\alpha$ -decreasing

\* The above figures shows tapered helical or conical spiral antennas in which the pitch angle is constant with diameter and turn spacing variable.

Two arm balanced conical spiral antenna.



\* The conducting signal spiral surface can be constructed conveniently using printed circuit technique, the conical antenna arms on dielectric cone which is also used as a support. The feed cable can be bonded to the metal arms which are wrapped around the cone as shown in the above figure.



\* The conical equi-angular spiral antenna is fed at the apex by means of a balanced transmission line carried up inside the cone along the axis of the cone.

\* The main difference between the conical spiral and planar antenna is that the conical spiral antenna provides unidirectional radiation in a single lobe towards the apex of the cone and with a maximum radiation along the axis.

\* The conical antennas, the circular polarization and relatively constant impedance are preserved over large bandwidth.

\* The input impedance is between 100 to 150 ohms for a pitch angle  $\alpha = 17^\circ$  and full angles  $20^\circ$  to  $60^\circ$ . The bandwidth depends on the ratio of base diameter to the truncated apex diameter and this ratio may be chosen arbitrarily larger such as 5:1 or more.

\* The planar spiral antenna produces a bidirectional beam of about  $50^\circ$  to  $60^\circ$ . On the other hand, for a narrow angled cone, it produces an unidirectional beam of about  $70^\circ$  to  $90^\circ$  width.

\* Conical spirals can be used in conjunction with a ground plane, but with a reduction in the bandwidth when they are flush mounted on the plane.

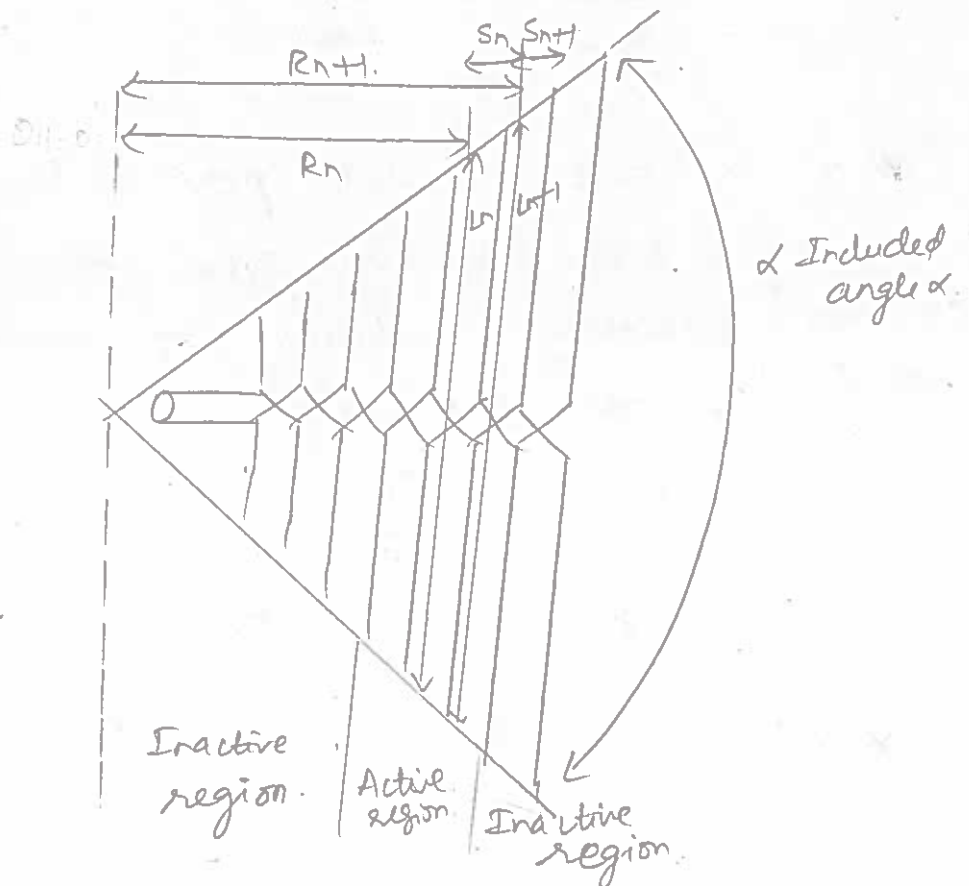


# LOG PERIODIC ANTENNA:-

\* A log periodic antenna is a broadband narrow beam antenna. It is a frequency independent antenna.

\* The geometry of an antenna structure is adjusted such that all the electrical properties of the antenna must repeat periodically with the logarithm of frequency. For every repetition, the structure size changes by a constant scale factor by which the structure can either be expanded or contracted. The log periodic principle can be understood with the help of the array of log periodic antenna known as log periodic Dipole Array (LPDA).

## Construction of LPDA:



\* The LPDA consists of a number of dipoles of different lengths and spacings. The array is fed using a balanced transmission line which is connected at narrow end or apex of the array. Also the transmission line is transposed between each adjacent pairs of terminals of dipoles.

\* The length of the dipoles increases from feed point towards other end such that the included angle  $\alpha$  remains constant. The dipole lengths and the spacings between two adjacent dipoles are related through parameter called design ratio or scale factor denoted by  $\tau$ . Thus the relationship between spacings 'S' and lengths  $L$  of adjacent elements are scaled as

$$\frac{S_n}{S_{n+1}} = \frac{L_n}{L_{n+1}} = \tau \quad \longrightarrow \textcircled{1}$$

\*  $\tau$  is also called periodicity factor which is always less than 1. The above expression can be written in terms of constant  $k$  with the radii of the arm as

$$\frac{R_{n+1}}{R_n} = \frac{S_{n+1}}{S_n} = \frac{L_{n+1}}{L_n} = \frac{1}{\tau} = k; k > 1$$

where  $n=1, 2, 3, \dots, n$ .

$\longrightarrow \textcircled{2}$

\* The spacing factor ( $\sigma$ ) is defined as

$$\sigma = \frac{S_n}{2L_n} \quad \longrightarrow \textcircled{3}$$

\* The ends of the dipoles lie along the straight lines on both the sides. These two straight lines meet at feed point or apex having an angle  $\alpha$  which is angle included by two straight line. (Typical value of  $\alpha = 30^\circ$  and  $\tau = 0.7$ ).

Working Principle of LPDA:-

\* The Analysis of a log periodic dipole array can be done by considering three region of an antenna which is classified according to the length of the dipoles. They are

- (i) Inactive transmission - line region ( $L < \lambda/2$ )
- (ii) Active region  $L \approx \lambda/2$ .
- (iii) Inactive reflective region ( $L > \lambda/2$ ):

(i) Inactive transmission line region ( $L < \lambda/2$ ):

\* It is the region in which the length of the dipoles is less than the resonant length  $\lambda/2$ . Therefore, the elements present relatively high capacitance impedance. The spacing between the elements are comparatively smaller.

\* The current in the region will be very small and hence it is considered as inactive region. These currents leads to the voltage supplied by the transmission line. Trans position of transmission introduces  $180^\circ$  phase shift between adjacent dipoles.

\* Hence currents in the elements of these regions are small and hence small radiation in backward direction (towards left).

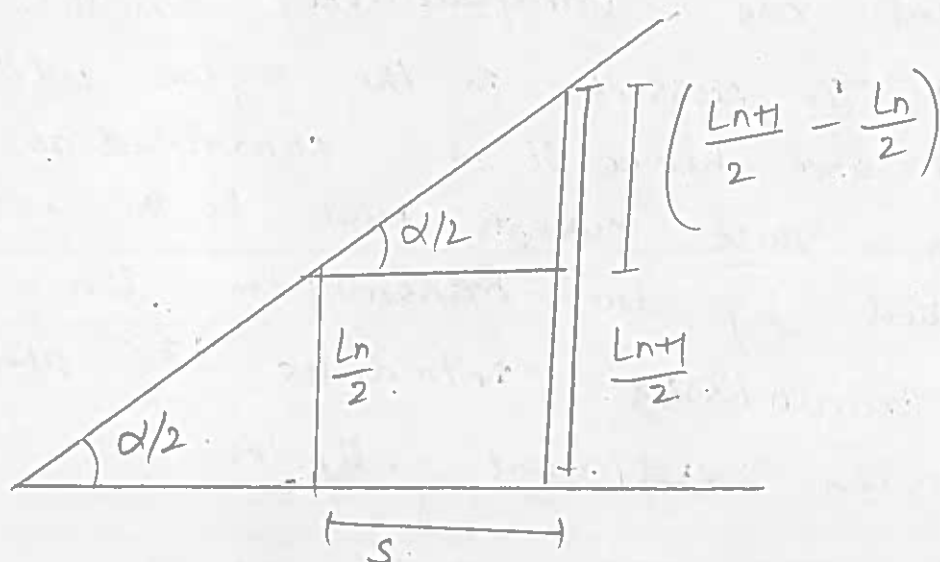
(ii) Active Region ( $L \approx \lambda/2$ ):

In this region, the dipole lengths are approximately equal to the resonant length ( $\lambda/2$ ). Therefore, the dipoles in this region offer a resistive impedance. Thus the element currents are of large value and are in phase with the base voltage. Hence, there is a strong radiation towards left in backward direction and a little radiation towards right.

(iii) Inactive Reflective Region ( $L > \approx \lambda/2$ ):

The element (dipoles) lengths are longer than the resonant length (i.e.)  $L > \approx \lambda/2$ . Hence the dipole offers an inductive impedance. The currents will be smaller in this region and also lags at the base voltage. Thus, any small amount of incident wave from an active region is reflected back towards the backward direction.

Design of LPDA:





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\* Consider a part of a log periodic array as shown in figure. The performance of a log periodic dipole array depends on the following parameters.

- (i) Apex angle ( $\alpha$ )
- (ii) Design ratio ( $\tau$ ) and
- (iii) Spacing factor ( $\sigma$ ).

\* From the figure

$$\tan(\alpha/2) = \frac{\frac{L_{n+1} - L_n}{2}}{S} \rightarrow (4)$$

$$\tan(\alpha/2) = \frac{L_{n+1} - L_n}{2S} = \frac{L_{n+1} \left[ 1 - \frac{L_n}{L_{n+1}} \right]}{2S} \rightarrow (5)$$

But  $\frac{L_{n+1}}{L_n} = k$  (i.e)  $\frac{L_n}{L_{n+1}} = \frac{1}{k} \rightarrow (6)$

\* By substituting the equation (6) in equation (5)

$$\tan(\alpha/2) = \frac{(1 - 1/k)L_{n+1}}{2S} \rightarrow (7)$$

For active region  $L_{n+1} = \lambda/2 \rightarrow (8)$

\* By substituting the equation (8) in (7)

$$\tan(\alpha/2) = \frac{(1 - 1/k)(\lambda/2)}{2S} = \frac{1 - 1/k}{4(S/\lambda)} = \frac{1 - 1/k}{4\sigma} \rightarrow (9)$$

where  $\sigma = S/\lambda =$  Spacing factor.

$\alpha \rightarrow$  Apex angle

$k \rightarrow$  Scale factor

But  $\tau = 1/k$

$$\tan(\alpha/2) = \frac{1 - \tau}{4\sigma} \rightarrow (10)$$



\* Out of the three parameters ( $\sigma$ ,  $\tau$ , and  $\alpha$ ) two are specified and the third is determined. The number of elements in an array ( $n$ ) can be obtained from an upper frequency ( $f_u$ ) and lower frequency ( $f_L$ ) and it is given as:

$$\log(f_u) - \log(f_L) = (n-1) \log\left(\frac{1}{\epsilon}\right) \rightarrow \textcircled{11}$$

### Uses of Log Periodic Antenna:

(i) It is mainly used in the field of HF communication where the multiband steerable (rotatable) and fixed antennas are generally used. It has an advantage that no power is wasted in terminating resistance.

(ii) It is used for TV reception. Only one log periodic design will suffice for all the channels even upto an UHF band.

(iii) It is best suited for all round monitoring (i.e) a simple log periodic antenna will receive all the higher frequency bands, when there is no problem with the cost of installation.

## Microstrip Antennas (or) patch antennas

The antenna which is made up of metal plates placed on dielectric and fed by microstrip (or) coplanar transmission line is called microstrip antenna. It is also called as patch antenna (or) microstrip patch antenna.

As MSA are directly printed on to the circuit boards so it is called as printed antenna.

### Applications

The MSA is generally preferred in high performance aircraft, spacecraft and missile applications where size, weight, cost, performance, ease of installation, and aerodynamic profile are the main constraints and low profile antennas are required.

### Construction

MSAs are constructed on a dielectric substrate using a process similar to lithography in which patterns are printed on the substrate while fabricating on printed circuit board (or) integrating circuits.

The MSA consists of a very thin ( $t \ll \lambda$ ) metallic strip (or) patch placed over a substrate. The substrate in between the patch and ground plane is a dielectric sheet or a dielectric constant are usually in the range of  $2.2 \leq \epsilon_r \leq 12$ .

The height is very small ( $h \ll \lambda$ ) as compared to the free space wavelength  $\lambda_0$

$$0.003 \lambda \leq h \leq 0.05 \lambda$$

$$\text{Length of the patch } \lambda/5 < L < \lambda/2$$

The size of MSA is inversely proportional to its frequency. At frequencies lower than for an AM radio at 1 MHz, the microstrip patch would be of the size of a football field for MSA designed to receive an fm radio at 100 MHz its length would be of the order of 1 m. At X-band, MSA size will be of the order of 1 cm.

### Rectangular Microstrip antenna

Rectangular shape is simplest and most widely used configuration for fabrication of microstrip antennas, dimension of length  $L$  of patch is always greater than the dimension of  $w$ .

\* The shapes are useful for low cross polarization radiation and radiation pattern can be easily analyzed. square patch can generate pencil beam while the rectangular patch can generate fan beam. Actually the circular patch is easy to fabricate but it is very difficult to calculate the current distribution in it.

\* To obtain linear and circular polarization either a single element (or) array of microstrip antenna can be used. To achieve greater directivity array of microstrip elements with single (or) multiple feed are used.

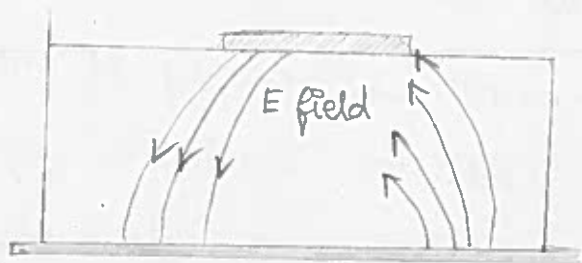
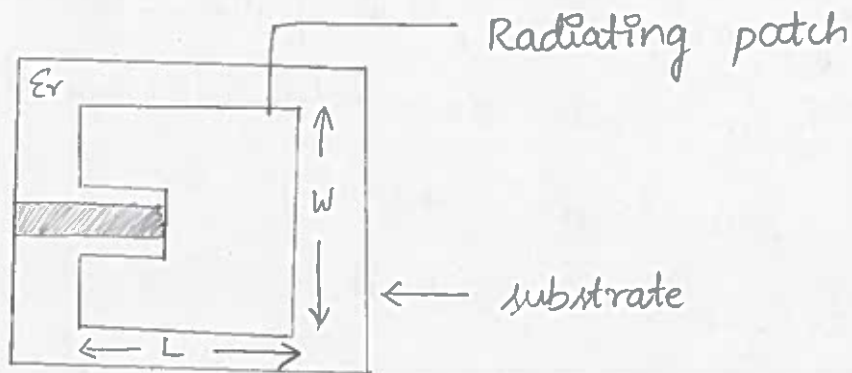
\* These E fields lines emerge out and Propagated in a direction which is normal to the substrate, they are row in the same direction. As the fields are in same phase both get added together.

\* The frequency of operation of the patch antenna is generally determined by the length L. The critical frequency

$$f_c \approx \frac{c}{2L\sqrt{\epsilon_r}} = \frac{1}{2L\sqrt{\epsilon_0\epsilon_r\mu_0}}$$



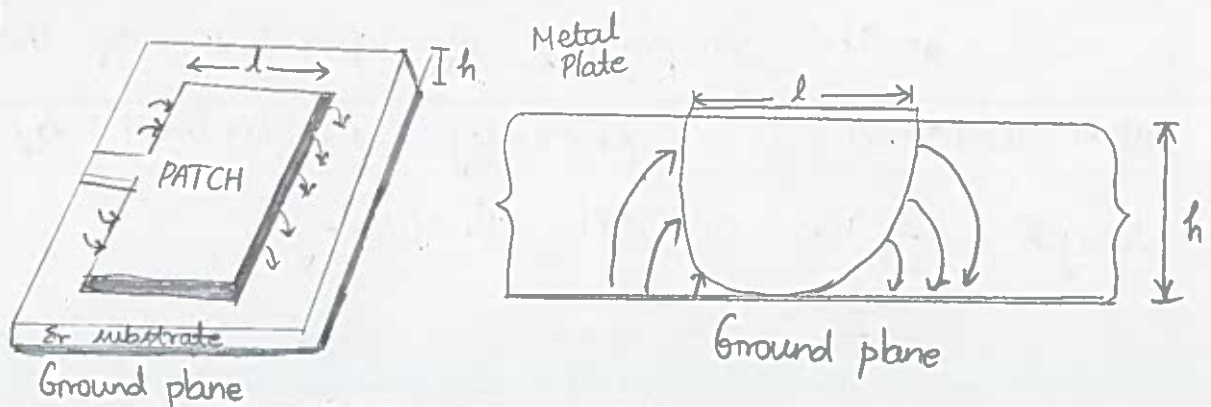
If patch length ( $L = \lambda/2$ ) the electric field produced under the edges of opposite polarity



Electric field of patch antenna

### Types of patches in MSA

The shapes of the radiating element (or) patch are out of the shapes, square, rectangular, triangular, dipole and circular are most commonly used shapes for the patch because of ease in fabrication.





where  $c$  - velocity of light

$\epsilon_0$  - permittivity of free space

$\epsilon_r$  - permittivity of dielectric substrate

$\mu_0$  - permittivity of free space.

The expression for frequency of operation of patch antenna ~~then~~ considering  $L$  &  $w$  is given by

$$f_{r, nm} = \frac{c}{2 \sqrt{\epsilon_{r, eff}}} \left[ \left\{ \frac{n}{L + 2\Delta L} \right\}^2 + \left\{ \frac{m}{W + 2\Delta W} \right\}^2 \right]^{1/2}$$

for dominating mode  $n=1, m=0$

$$f_{r, nm} = \frac{c}{2(L + 2\Delta L) \sqrt{\epsilon_{r, eff}}}$$

$w$  is the important parameter as it controls the i/p impedance of the antenna.

For square patch antenna ( $L = w$ ), i/p impedance is typically same.



Square



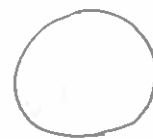
Rectangular



ellipse



Triangular



Circular



Elliptical



Circular ring



Disc scalar

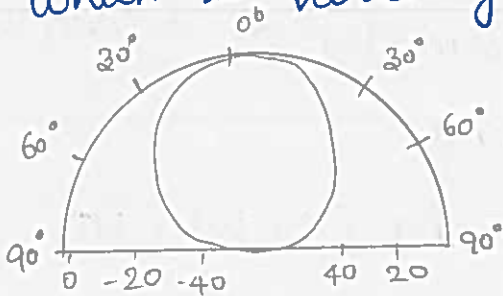


Ring sector

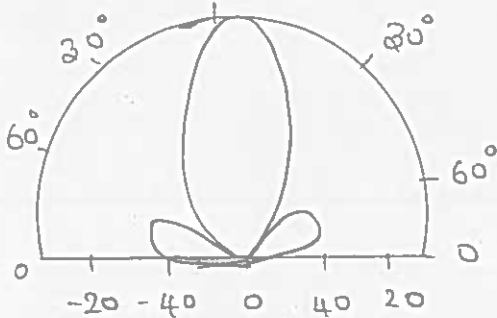
### Various shapes of patches

### Radiation pattern of MSA

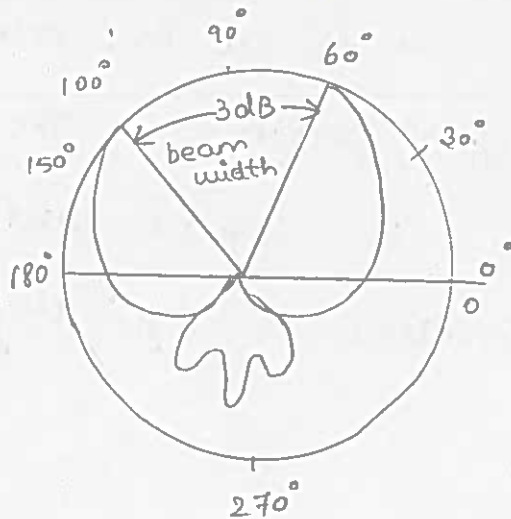
Consider a substrate of height  $h \ll \lambda \approx 0.05\lambda$  and length  $L = 0.5\lambda$ . The structure radiates from the fringing fields exposed above the substrate at the edges of the patch. The patch acts as resonator cavity with an electric field perpendicular to the patch. The magnetic field has tangential components which is vanishing at the four edges of the patch.



a)  $\phi = 0^\circ$



b)  $\phi = 90^\circ$



c)  $R_p$  for linearly polarized MSA

# feed methods of MSA

## 1) contacting feed

In this method, the RF power is fed directly to the radiating patch which uses a connecting element such as microstrip (or) co-axial line. The commonly used feeds are microstrip feed and coaxial feed.

## 2) Non-contacting feed

Here electromagnetic coupling is done to transfer the power from feed line to the radiating patch. The most commonly used non contacting feed methods are aperture the fields of the lowest resonate mode  $L \gg \lambda$  are given by

$$E_z = -E_0 \sin\left(\frac{\pi x}{L}\right) \quad -\frac{L}{2} \leq x \leq \frac{L}{2}$$

Most of the radiation from the MSA comes from sides 1 & 3. The sides 2 & 4 contribute little to the total radiation and they are normally negligible.

The normalised gain is given by

$$G(\theta, \phi) = \frac{|E(\theta, \phi)|^2}{|E(\theta, \phi)|_{max}^2}$$

The expression for E field components  $E_\theta$  and  $E_\phi$  are given by

$$E_\theta = \frac{\sin[(kw \sin\theta \sin\phi)/2]}{(kw \sin\theta \sin\phi)/2} \cos\left[\frac{kl \sin\theta \cos\phi}{2}\right]$$

$$E_\phi = \frac{\sin[(kw \sin\theta \sin\phi)/2]}{(kw \sin\theta \sin\phi)/2} \frac{\cos(kl \sin\theta \cos\phi)}{2} \sin\phi$$

where  $\phi$  and  $\theta$  are elevation and angle of radiation pattern.

$$k = \frac{2\pi}{\lambda} = \text{wave number}$$

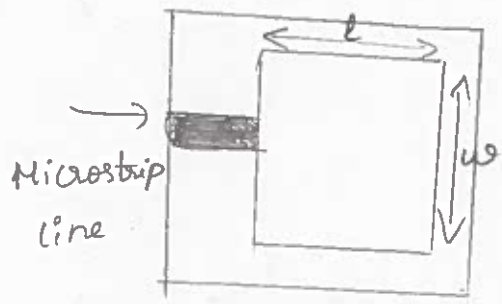
Thus the resultant field at any point is given by

$$E(\theta, \phi) = \sqrt{E_\theta^2 + E_\phi^2}$$

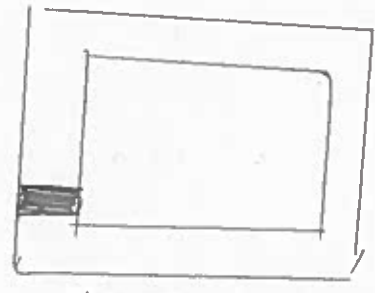
The normalised Radiation pattern is obtained by  $L = W = \lambda/2$   $\sin\theta = 0$ ,  $\phi = 90^\circ$  plane and linear polarized patch antenna

Coupling and proximity coupling

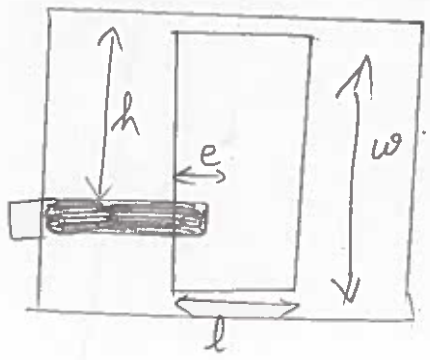
The microstrip feed methods are further subdivided into four main classes.



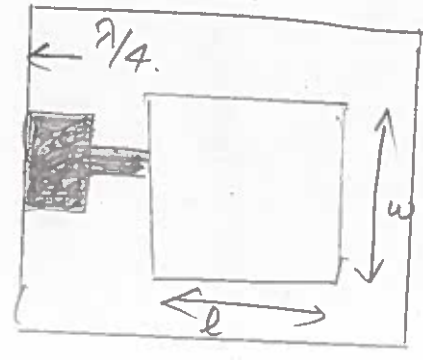
(a) Center feed.



(b) Offset feed.



(c) Inset feed.



(d) Quarter wave line feed.

Advantages:-

- \* MSA are low profile antennas. They are smaller in size, light and weight antenna.
- \* Low fabrication cost.
- \* Simple and inexpensive.

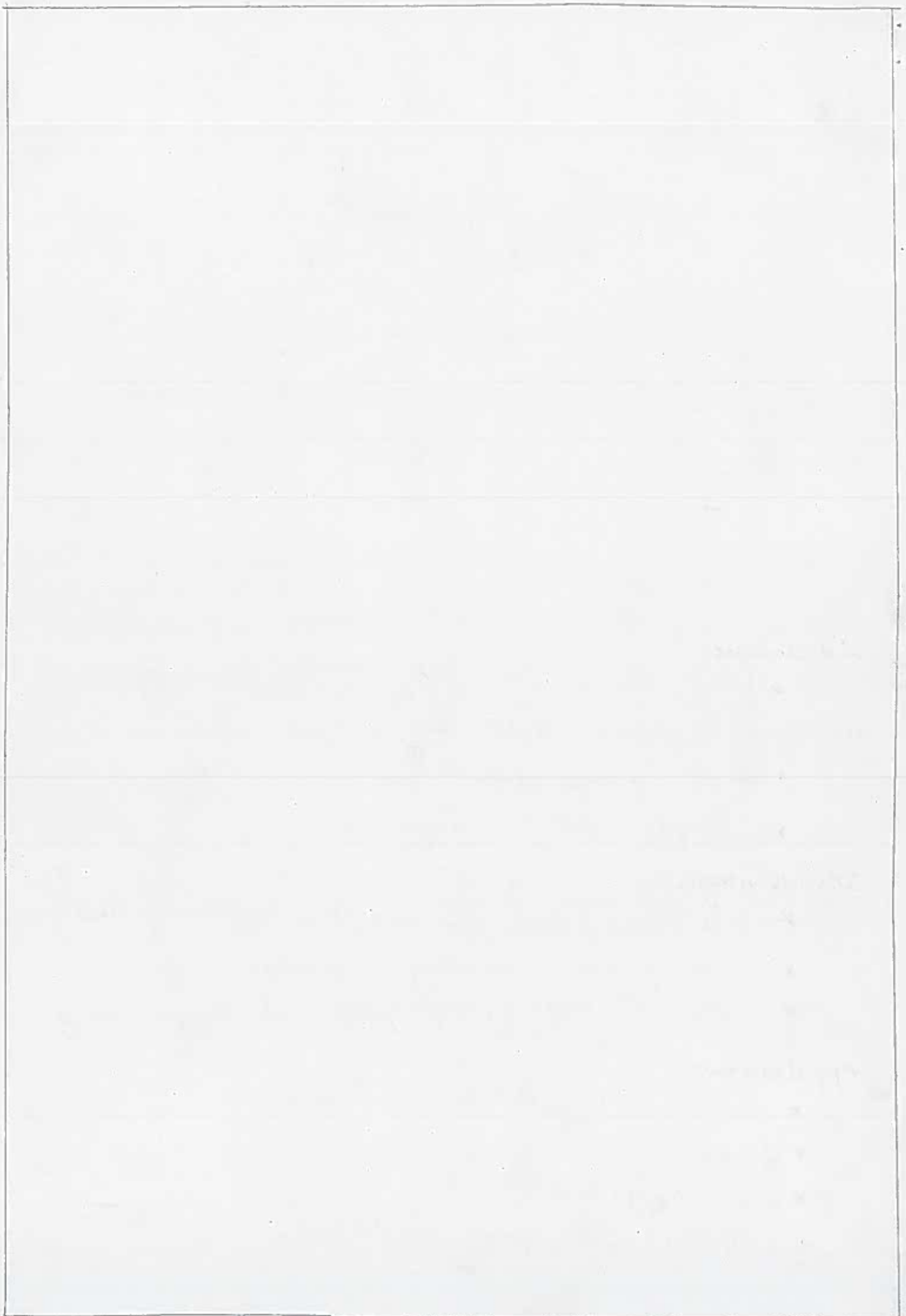
Disadvantages:-

- \* MSA's are low gain and low efficiency antennas.
- \* Low power handling capacity.
- \* Size is inversely proportional to frequency.

Applications:-

- \* Mobile and satellite communication.
- \* RFID.
- \* Radar application.
- \* Military and space application.





## UNIT-III

### ANTENNA ARRAYS AND APPLICATIONS

#### Antenna Array:

An antenna array is simply defined as a system of similar antennas oriented similarly to get the greater directivity in a desired direction.

#### Linear Array:

The individual antenna of an antenna array system is termed as an 'element'. An antenna array is said to be linear, if the elements of an antenna array are equally spaced along a straight line.

#### Uniform Linear Array:

The linear antenna array is said to be an uniform linear array, if all the elements are fed with a current of equal magnitudes with the progressive uniform phase shift along the line.

#### Advantages:

- High directivity is obtained
- High SNR is obtained
- Increase in overall gain
- Power wastage is reduced
- Better performance

### Disadvantages:

- Mounting and maintenance is difficult
- Large space required for placing antennas
- High resistive losses.

\* Practically, the various forms of the antenna array are used as radiating systems. Some of the practically used forms are as follows:

- Broadside array
- Endfire array
- Collinear array
- Parasitic array.

### Two-Element Array: Arrays of Two Point Sources:

Array of two driven  $\lambda/2$  elements.

\* This is the simplest situation in the arrays of isotropic point sources in which it is assumed that the two point sources are separated by a distance (say 'd') and also have the same polarization.

\* According to antenna theory, the superposition or addition of fields from the various sources at a great distance with the due regard to phases.

\* Arrays of two isotropic point sources are different cases as follows:

- Equal amplitude and phase
- Equal amplitude and opposite phase
- Unequal amplitude and opposite phase.

# (1) Arrays of Two point sources with Equal Amplitude and Phase:

## BROAD SIDE ARRAY:

\* The Broadside array is defined as, "the array of antennas in which all the identical antennas are placed parallel to each other along axis of antenna array and each element is perpendicular to the axis of antenna array.

\* The direction of maximum radiation is always perpendicular to the plane consisting elements (antenna array axis)."

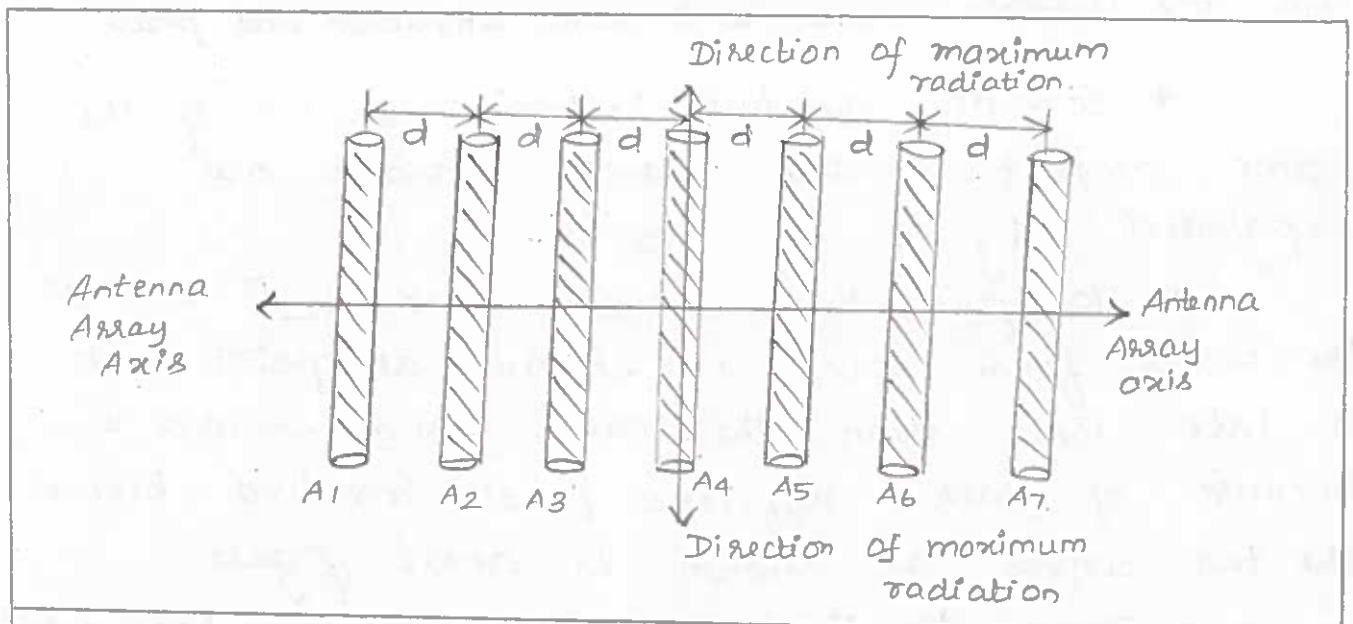


Fig. Broadside array of Antennas

\* All individual antennas are spaced equally along the axis of antenna array. The spacing between any two elements is denoted by 'd' and they are fed with the currents of equal magnitude and same phase and the radiation pattern for the broadside array is bidirectional.

### (i) Field Pattern:-

\* Two isotropic point sources symmetrically situated with respect to an origin in the cartesian coordinate system as shown below.

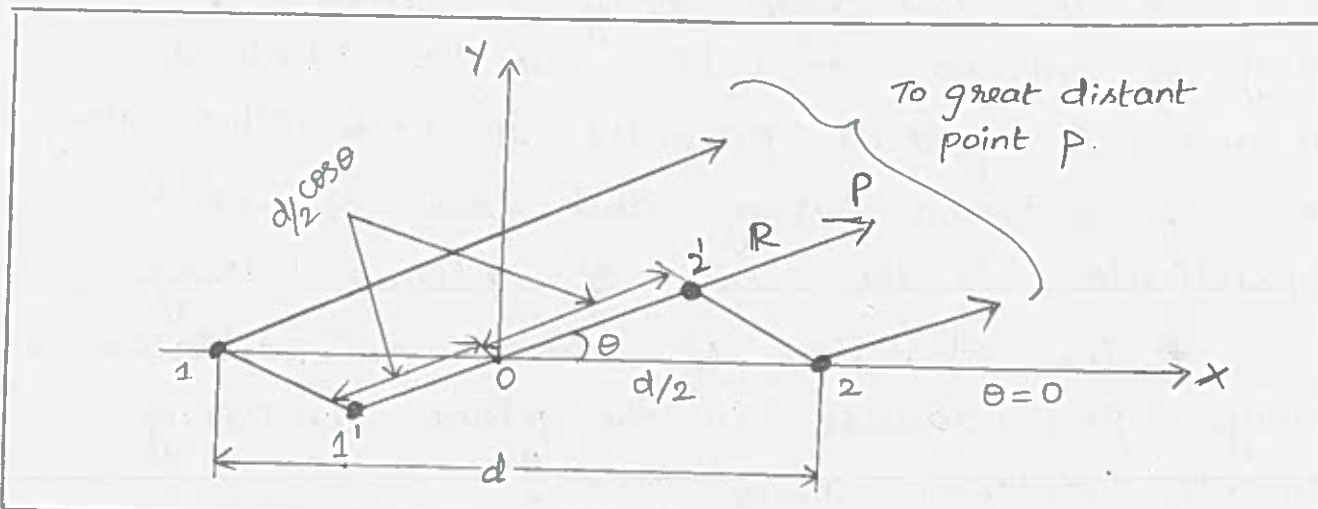


Fig. Two isotropic sources with same amplitude and phase

\* Consider the two isotropic sources of the same amplitude and phase 1 and 2 are separated by a distance 'd'.

\* To find the resultant field at P consider the waves from source 1 reaches at point P at a later time than the waves from source 2 because of path difference (1'2') involved between the two waves as shown in above figure.

\* Thus the fields due to source 1 lags while that due to source 2 leads.

\* Path difference between the two waves is expressed as,

$$= \frac{d}{2} \cos \theta + \frac{d}{2} \cos \theta \quad [ \because (1'2') ]$$
$$= d \cos \theta \text{ (meters)}$$

$$\text{Path difference} = \frac{d}{\lambda} \cos \theta \text{ (wavelengths)} \longrightarrow \textcircled{1}$$



$$\begin{aligned}
 * \text{ Phase angle } (\psi) &= 2\pi \times \text{path difference} \\
 &= 2\pi \times \frac{d}{\lambda} \cos \theta \\
 &= \frac{2\pi}{\lambda} d \cos \theta \text{ (radians)}
 \end{aligned}$$

We know that  $\beta = \frac{2\pi}{\lambda}$

$$\therefore \boxed{\psi = \beta d \cos \theta \text{ radians.}} \longrightarrow \textcircled{2}$$

Now,

Let  $E_1 \rightarrow$  Far Electric Field at distance  $P$ , due to source 1.

$E_2 \rightarrow$  Far Electric field at distance  $P$ , due to source 2.

$E \rightarrow$  Total Electric field at distance point.

\* Then, total far field at distance point  $P$ , in the direction  $\theta$  is given by

$$E = E_1 e^{-j\psi/2} + E_2 e^{+j\psi/2} \longrightarrow \textcircled{3}$$

where  $E_1 e^{-j\psi/2} \rightarrow$  Field component due to source 1.

$E_2 e^{+j\psi/2} \rightarrow$  Field component due to source 2.

\* But in this case it is assumed that amplitudes are same (i.e)

$$E_1 = E_2 = E_0 \longrightarrow \textcircled{4}$$

Then, equation  $\textcircled{3}$  becomes.

$$\begin{aligned}
 E &= E_0 (e^{-j\psi/2} + e^{+j\psi/2}) \\
 &= 2E_0 \left( \frac{e^{-j\psi/2} + e^{+j\psi/2}}{2} \right) \quad \left[ \because \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \right]
 \end{aligned}$$

$$E = 2E_0 \cos(\psi/2) \longrightarrow \textcircled{5}$$

$$E = 2E_0 \cos\left(\frac{\beta d \cos \theta}{2}\right) \longrightarrow \textcircled{6}$$

\* This above equations are the equations of far-field pattern of two isotropic point sources of the same amplitude and phase.

\* Here the total amplitude  $2E_0$  whose maximum value may be 1. By substituting  $2E_0=1$  or  $E_0=\frac{1}{2}$ , the pattern is said to be normalised. Thus equation (6) becomes.

$$E = \cos\left(\frac{\beta d \cos\theta}{2}\right) \\ = \cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \frac{\cos\theta}{2}\right)$$

$$E = \cos\left(\frac{\pi}{2} \cos\theta\right) \longrightarrow (7)$$

\* In order to draw the field pattern, the direction of maxima, minima and half power points must be known, which can be calculated with the help of equation (7) as follows:

(ii) Maxima Direction for Major lobe:

$E$  is maximum, when  $\cos\left(\frac{\pi}{2} \cos\theta\right)$  is maximum ( $\pm 1$ ).

$$\cos\left(\frac{\pi}{2} \cos\theta\right) = \pm 1$$

$$\frac{\pi}{2} \cos\theta = \cos^{-1}(\pm 1)$$

$$\frac{\pi}{2} \cos\theta_{\max} = \pm N\pi \quad \text{where } N=0, 1, 2, \dots$$

If  $N=0$ .  $\frac{\pi}{2} \cos(\theta_{\max})_{\text{major}} = \pm N\pi$  [Here  $N=0$ ]

$$\cos(\theta_{\max})_{\text{major}} = 0.$$

$$(\theta_{\max})_{\text{major}} = \cos^{-1}(0)$$

$$(\theta_{\max})_{\text{major}} = 90^\circ \text{ and } 270^\circ \longrightarrow (8)$$

(iii) Minima Directions:-

E is minimum when  $\cos(\frac{\pi}{2} \cos \theta)$  is minimum and its minimum value is 0.

$$\cos(\frac{\pi}{2} \cos \theta) = 0.$$

$$\frac{\pi}{2} \cos(\theta_{min}) = \pm (2N+1)\frac{\pi}{2} \text{ when } N=0,1,2,\dots$$

If  $N=0,$

$$\frac{\pi}{2} \cos(\theta_{min}) = \frac{\pi}{2}.$$

$$\cos(\theta_{min}) = \pm 1$$

$$\theta_{min} = 0^\circ \text{ and } 180^\circ \rightarrow \textcircled{9}$$

(iv) Half Power Point Directions:-

When the power is half, the voltage or current is  $\frac{1}{\sqrt{2}}$  times the maximum value. Hence the condition for half power point is given by

$$\cos\left(\frac{\beta d \cos \theta}{2}\right) = \pm \frac{1}{\sqrt{2}}.$$

Let  $d = \lambda/2$  and  $\beta = \frac{2\pi}{\lambda}$ , then

$$\cos\left(\frac{\pi}{2} \cos \theta\right) = \pm \frac{1}{\sqrt{2}}.$$

$$\frac{\pi}{2} \cos(\theta_{HPPD}) = \pm (2N+1)\frac{\pi}{4} \text{ where } N=0,1,2,\dots$$

If  $N=0 \Rightarrow \frac{\pi}{2} \cos(\theta_{HPPD}) = \pm \frac{\pi}{4}.$

$$\cos(\theta_{HPPD}) = \pm \frac{1}{2}$$

$$\theta_{HPPD} = \cos^{-1}\left(\pm \frac{1}{2}\right)$$

$$\theta_{HPPD} = 60^\circ \text{ or } 120^\circ \rightarrow \textcircled{10}$$

\* Now the field pattern between  $E$  and  $\theta$  is drawn for the case  $d = \lambda/2$  then the below figure is obtained which is bidirectional,  $360^\circ$  rotation of this figure around  $x$ -axis will generate the 3-dimensional space pattern - a doughnut shape.

\* This is the simplest type of "Broadside array" and it is also known as "Broad side couplet" as two isotropic radiators in phase.

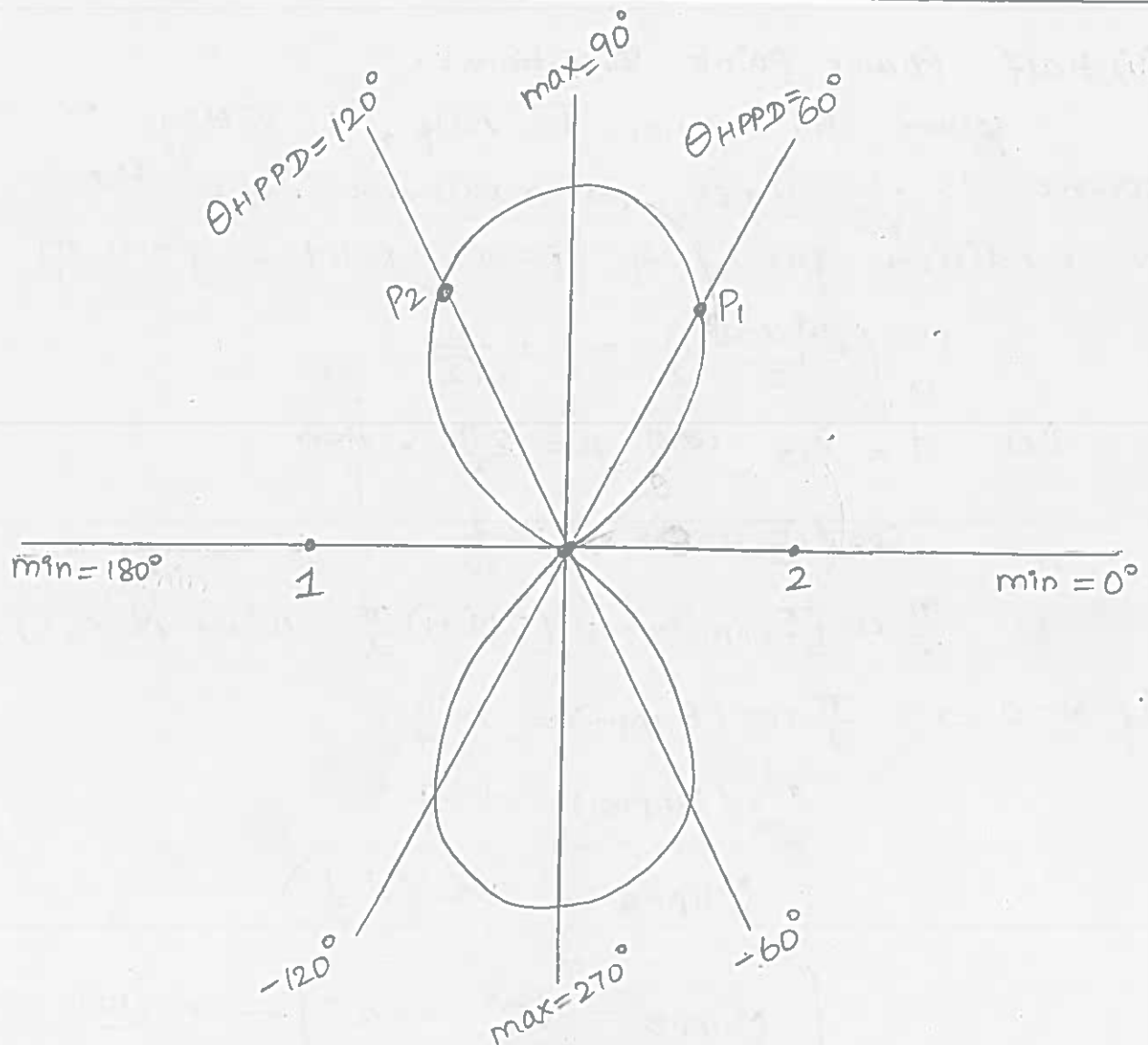


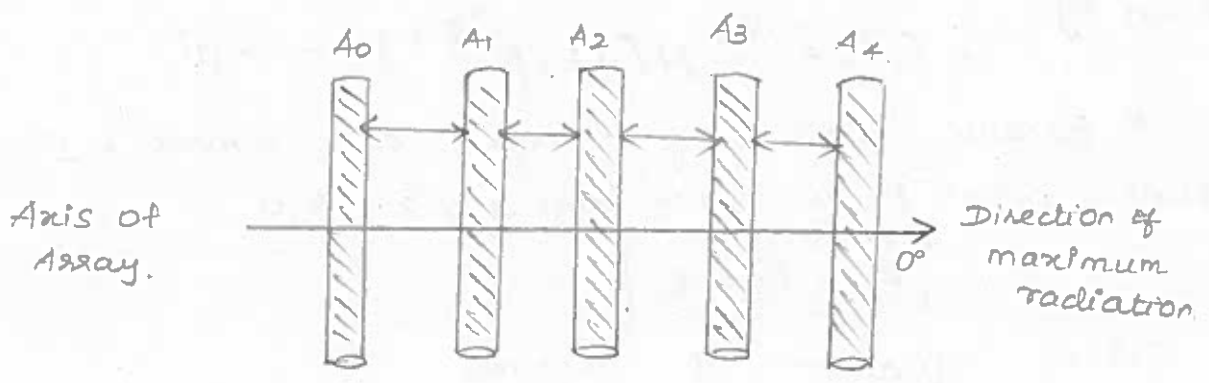
Fig. Field patterns of broadside array of two element with in-phase.

# Arrays of two point sources with Equal Amplitude and Opposite Phase :

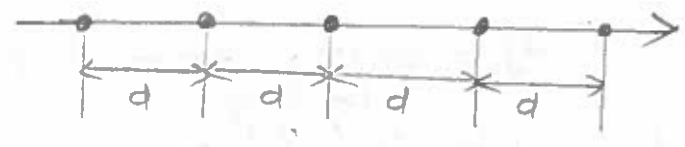
## END FIRE ARRAY:

\* An array is said to be end fire, if the direction of maximum radiation coincides with an axis to get unidirectional radiation.

\* In an end fire array, the number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to make the entire arrangement to get unidirectional radiation along the axis of the array.



(a) Top View



(b) Front View

FB End Fire Array.



(i) Field Pattern:

\* Consider an array of two centre-fed vertical  $\lambda/2$  elements (dipoles) in free-space arranged side by side with a spacing 'd' and equal currents in opposite phase (i.e. point source 1 is out of phase or opposite phase ( $180^\circ$ ) to as in below figure.

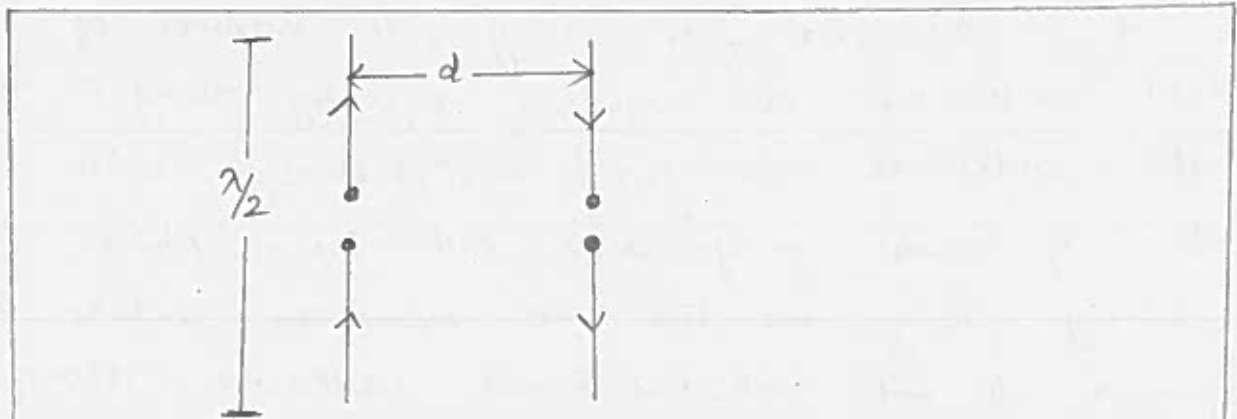


Fig. End fire array of two linear  $\lambda/2$  elements with currents of equal magnitude but opposite phase.

\* The total far field at a distant point 'P' is given by.

$$E = (-E_1 e^{-j\psi/2}) + (+E_2 e^{+j\psi/2}) \rightarrow (11)$$

\* Because, phase of source 1 and source 2 at distant point P is  $-\psi/2$  and  $+\psi/2$ . But

$$E_1 = E_2 = E_0$$

Then equation (11) becomes.

$$E = E_0 2j \left( \frac{e^{+j\psi/2} - e^{-j\psi/2}}{2j} \right)$$

$$E = 2j E_0 \sin\left(\frac{\psi}{2}\right) \rightarrow (12)$$

$$E = 2j E_0 \sin\left(\frac{\beta d}{2} \cos\theta\right) \rightarrow (13)$$

\* The operator 'j' simply means that an opposite phase brings a phase shift of  $90^\circ$  in the total field.

Let  $d = \lambda/2$  and  $2E_0 j = 1$ .

$$E_{norm} = \sin\left(\frac{\beta d}{2} \cos\theta\right) \quad \left[\because \beta = \frac{2\pi}{\lambda} \text{ and } d = \lambda/2\right]$$

$$= \sin\left(\frac{2\pi}{\lambda} \times \lambda/4 \cos\theta\right)$$

$$E_{norm} = \sin\left(\frac{\pi}{2} \cos\theta\right) \longrightarrow (14)$$

(ii) Maximum Directions:-

The maximum value of sine function is  $\pm 1$ .

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = \pm 1.$$

$$\frac{\pi}{2} \cos(\theta_{max}) = \sin^{-1}(\pm 1)$$

$$\frac{\pi}{2} \cos(\theta_{max}) = \pm 1 (2N+1) \frac{\pi}{2} \quad \text{where } N=0,1,2,\dots$$

$$\text{If } N=0 \Rightarrow \cos(\theta_{max}) = \pm 1$$

$$\theta_{max} = 0^\circ \text{ and } 180^\circ \longrightarrow (15)$$

(iii) Minima Directions:-

Minimum value of a sine function is 0.

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = 0$$

$$\frac{\pi}{2} \cos(\theta_{min}) = \pm N\pi \quad \text{where } N=0,1,2,\dots$$

$$\cos(\theta_{min}) = 0$$

$$\theta_{min} = 90^\circ \text{ and } 270^\circ \longrightarrow (16)$$

(iv) Half Power Point Directions:-

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = \pm \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2} \cos(\theta_{HPPD}) = \sin^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2} \cos(\theta_{HPPD}) = \pm (2N+1) \frac{\pi}{4} \quad \text{where } N=0,1,2,\dots$$

$$\text{If } N=0 \Rightarrow \cos(\theta_{HPPD}) = \pm \frac{1}{2}$$

$$\theta_{HPPD} = \cos^{-1}\left(\pm \frac{1}{2}\right)$$

$$\theta_{HPPD} = 60^\circ, \pm 120^\circ \longrightarrow (16)$$

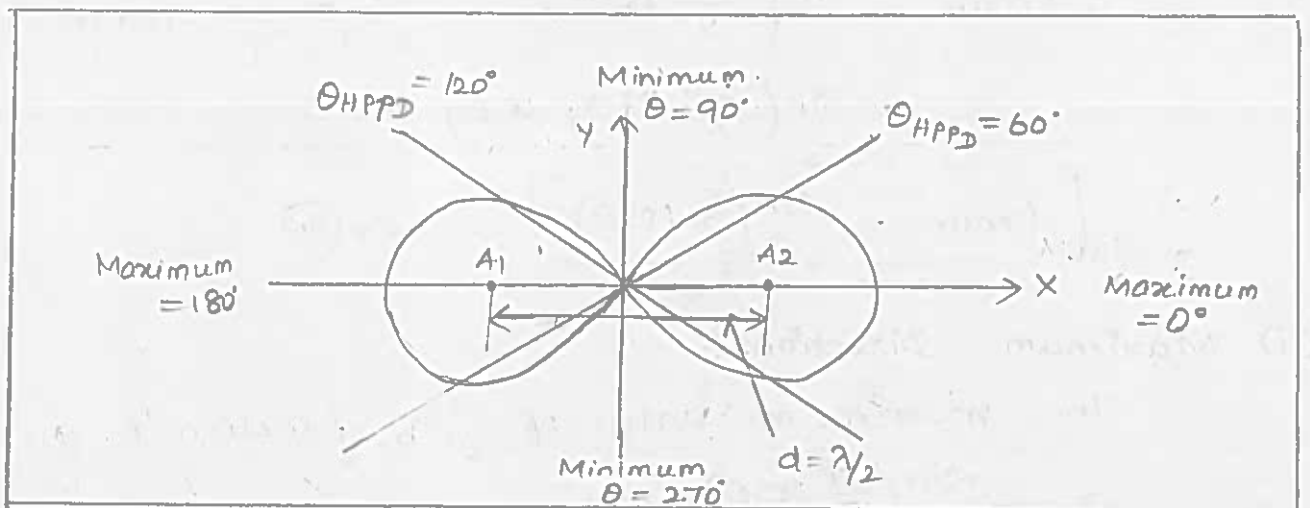


Fig. Field pattern for two point sources with spacing  $d = \lambda/2$  and fed with current equal in magnitude but out of phase by  $180^\circ$ .

Arrays of Two Points Sources with Unequal Amplitude and Any Phase: Equal Currents of any phase Relation.

\* Consider two points sources with an unequal amplitude and hence any phase difference say  $\alpha$ . Assume the source 1 is taken as a reference for phase and amplitude.

\* Fields due to source 1 and 2 at a distant point P and  $E_1$  and  $E_2$  where  $E_1$  is greater than  $E_2$  ( $E_1 > E_2$ ).

Then the total phase difference between the radiations of two sources at a point P is given by

$$\psi = \frac{2\pi}{\lambda} d \cos \theta + \alpha \quad \longrightarrow (17)$$

where  $\alpha$  is the phase angle by which the current ( $I_2$ ) of source 2 leads to the current ( $I_1$ ) of source 1.

If  $\alpha = 0^\circ$  or  $180^\circ$  and  $E_1 = E_2 = E_0$ , then the total fields at P is given by

$$E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 (1 + E_2/E_1 e^{j\psi}) \quad \left[ \because e^{j0} = e^0 = 1 \right]$$

$$E = E_1 (1 + k e^{j\psi}) \quad \longrightarrow (18)$$

where  $k = \frac{E_2}{E_1}$  and  $E_1 > E_2$ . Then  $k < 1$  (i.e)  $0 \leq k \leq 1$ .

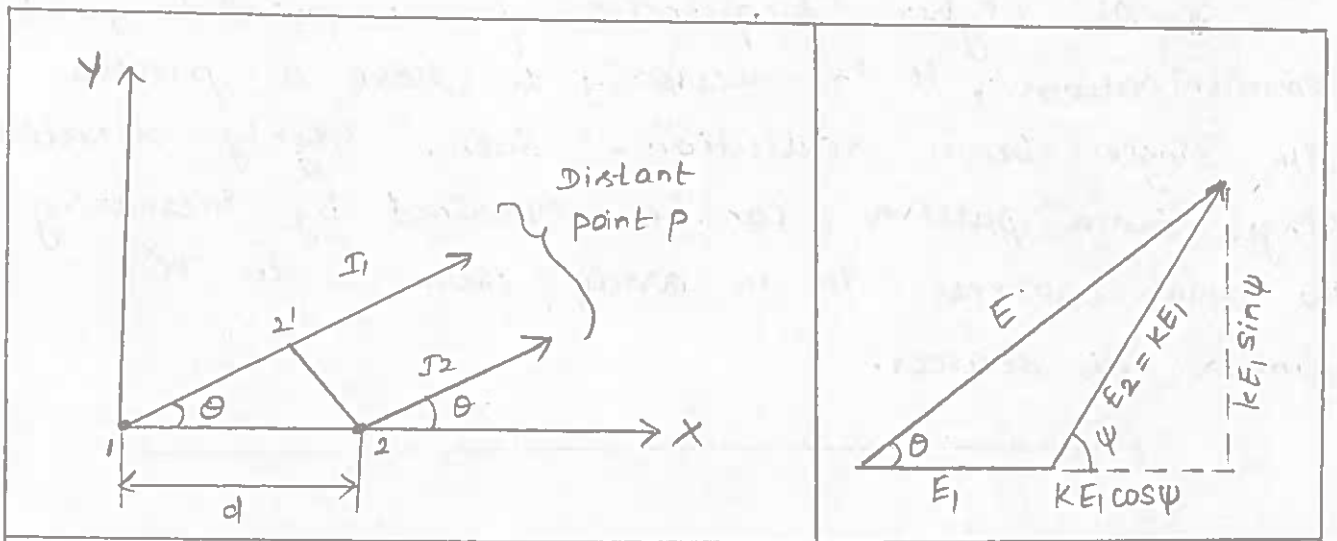


Fig (a). Two point sources with unequal amplitude and any phase difference  $\alpha$ .

Fig (b) Vector diagram of fig (a).

From equation (18)

Magnitude and phase angle ( $\theta$ ) at point P is given by taking its modulus.

$$E = |E_1 \{ 1 + k(\cos \psi + j \sin \psi) \}|$$

(or)

$$E = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2} \angle \theta \rightarrow (19)$$

where

Phase angle at P

$$\theta = \tan^{-1} \left( \frac{k \sin \psi}{1 + k \cos \psi} \right) \rightarrow (20)$$



## N-ELEMENT LINEAR ARRAY : UNIFORM AMPLITUDE AND SPACING.

\* At higher frequencies, for a point to point communications, it is necessary to have a pattern with single beam radiation. Such, highly directive single beam pattern can be obtained by increasing the point sources in an array from 2 to 'n' number of sources.

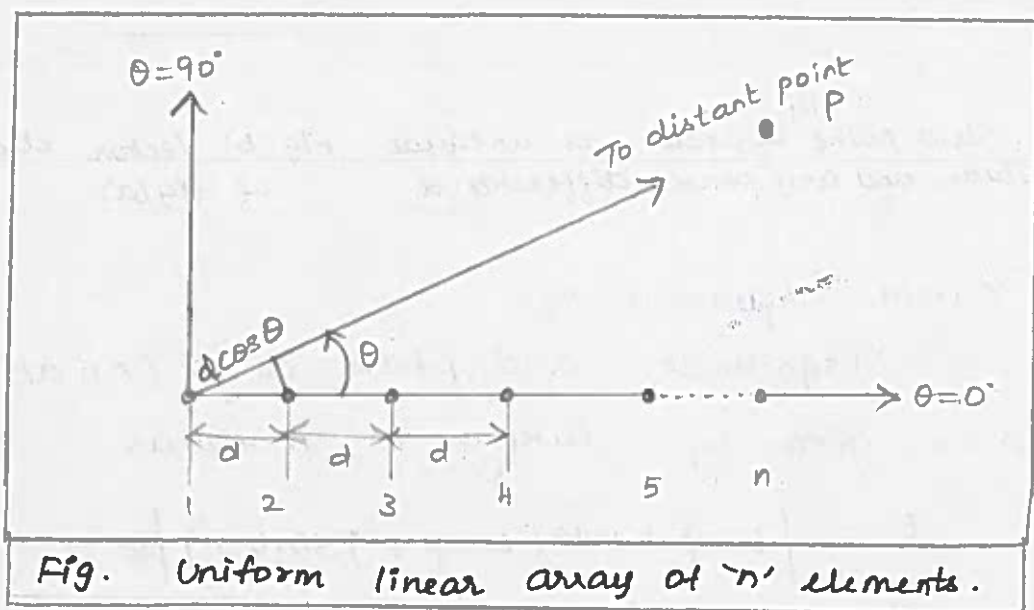


Fig. Uniform linear array of 'n' elements.

\* An array of 'n' elements is said to be a linear array, "when all the individual elements are spaced equally along a line"

\* An array is said to be a uniform array, "when the elements in the array are fed with the currents of equal magnitudes and uniform progressive phase shift along a line".

\* Consider 'n' isotropic point sources of equal amplitude and spaced equally 'd' as a linear array. Here, sources are fed with phase currents of equal amplitudes ( $E_0$ ).



\* Total far-field at a distant point 'P' is obtained by adding the vectorially individual fields of 'n' sources as,

$$E_T = E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} + \dots + E_0 e^{jn\psi}$$

$$E_T = E_0 (1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}) \rightarrow \textcircled{1}$$

\*  $\psi$  is the total phase difference of the fields at a distant point 'P' from an adjacent sources and it is expressed as,

$$\psi = \beta d \cos \theta + \alpha \text{ (radian)} \rightarrow \textcircled{2}$$

where  $\alpha$  = phase difference in adjacent point sources.

$\beta d \cos \theta$  = phase difference due to path difference and

$$\beta = \frac{2\pi}{\lambda} = \text{propagation constant.}$$

\* Consider, the source 1 is a phase reference. The field from source 2 is advanced in phase with respect to source 1 by  $\psi$ . The field from 3 is advanced in phase with respect to the source 1 by  $2\psi$  etc. Multiplying equation  $\textcircled{1}$  by  $e^{j\psi}$  becomes,

$$E_T e^{j\psi} = E_0 (e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + \dots + e^{jn\psi}) \rightarrow \textcircled{3}$$

By subtracting equation  $\textcircled{3}$  from an equation  $\textcircled{1}$ , we get.,

$$E_T - E_T e^{j\psi} = E_0 \left\{ [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{jn\psi}] - [e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}] \right\}$$

$$E_T (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$E_T = E_0 \left( \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right) \rightarrow \textcircled{4}$$

Equation (4) may be written as,

$$E_T = E_0 \left( \frac{1 - e^{jn\psi/2} \cdot e^{jn\psi/2}}{1 - e^{j\psi/2} \cdot e^{j\psi/2}} \right)$$

$$= E_0 \left( \frac{e^{jn\psi/2} \cdot e^{-jn\psi/2} - e^{jn\psi/2} \cdot e^{jn\psi/2}}{(e^{j\psi/2} \cdot e^{-j\psi/2} - e^{j\psi/2} \cdot e^{j\psi/2})} \right)$$

$$E_T = E_0 \left[ \frac{e^{j\frac{n\psi}{2}} (e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}})}{e^{j\frac{\psi}{2}} (e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}})} \right]$$

According to trigonometric identity,

$$e^{j\theta} - e^{-j\theta} = -2j \sin \theta \longrightarrow (5)$$

\* Using the equation (5), then the resultant field in equation (5) becomes,

$$E_T = E_0 \left[ \frac{(-2j \sin \frac{n\psi}{2}) e^{j\frac{n\psi}{2}}}{(-2j \sin \frac{\psi}{2}) e^{j\frac{\psi}{2}}} \right]$$

$$E_T = E_0 \cdot e^{j\left(\frac{n-1}{2}\right)\psi} \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \longrightarrow (6)$$

\* Using equation (2) the phase angle of the resultant field at point P is given as,

$$\phi = \frac{(n-1)}{2} \psi = \left(\frac{n-1}{2}\right)(\beta d \cos \theta + \alpha) \longrightarrow (7)$$

Then the equation (6) becomes.

$$E_T = E_0 \left[ \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] e^{j\phi} = E_0 \left[ \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] (\cos \phi + j \sin \phi)$$

$$E_T = E_0 \left[ \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \angle \phi \longrightarrow (8)$$

\* This equation (8) indicates the resultant field due to 'n' element linear array at a distant point P. The magnitude of the resultant field is given as,

$$E_T = E_0 \left[ \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \longrightarrow (9)$$

\* If the reference point (source 1) is shifted to the centre of an array, then the phase angle  $\phi$  is automatically eliminated from an equation (8) and it is reduced to,

$$E_T = E_0 \left[ \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \longrightarrow (10)$$

#### ANTENNA ARRAY FACTOR (AF):

\* AF is the ratio of the magnitude of the resultant field to the magnitude of the maximum field and it is given as

$$\text{Array factor} = \frac{|E_T|}{|E_{\max}|} \longrightarrow (11)$$

\* When,  $\psi \rightarrow 0$ , the field from an array is maximum in any direction. Thus the maximum value of  $E_T$  is 'n' times the field from a single source.

$$E_T(\max) = E_0 n \longrightarrow (12)$$

\* If  $E_0$  is assumed to be unity for normalized ( $E_0=1$ ) then an equation (12) becomes,

$$E_T(\max) = n.$$

\* The normalized field pattern ( $E_{Nor}$ ) an antenna array factor may be obtained from the equations (10) and (12), we get

$$E_{Nor} = \frac{E_T}{E_{T(Max)}} \\ = \frac{E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}}{E_0 n}$$

$$E_{Nor} = \frac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} = (\text{Array Factor})_n \rightarrow (13)$$

### PATTERN MULTIPLICATION:

\* Multiplication of pattern or simply pattern multiplication in general can be stated as follows:

"The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources."

\* The total field pattern of an array of non-isotropic but similar sources may be expressed as

$$\text{Total Field } (E_T) = (\text{Multiplication of field pattern}) \times (\text{Addition of phase pattern}).$$

$$E_T = \{E_i(\theta, \phi) \times E_a(\theta, \phi)\} \times \{E_{pi}(\theta, \phi) + E_{pa}(\theta, \phi)\} \rightarrow (1)$$

where  $E_i(\theta, \phi)$  = Field pattern of individual source

$E_a(\theta, \phi)$  = Field pattern of array of isotropic point sources.

$E_{pi}(\theta, \phi)$  = Phase pattern of individual source

$E_{pa}(\theta, \phi)$  = Phase pattern of array of isotropic point sources

$\theta$  - Polar angles  
 $\phi$  - Azimuth angles.

\* This principle may be applied to any number of sources but they are similar. The word similar is used here to indicate the variation with absolute angle ' $\phi$ ' of both the amplitude and phase of the field is the same.

\* The maximum amplitudes of an individual source may be equal. If their maximum amplitudes are equal, then the sources are not only similar but are also identical.

Advantages:-

Advantages of pattern multiplication are,

- (i) It is a speedy method for sketching the pattern of complicated arrays just by inspection.
- (ii) It is a useful tool in the design of antenna arrays and.
- (iii) The secondary lobes are determined from the number of nulls in the resultant pattern.



Radiation Pattern of 4- isotropic elements fed in phase, spaced  $\lambda/2$  Apart.

\* Consider a four element of a isotropic (or) non-directive radiators are in a linear array antennas and are placed as shown in figure, in which the spacing between the elements is  $\lambda/2$  and the currents are in phase (i.e)  $\alpha = 0$ .

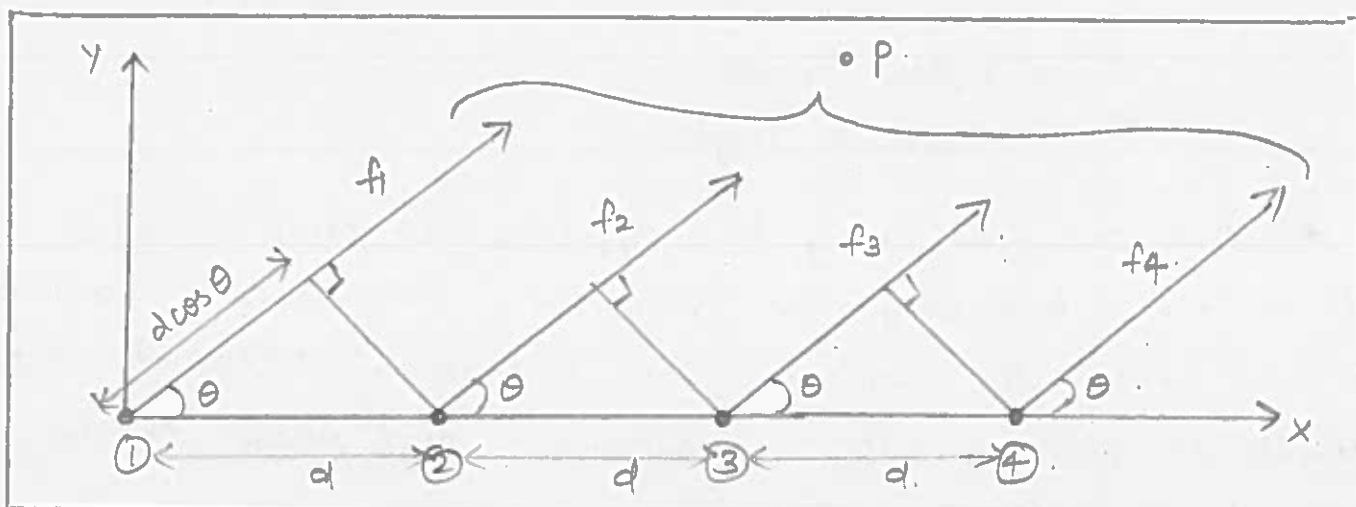


Fig. Linear array of 4 isotropic elements spaced  $\lambda/2$  apart, fed in phase.

\* One of the method to get the radiation pattern of an array is adding the fields of an individual four elements at a distance point 'P' vectorially. However, the same radiation pattern can be obtained by pattern multiplication in the following manner.

\* Now the elements 1 and 2 are considered as unit one (1) and this new unit is considered to be placed between the midway of elements 1 and 2. Similarly, the elements 3 and 4 are considered as unit two (2) and these are placed between the elements 3 and 4 and shown in figure.

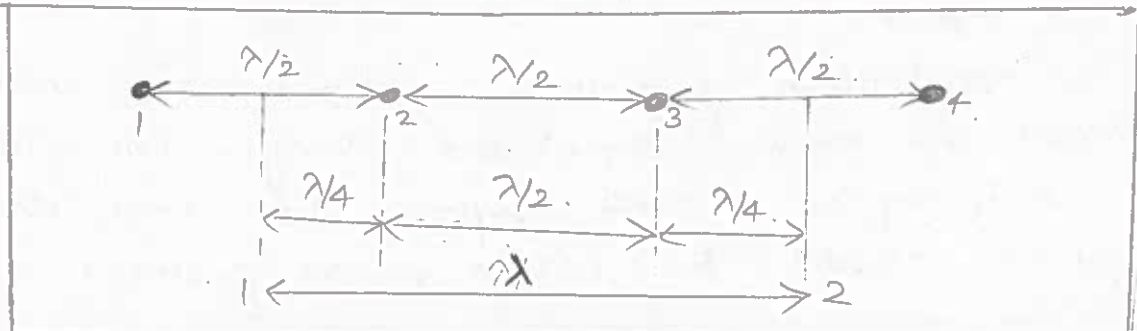


Fig. Two Units array spaced at  $\lambda$ .

\* Now the four elements are spaced at  $\lambda/2$  and have been replaced by a 2 units spaced  $\lambda$  and therefore the problem of determining the radiation of 4 elements has been reduced to find out the radiation pattern of 2 elements.

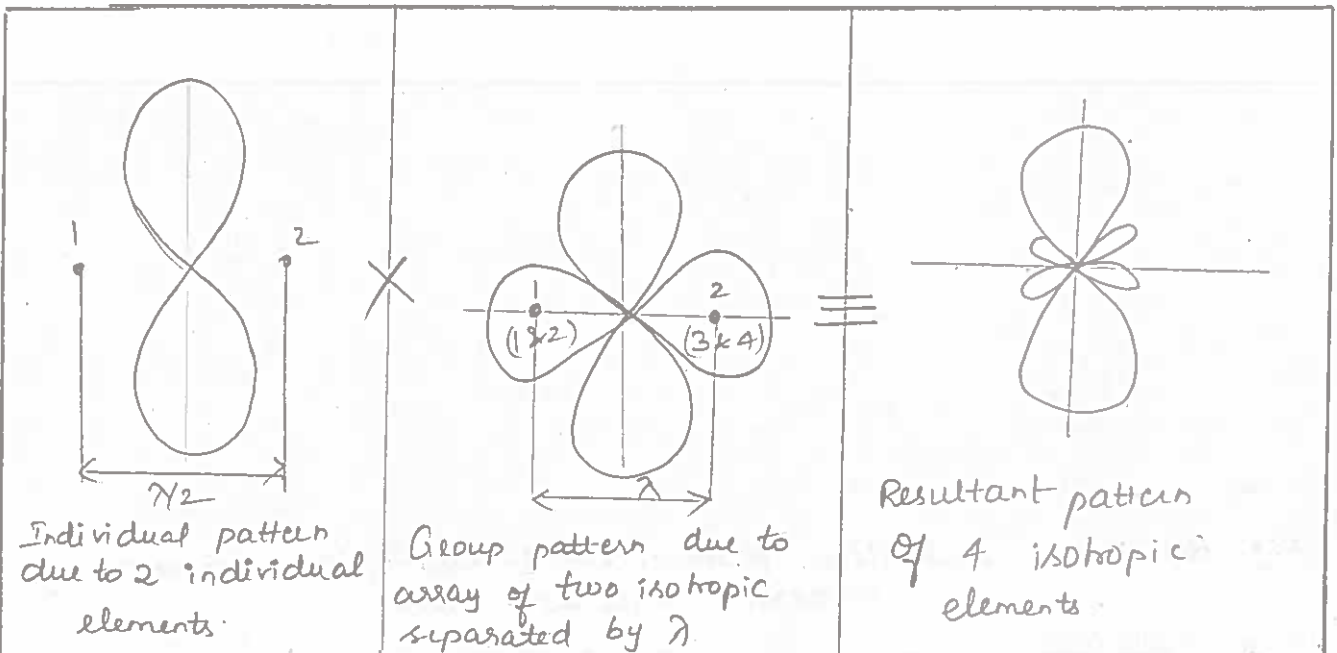


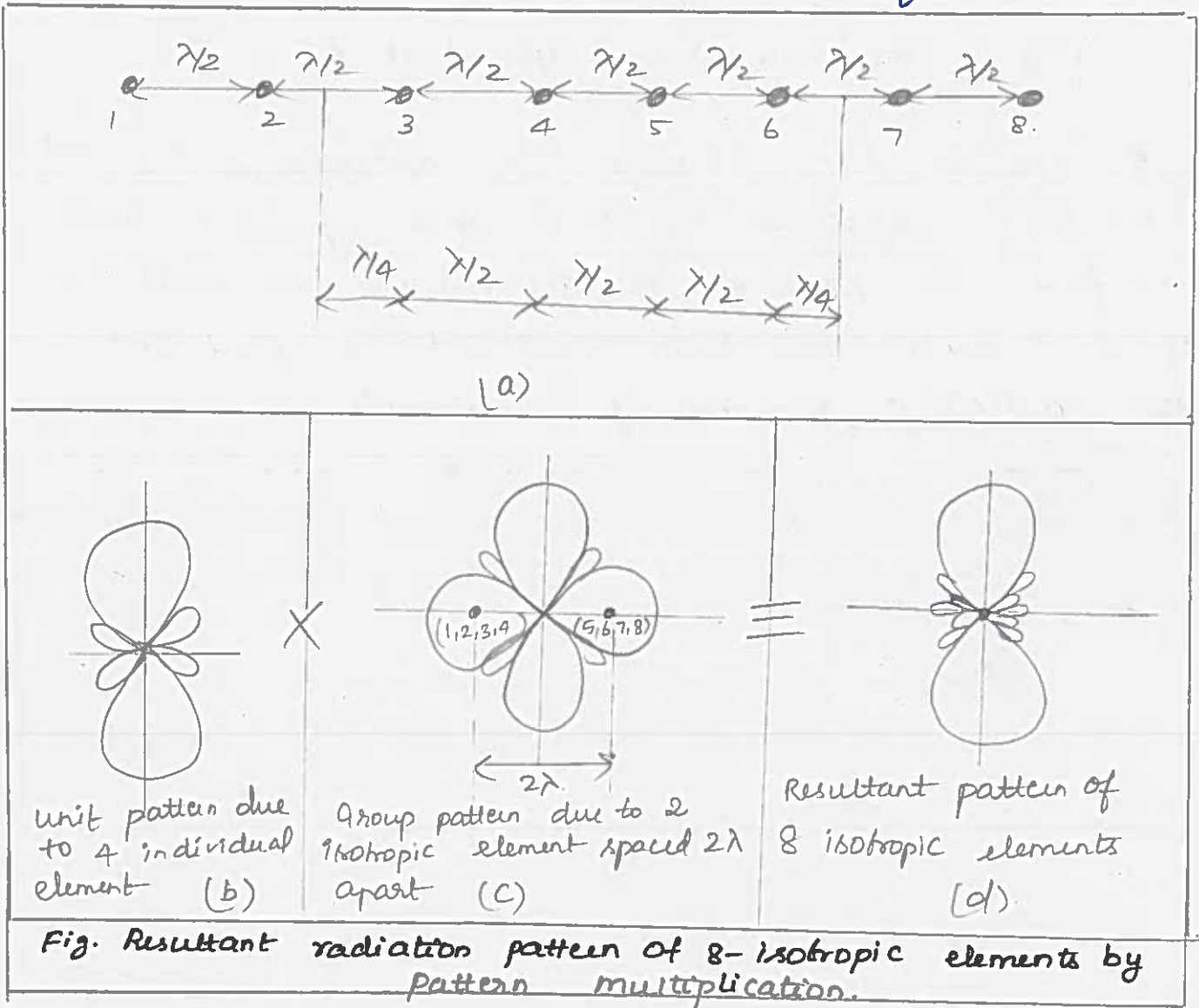
Fig. Resultant radiation pattern of 4 isotropic elements by pattern multiplication.

\* Two isotropic point source spaced  $\lambda/2$  apart fed in phase provides a bidirectional pattern. According to this pattern multiplication, the radiation pattern of 4 elements is obtained as,

$$\therefore \left( \text{Resultant radiation pattern of 4 elements} \right) = \left\{ \text{Radiation pattern of individual elements} \right\} \times \left\{ \text{Array of two units spaced } \lambda \right\}$$

Radiation Pattern of 8-isotropic elements fed in Phase, Spaced  $\lambda/2$  Apart.

\* The application of pattern multiplication can now be extended to more complicated arrays. For example, consider 8 isotropic elements spaced  $\lambda/2$  apart and fed in phase would be obtained as follows.



\* Consider four elements (1, 2, 3 and 4) as one unit (1) and another four elements (5, 6, 7 and 8) as unit two (2). Now, the combined 8 elements array has been reduced to 2 elements (units) array spaced at a distance  $2\lambda$  apart as shown in figure above. The resultant pattern for the 8 element array is obtained as shown in figure.

### PHASE ARRAY:

\* Phased array means an array of many elements with the phase of each element being a variable that provides the control of the beam direction, that is, maximum radiation in any desired direction and pattern shape including the side lobes.

\* The applications for large phased arrays are mostly in the advanced radar systems and in radio astronomy.

\* Smaller phased arrays and beam-forming arrays are used as feed systems to illuminate a reflector in satellite communication systems, when it is necessary to provide several spot beams, scanning beams, and/or wide-angle coverage beams from an one-antenna system.

- \* Some of the specialized phased arrays are
- (i) Frequency Scanning array,
  - (ii) Retroarray and
  - (iii) Adaptive array.

\* In the frequency scanning array or scanning array, phase change is accomplished by varying the frequency. It is one of the simplest phased arrays since no phase control is required at an each element.

\* A retroarray or self-focussing array is an array that will receive a signal from any direction in space and automatically reflects an incoming signal back toward its source, usually after suitable modulation and amplitude.



## Phased Array Designs:-

### Objectives:

The objectives of the phase array are:

(i) A phased array has to accomplish a beam steering without the mechanical and inertial problems of rotating an entire array and.

(ii) It has to provide a beam control at a fixed frequency (or) at any number of frequencies within a certain bandwidth in the frequency-independent manner.

\* In the simplest form of a phased array, beam steering can be done by the mechanical switching. Consider a basic 3-element array and each element be a  $\lambda/2$  dipole antenna as in figure below.

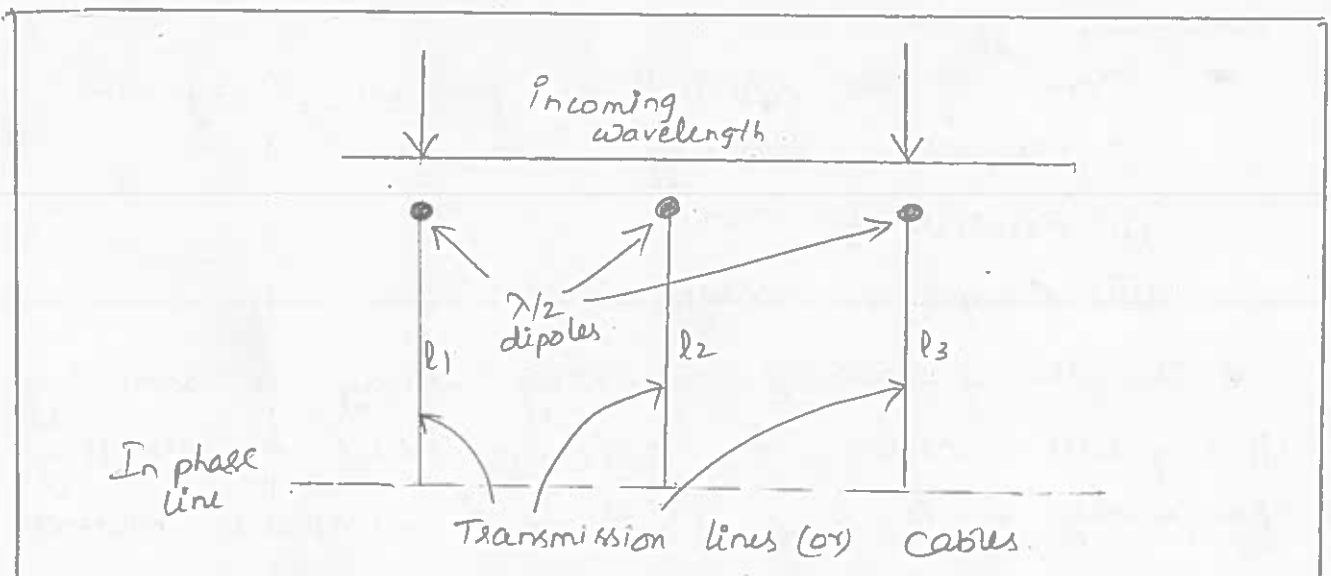


Fig. A simple based array of three  $\lambda/2$  dipoles.

\* An incoming wave will induce the voltages in all transmission lines in the same phase so that if all cables are of the same length ( $l_1 = l_2 = l_3$ ). Then the voltages will be in phase at the inphase line.



\* All three transmission cables are joined as a common point and this 3-element array will be operated as a broadside array.

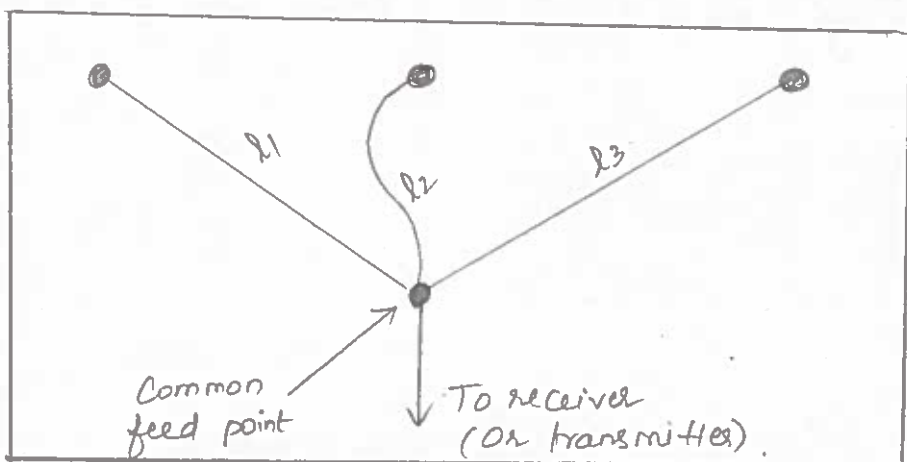
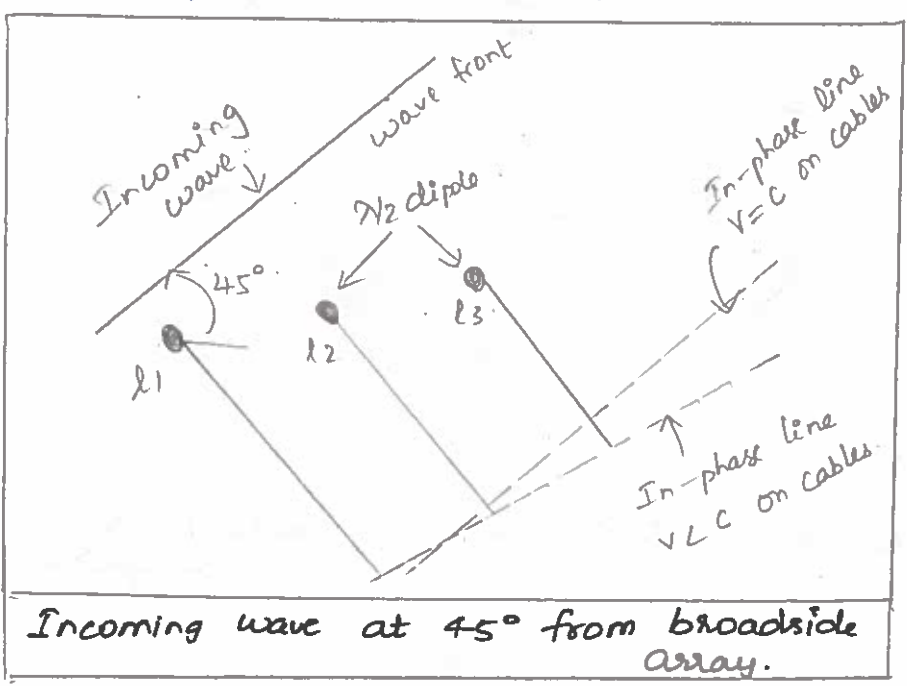


Fig. A Simple equal length cables joined in a common point.

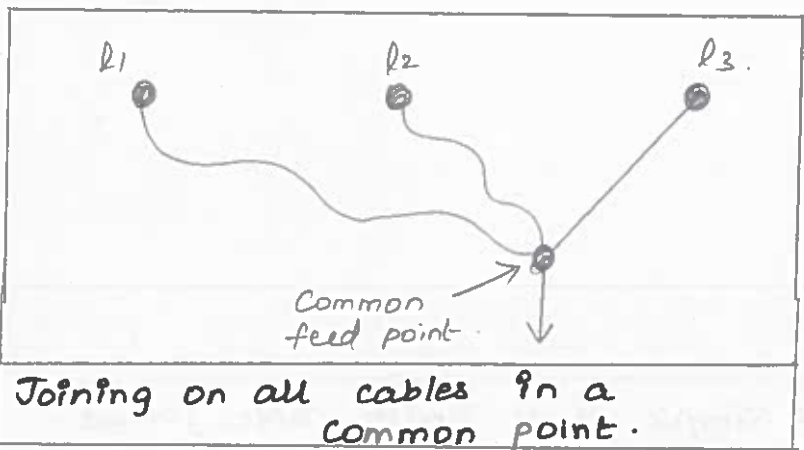
\* For an impedance matching, the cable to the receiver or transmitter should be  $\frac{1}{3}$  of the impedance of these three cables, or a 3 to 1 impedance transformer can be inserted at the common junction point with all the cables of the small impedance.



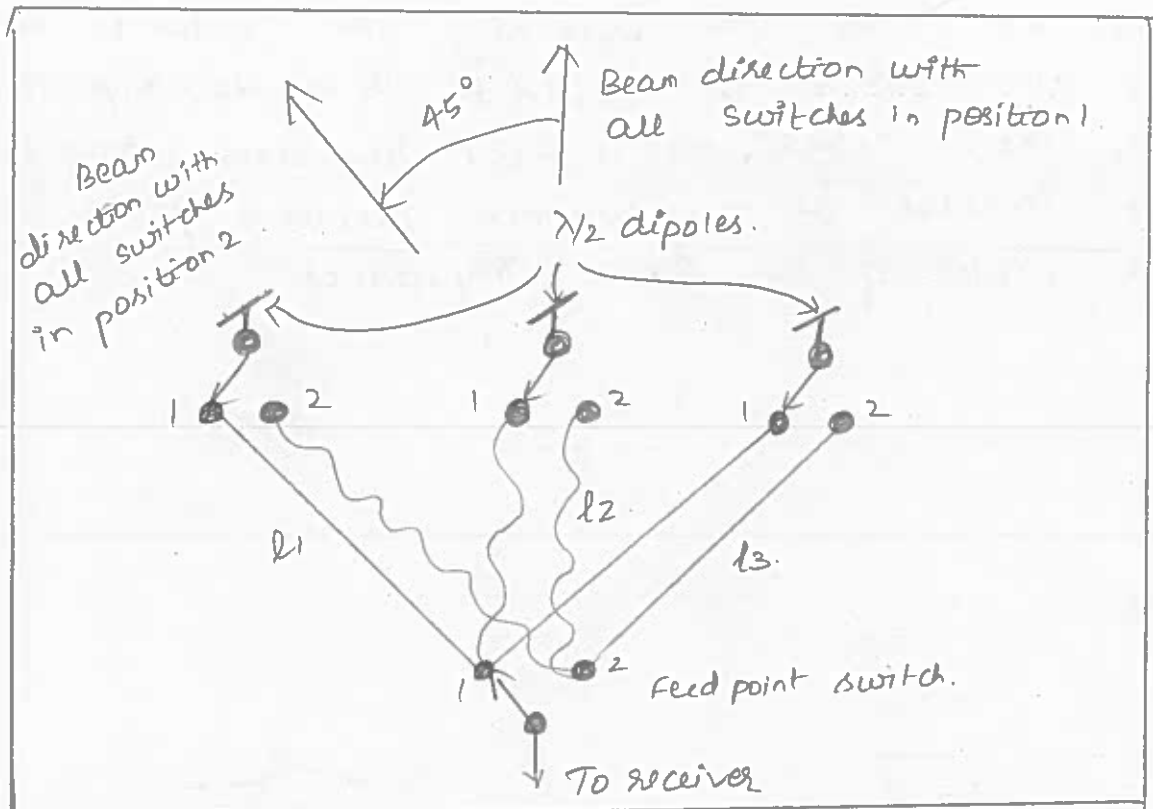
Incoming wave at  $45^\circ$  from broadside array.

\* Now, consider a wave arriving at an angle of  $45^\circ$  from broadside as in figure above. If the wave velocity  $v=c$  (light velocity =  $3 \times 10^8$  m) on the cables, then the inphase line is parallel to the wave front of the incoming wave.

\* If  $v < c$ , the length  $l_2$  and  $l_3$  must be increased in order for all phases to be the same. Then the cables of these lengths are joined as shown below and the 3-element array will have its beam  $45^\circ$  from broadside.



Joining on all cables in a common point.



Switches for shifting from broadside to  $45^\circ$  reception

\* By installing a switch at each antenna element and one at the common feed point as in above figure and mechanically ganging all the switches together, the beam can be shifted from broadside to  $45^\circ$  by operating the ganged switch.

\* By adding more switch points and more cables of an appropriate length, the beam can be steered to an arbitrarily large number of directions. With more elements the narrower beams can be formed.

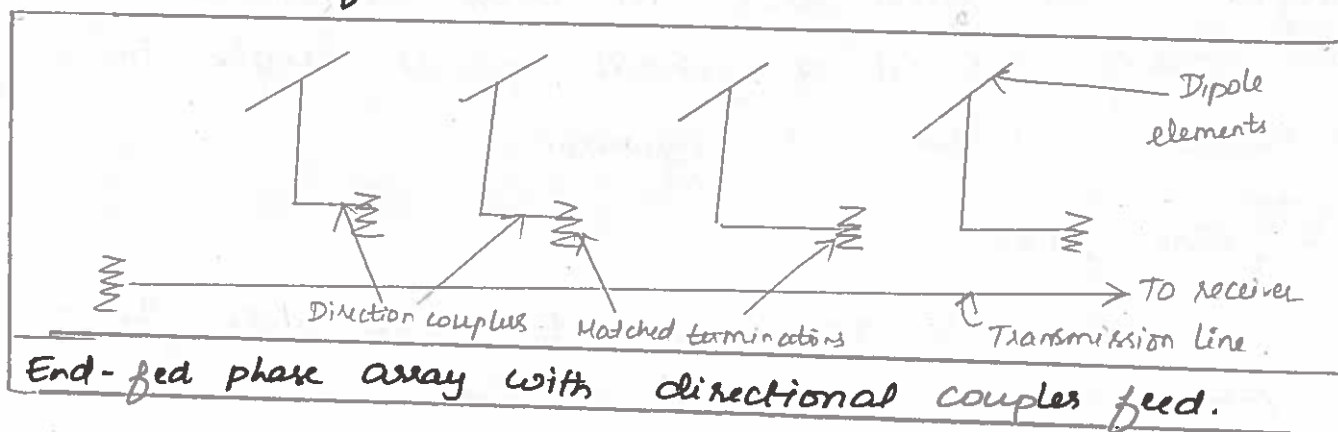
\* With diodes (PIN type) in place of mechanical switches, control can be electronics. Computers can do the same thing by an appropriate programming of sampled signals.

\* Instead of controlling the beam by switching the cables, a phase shifter can be installed at an each element. Thus, the phase shifting may be accomplished by a ferrite device.

\* Insertion of cables of about  $\lambda/4$ ,  $\lambda/2$ ,  $3\lambda/4$  by electronics switching provides the phase increments of  $90^\circ$ . For more precise phasing, cables with smaller incremental differences are used.

Different types of feed using phase array.

(i) Coupler feed.

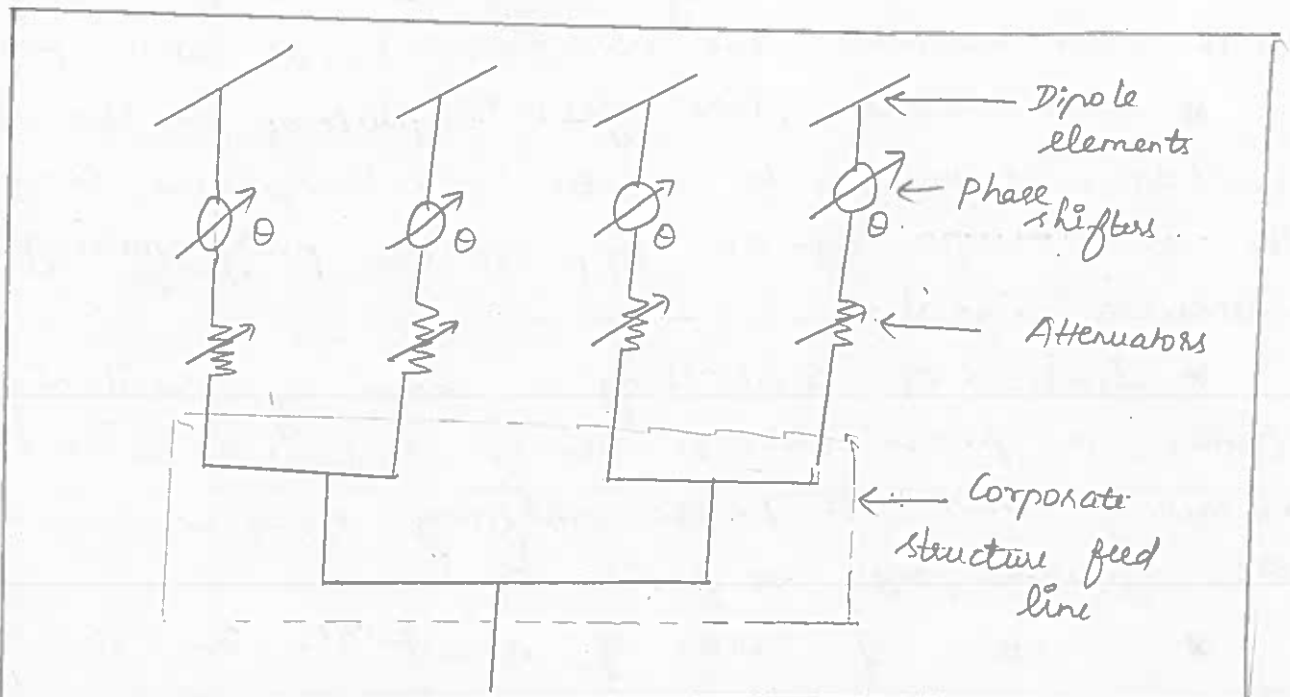


\* The transmission line has a matched termination for zero reflection so that they (ideally) pass a pure travelling wave on the line.

\* Phasing is accomplished by physically sliding the directional couplers along the line. The element amplitude is controlled by changing the closeness of coupling.

## ~~Different~~ Types of Feed using Phased Array.

### (ii) Corporate Structure:-

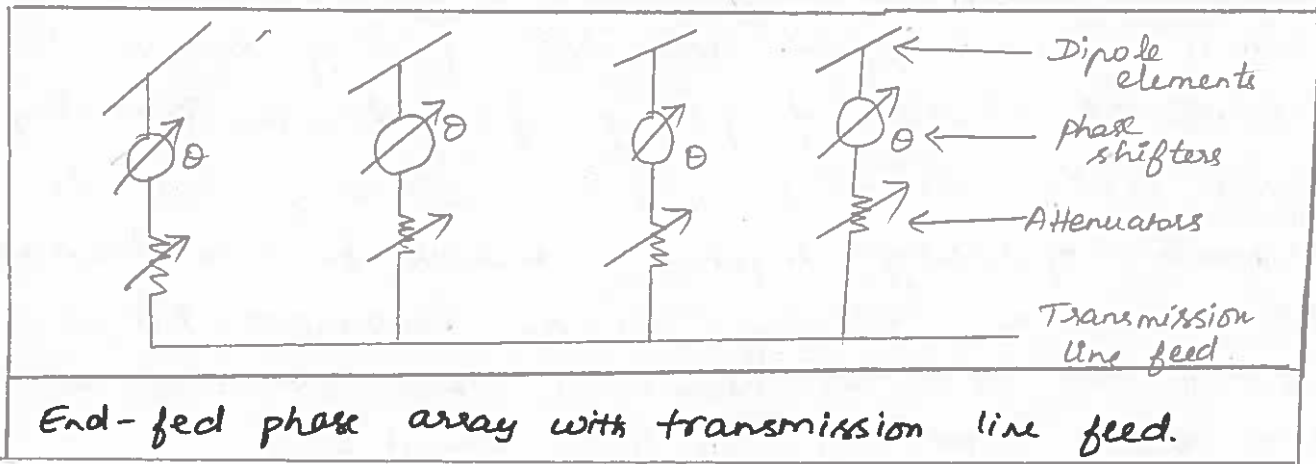


Schematic of phased array feed by corporate structure.

The above figure shows the schematic diagrams of a phased array with a phase shifter and attenuator at each element. The feed cables are all of about equal length in a corporate structure arrangement.

### (iii) Line feed:-

All individual elements have phase the shifter and an attenuator. Since, a progressive phase shift is introduced between elements with a frequency change, the phase shifters must introduce an opposing change to compensate in addition to making the desired phase changes.



### ADAPTIVE ARRAYS AND SMART ANTENNAS:

#### Adaptive arrays:-

\* Adaptive arrays are arrays that can automatically self-adapt to various incoming signals conditions so as to maximize the signal from a particular source or to null out interfering signals.

\* The self phasing array is mainly designed to bring all the signals which are received by the various elements from a particular source into phase.

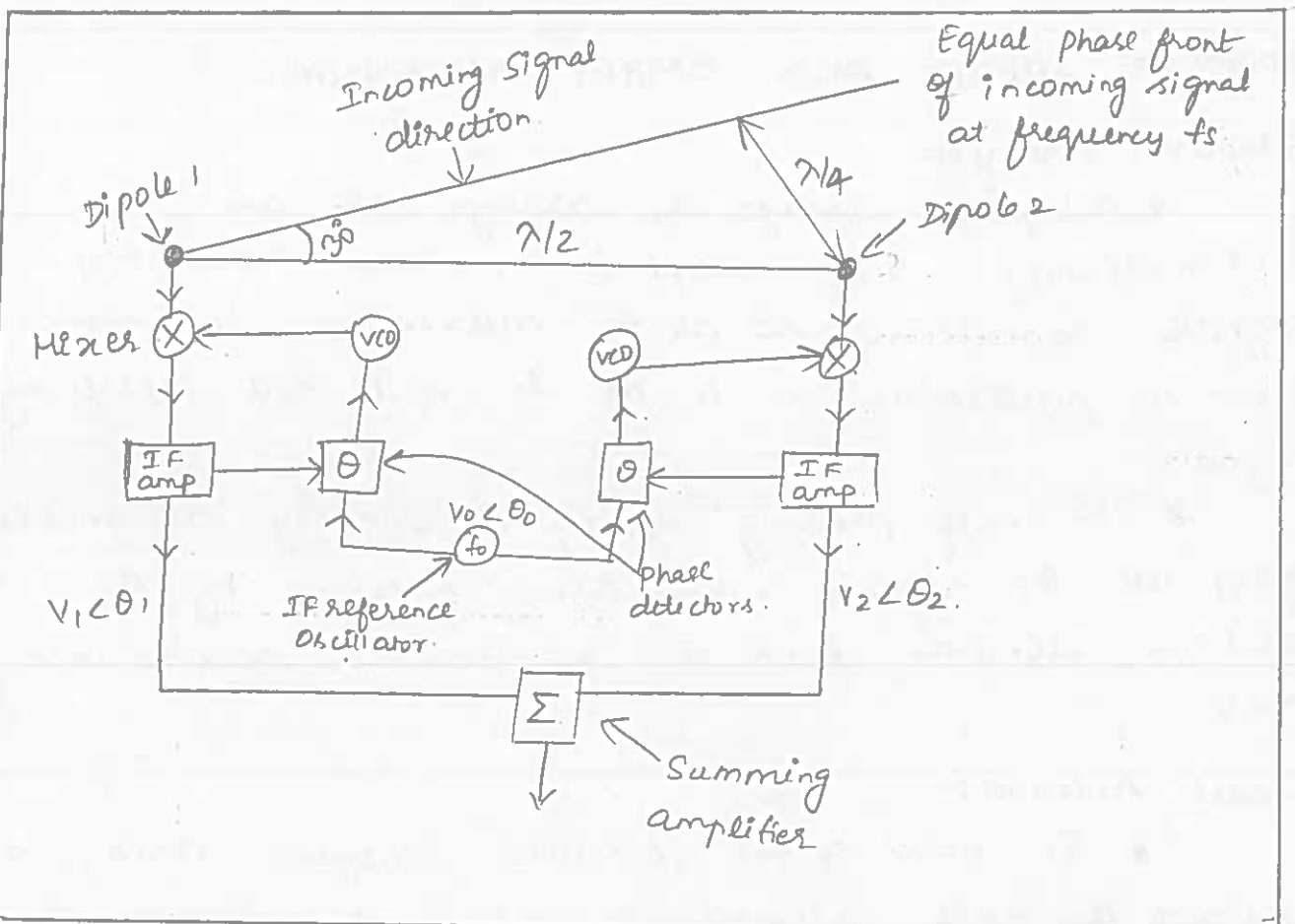
#### Smart Antenna:-

\* In most of the versatile adaptive array, an output of each element is sampled, digitized and processed by a computer which can be programmed to accomplish the tasks. Such an array may be called as "Smart antenna".

\* Multiple beams may be simultaneously directed toward many signals arriving from different directions within the field of an antenna. These antennas are sometimes called as, "Digital Beam Forming (DBF) antennas".



\* Considering a simple 2-element adaptive array as shown in below figure with  $\lambda/2$  spacing between the elements at signal frequency  $f_s$ . Now the incoming signal arrive at  $30^\circ$  from broadside that is all the elements operating in phase and the beam is broadside and the wave arriving at an element 2 travels  $\lambda/4$  further than to element 1, thus retarding the phase of the signal by  $90^\circ$  at element 2.



Two-element adaptive array with signal processing circuit.

\* The phase detector compares the phase of the downshifted signal with the phase of reference oscillator and produces an error voltage  $V_0$  with a magnitude proportional to the phase error between them that advances or retards the phase of VCO output so as to reduce the phase difference to zero. These objectives are achieved by using PLL principles.

\* Now, all the signals at an intermediate frequency (IF) are in phase and may be added together in a summing amplifier.

\* The voltage for the VCO of element 1 would ideally be equal in magnitude but of an opposite sign to the voltage for the VCO of element 2 so that the downshifted signals from both elements are locked in phase (i.e.).

$$\theta_1 = \theta_2 = \theta_0.$$

where

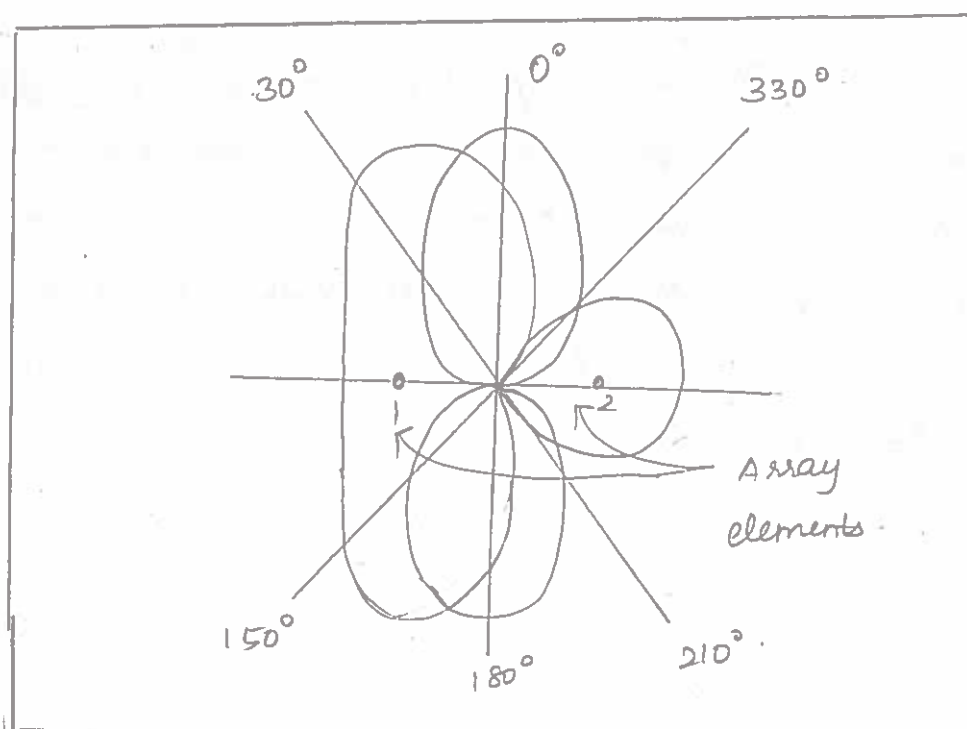
$\theta_1 \rightarrow$  phase of downshifted <sup>signal</sup> from element 1.

$\theta_2 \rightarrow$  phase of downshifted signal from element 2

$\theta_0 \rightarrow$  phase of reference oscillator.

\* An equal gain from both IF amplifiers and the voltages  $V_1$  and  $V_2$  from both the elements should be equal so that,

$$V_1 \angle \theta_1 = V_2 \angle \theta_2.$$



Pattern of 2-element adaptive array for signals from  $0^\circ$  and  $30^\circ$ .

\* The output from the summing amplifier is  $2V_1 (=2V_2)$  and maximizing the response of an array to the incoming signal by steering the beam onto the incoming signal. For example, the beam will be in the  $0^\circ$  direction for a signal from the  $0^\circ$  direction and at  $30^\circ$  for a signal from that direction.

\* For the  $0^\circ$  signal, nulls are at  $90^\circ$  and  $270^\circ$  while for the  $30^\circ$  signal, nulls are at  $210^\circ$  and  $330^\circ$ .

## NON-UNIFORM EXCITATION AMPLITUDES: BINOMIAL ARRAYS

### Need of Binomial Array:

The binomial array is needed for the following reasons:

(i) When the uniform linear array length is increased to increase the directivity, at that time secondary (or) minor lobes also appear along the desired radiation pattern.

(ii) In some of the special applications, it is desired to have single main lobe with no minor lobes. That means the minor lobes should be eliminated completely or reduced to minimum level as compared to main lobe because considerable amount of power is wasted in these directions.

\* To reduce the sideband level, we use binomial array which deals with the non-uniform amplitude of elements. Here the amplitudes of the radiating sources are arranged to the coefficients of successive terms of binomial series.

The binomial series is

$$(a+b)^{(n-1)} = a^{(n-1)} + \frac{n-1}{1!} a^{(n-2)} \cdot b + \frac{(n-1)(n-2)}{2!} a^{(n-3)} \cdot b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{(n-4)} \cdot b^3 + \dots \rightarrow \textcircled{1}$$

where  $n \rightarrow$  Number of radiating sources in the array.

### Concepts of Binomial array:

\* If an array is arranged in such a way that radiating sources are in the centre of the broadside array radiates more strongly than the radiating sources at the edges. The secondary lobes can be eliminated entirely, when the following two conditions are satisfied.

- (i) The space between the two consecutive radiating sources does not exceed  $\lambda/2$  and
- (ii) The current amplitudes in radiating sources (from outer, towards centre source) are proportional to the coefficients of the successive terms of the binomial series.

\* The above conditions are necessarily satisfied in the binomial arrays and the coefficients which corresponds to the coefficients of the successive terms of the amplitude of the sources are obtained by putting  $n = 1, 2, 3, \dots$   
 $\hookrightarrow$  in eqn  $\textcircled{1}$ .

\* For example the relative amplitudes for the arrays of 1 to 10 radiating sources are given as follows:

Number of sources	Relative Amplitude.
$n=1$	1
$n=2$	1 1
$n=3$	1 2 1
$n=4$	1 3 3 1
$n=5$	1 4 6 4 1
$n=6$	1 5 10 10 5 1
$n=7$	1 6 15 20 15 6 1
$n=8$	1 7 21 35 35 21 7 1
$n=9$	1 8 28 56 70 56 28 8 1
$n=10$	1 9 36 84 126 126 84 36 9 1

\* These coefficients for any number of radiating sources can also be obtained from Pascal's triangle, where each internal integer is the sum of above adjacent integers.

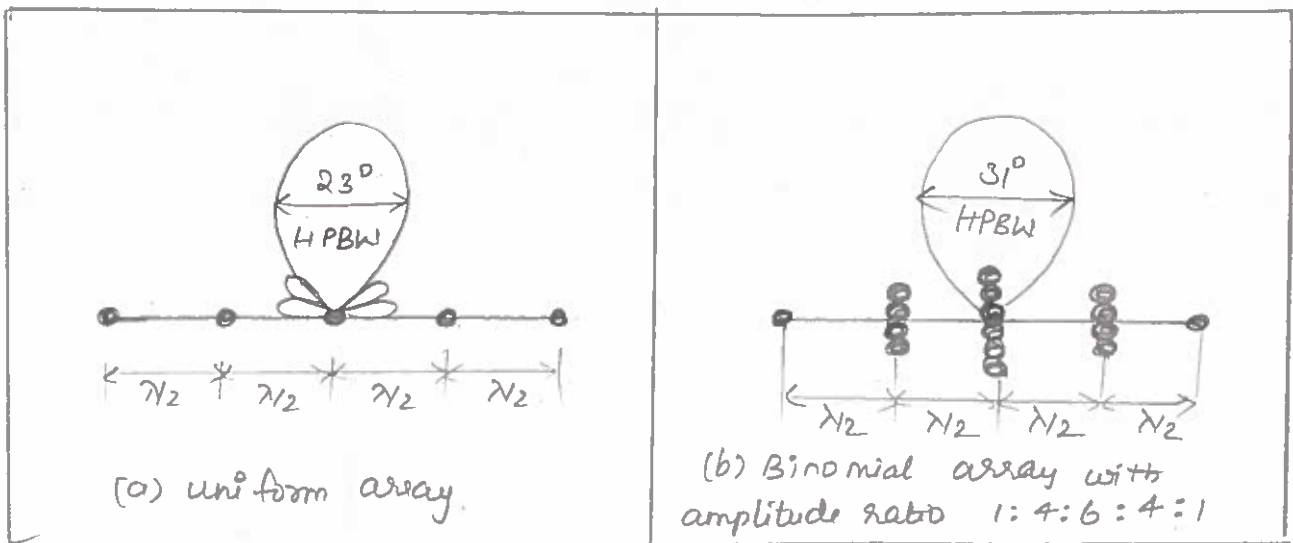
### PASCAL'S TRIANGLE.

1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	28	8	1	
1	9	36	84	126	126	84	36	9	1



\* However the elimination of secondary lobes in a binomial array takes place in the cost of directivity. HPBW of binomial array is more than that of uniform array for the same length of an array.

\* For example, consider  $n=5$ ,  $d=\lambda/2$ , HPBW of binomial array is  $31^\circ$  and HPBW of a uniform array is  $23^\circ$  as shown in figure.



\* If we reduce the spacing between the two elements to one half wavelength then only the primary lobes will obtain. The resultant pattern can be obtained by using the concept of the pattern multiplication. In general, the far field pattern for the binomial array of 'n' sources are

$$E_{\theta} \propto \cos^{(n-1)} \left( \frac{\pi}{2} \cos \theta \right) \rightarrow \textcircled{2}$$

**Disadvantages:**

Disadvantages of binomial arrays are.

(i) HPBW increases and hence the directivity decreases.

(ii) For the design of a large array, the larger amplitude ratio of sources is required.

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## UNIT-IV

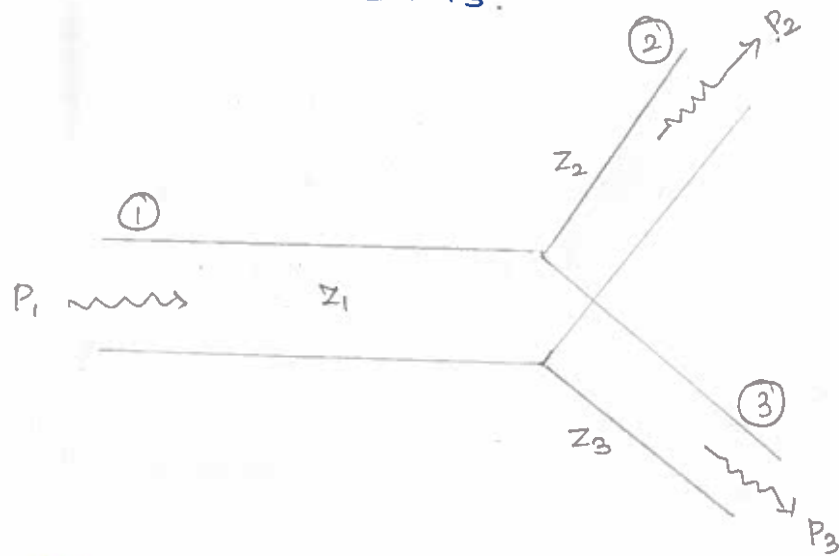
### PASSIVE AND ACTIVE MICROWAVE DEVICES.

#### THREE PORTS JUNCTIONS (T-JUNCTIONS)

\* A simple power divider is a T-junction network. When we are using as power divider, the port 1 acts as an input port and ports 2 and 3 all act as an output ports. The power is divided among the ports equally and it is expressed in terms of lossless as,

$$\text{Input power} = \text{Output power}$$

$$P_1 = P_2 + P_3.$$



\* When used as power combiner, the port 1 is act as an output port and the ports 2 and 3 all act as an input ports and it is expressed as

$$P_2 + P_3 = P_1$$

\* The scattering matrix for an arbitrary three port network is given as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \rightarrow \textcircled{1}$$

For a lossless, the reciprocal three port junction is one where all three ports can be perfectly matched then  $S_{11} = S_{22} = S_{33} = 0$  and its scattering matrix will be symmetric. Now the scattering matrix is given by,

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \rightarrow \textcircled{2}$$

The scattering matrix of a reciprocal four port network that all the ports are perfectly matched and it is expressed as,

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \textcircled{3}$$

### Applications :-

\* Used in the radiating elements of an array antenna.

\* Used in the balanced amplifiers both as power dividers and power combiners.

## FOUR PORTS NETWORKS : DIRECTIONAL COUPLERS

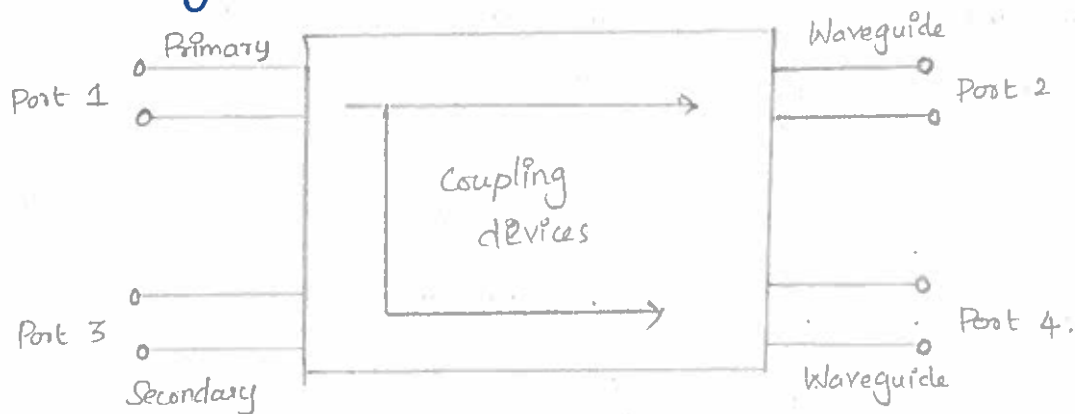
### Definition :

A directional coupler is a four port passive device commonly used for coupling a known fraction of the microwave power to a port in an auxiliary line while the power is flowing from an input port to an output port in the main line. The remaining port is ideally isolated port and matched terminated.

\* Here, portions of the forward and reverse travelling waves on a line are separately coupled to two of the other ports.

\* They can be designed to measure an incident and reflected power, SWR values, provide a signal path to a receiver or perform the other desirable operations.

\* They can be unidirectional or bi-directional powers.



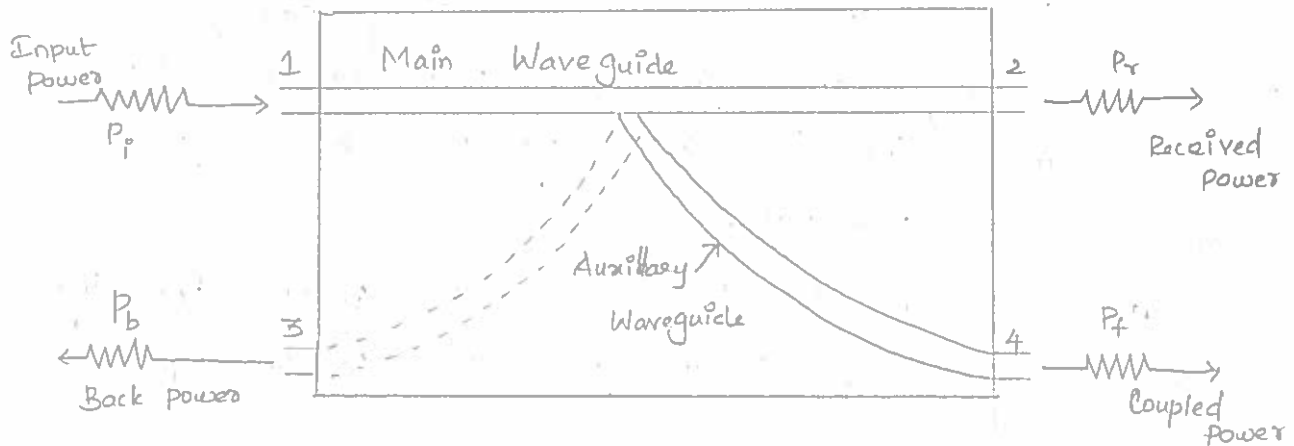
### Properties :-

\* A portion of power travelling from port 1 and port 2 is coupled to port 4, not to port 3.

\* A portion of power travelling from port 2 and port 1 is coupled to port 3, not to port 4.



\* A portion of power incident on port 3 is coupled to port 2 but not to port 1 and a portion of power incident on port 4 is coupled to port 1 but not to port 2. Also ports 1 and 3 are decoupled as ports 2 and 4.



**Coupling factor (C):-**

\* It is defined as the ratio of an incident power to the forward power  $P_f$  which is measured in dB.

$$C \text{ (dB)} = 10 \log_{10} \left( \frac{P_i}{P_f} \right)$$

\* The coupling factor is a measure of how much of an incident power is being sampled.

**Directivity (D):-**

\* It is defined as, the ratio of forward power to the backward power  $P_b$  as expressed in dB.

$$D \text{ (dB)} = 10 \log_{10} \frac{P_f}{P_b}$$

\* Directivity is a measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.

**Isolation (I):-**

\* It is defined as the ratio of an incident power to the back power as expressed in dB.

$$I \text{ (dB)} = 10 \log_{10} P_i / P_b$$

②

\* The term Isolation is sometimes used to describe the directive properties of a coupler. Isolation (dB) equals coupling plus directivity.

$$\text{Isolation (I)} = \text{Coupling factor (C)} + \text{Directivity (D)}$$

Scattering Matrix of a Directional Coupler.

\* Directional coupler is a four port network. Hence [S] is 4x4 matrix and it is expressed as,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow \textcircled{1}$$

\* In a directional coupler all the four ports are perfectly matched to the junction. Hence, all the diagonal elements are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \longrightarrow \textcircled{2}$$

\* From symmetric property,  $S_{ij} = S_{ji}$

$$S_{12} = S_{21}, S_{23} = S_{32}, S_{13} = S_{31}, S_{24} = S_{42}, S_{34} = S_{43}, S_{41} = S_{14} \longrightarrow \textcircled{3}$$

\* There is no coupling between port 1 and port 3.

$$S_{13} = S_{31} = 0 \longrightarrow \textcircled{4}$$

\* Also there is no coupling between port 2 and port 4.

$$S_{24} = S_{42} = 0 \longrightarrow \textcircled{5}$$

\* By substituting  $\textcircled{2}, \textcircled{3}, \textcircled{4}, \textcircled{5}$  in  $\textcircled{1}$ ,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \longrightarrow \textcircled{6}$$

By applying an unity property of  $[S]$  matrix for (6), we then get,

$$[S][S^*] = I$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{12}|^2 + |S_{14}|^2 = 1 \longrightarrow (7)$$

$$R_2 C_2 = |S_{12}|^2 + |S_{23}|^2 = 1 \longrightarrow (8)$$

$$R_3 C_3 = |S_{23}|^2 + |S_{34}|^2 = 1 \longrightarrow (9)$$

By using zero property of  $[S]$  matrix

$$R_1 C_3 = S_{12} S_{23}^* + S_{14} S_{34}^* = 0 \longrightarrow (10)$$

By comparing (7) and (8)

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$$

$$\boxed{S_{14} = S_{23}} \longrightarrow (11)$$

lly (8) and (9)

$$|S_{12}|^2 + |S_{23}|^2 = |S_{23}|^2 + |S_{34}|^2$$

$$\boxed{S_{12} = S_{34}} \longrightarrow (12)$$

Let us assume that  $S_{12}$  is real and positive = 'p'

$$S_{12} = S_{34} = P = S_{34}^* \longrightarrow (13)$$

Sub (13) and (11) in (10)

$$S_{12} S_{23}^* + S_{14} S_{34}^* = 0 \quad [\because S_{14} = S_{23}]$$

$$P (S_{23} + S_{23}^*) = 0$$

$$S_{23} + S_{23}^* = 0$$

$$\boxed{S_{23} = -S_{23}^*} \longrightarrow (14)$$

From (14), it is clear that  $S_{23}$  must be imaginary

$$S_{23} = jQ$$

$$S_{23}^* = -jQ$$

From (11) & (12)

$$S_{12} = S_{24} = P \longrightarrow (15a)$$

$$S_{23} = S_{14} = jQ \longrightarrow (15b)$$

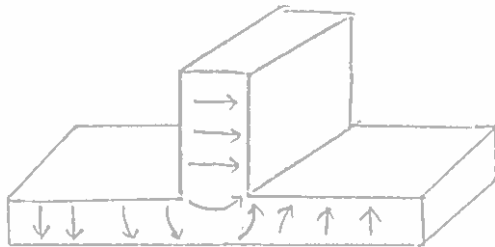
By substituting (15a) & (15b) in (7),

$$P^2 + Q^2 = 1 \longrightarrow (16)$$

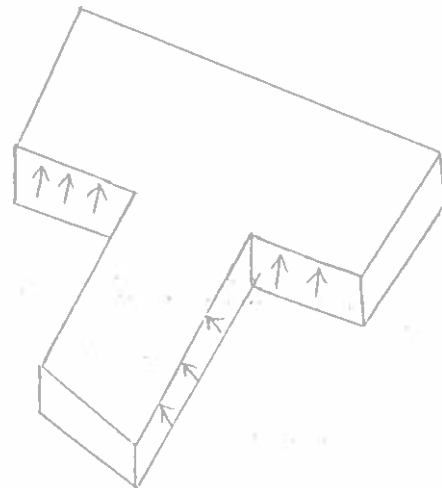
By substituting (15a) and (15b) in (6) then [S] matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jQ \\ P & 0 & jQ & 0 \\ 0 & jQ & 0 & P \\ jQ & 0 & P & 0 \end{bmatrix} \longrightarrow (17)$$

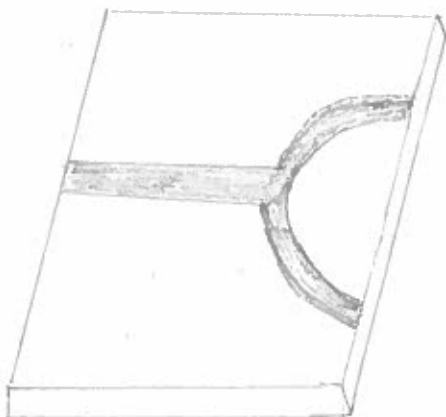
T-JUNCTION POWER DIVIDER.



(a) E-plane waveguide T



(b) H-plane waveguide T



(c) Microstrip line T-junction divider

\* The T-junction power divider is a simple three port network that can be used for power division or power combining and it can be implemented in virtually any type of transmission line medium.

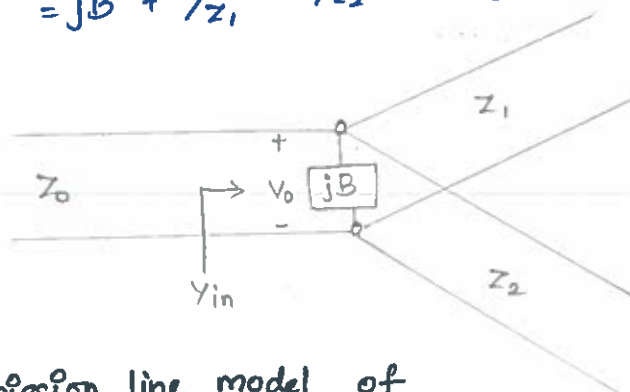
\* Figure shows some commonly used T-junctions in waveguide and microstrip line or stripline form which are in the absence of transmission line loss, lossless junctions.

### Lossless Divider

\* In general, there may be fringing fields and higher order modes associated with the discontinuity at such a junction, leading to the stored energy that can be accounted for by a lumped susceptance,  $B$ .

\* In order for the divider to be matched to an input line of characteristic impedance  $Z_0$ , we must have

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \rightarrow \textcircled{1}$$



Transmission line model of a lossless T-junction divider.

\* If the transmission lines are assumed to be lossless, then the characteristic impedances are real. If we also assume  $B=0$ , then  $\textcircled{1}$  reduces to

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0} \rightarrow \textcircled{2}$$

\* The output line impedances  $Z_1$  and  $Z_2$  can be selected to provide various power division ratios. Thus for a  $50\Omega$  input line, a 3dB power divider can be made by using the two  $100\Omega$  output lines.



①

\* If necessary, quarter-wave transformers can be used to bring an output line impedances back to the desired levels. If the output lines are matched, then an input line will be matched and there will be no isolation between the two output ports.

### Resistive Divider.

\* If a three port divider contains lossy components, it can be made to be matched at all ports although the two output ports may not be isolated.

\* The resistive divider can easily be analyzed using circuit theory. Assuming that all ports are terminated in the characteristic impedance  $Z_0$ , then the impedance  $Z$ , seen looking into the  $Z_0/3$  resistor followed by a terminated output line is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3} \longrightarrow \textcircled{3}$$

\* Then input impedance of the divider is

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0 \longrightarrow \textcircled{4}$$

\*  $\textcircled{4}$  shows that an input is matched to the feed line.

Because the network is symmetric from all three ports, the output ports are also matched. Thus  $S_{11} = S_{22} = S_{33} = 0$ .

\* If the voltage at port 1 is  $V_1$ , then by voltage division the voltage  $V$  at the center of the junction is given as

$$V = V_1 \frac{2Z_0/3}{\frac{Z_0}{3} + \frac{2Z_0}{3}} = \frac{2}{3} V_1$$

the output voltages by using voltage division are obtained as

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + \frac{Z_0}{3}} = V \times \frac{Z_0}{\frac{3Z_0 + Z_0}{3}} = V \times \frac{3}{4} \longrightarrow \textcircled{6}$$

Sub  $\textcircled{5}$  in  $\textcircled{6}$

$$V_2 = \frac{3}{4} \times \frac{2}{3} V_1 = \frac{1}{2} V_1 \longrightarrow \textcircled{7}$$

Thus  $S_{21} = S_{31} = S_{23} = \frac{1}{2}$ , so output powers are 6 dB below an input power level. The network is reciprocal, so the scattering matrix is symmetric but not a unitary matrix and it can be written as

$$|S| = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \longrightarrow \textcircled{8}$$

\* The power delivered to the input of the divider is

$$P_{in} = \frac{1}{2} \frac{V_i^2}{Z_0} \longrightarrow \textcircled{9}$$

while the output powers are

$$P_2 = P_3 = \frac{1}{2} \frac{(\frac{1}{2} V_i)^2}{Z_0} \\ = \frac{1}{8} \frac{V_i^2}{Z_0} = \frac{1}{4} P_{in} \longrightarrow \textcircled{10}$$

\* Equations  $\textcircled{10}$  represents that half of the supplied power is dissipated in the resistors.

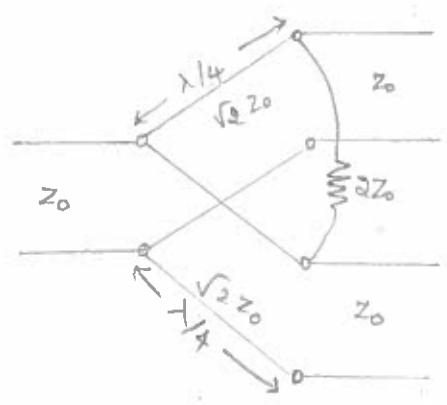
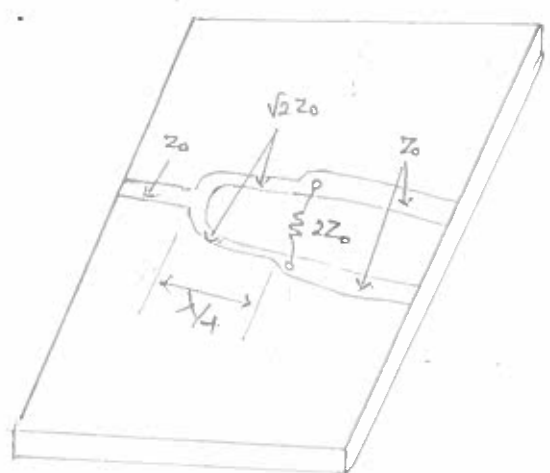
### WILKINSON POWER DIVIDER.

(i) The lossless T-junction divider is not being matched at all the ports and it does not have any isolation between output ports.

(ii) The resistive divider can be matched at all ports, but even though it is not lossless, isolation is still not achieved.

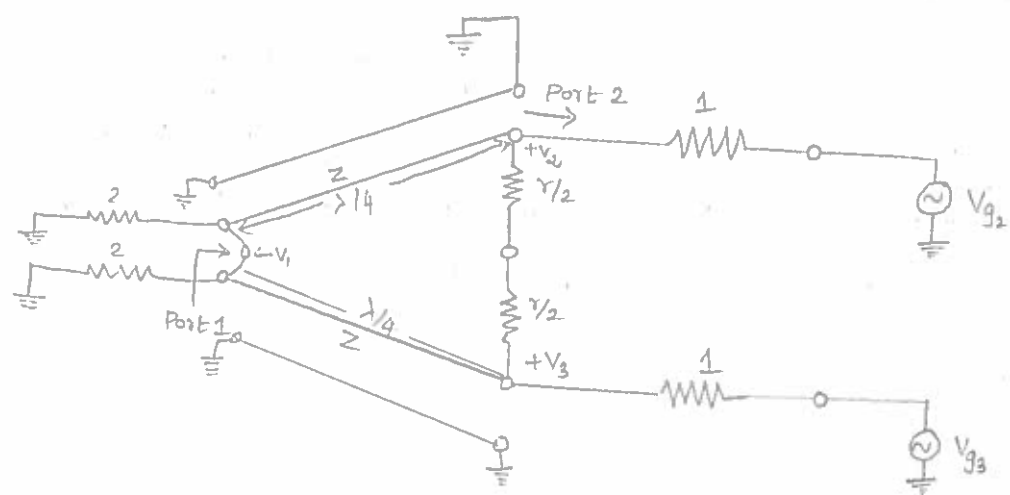
\* The wilkinson power divider is a network with the useful property of an appearing lossless when the output ports are matched; that is only the reflected power from the output ports is dissipated.

\* The wilkinson power divider can be made with the arbitrary power division which is often made in microstrip line or stripline form, as depicted in figure.



(b) Equivalent transmission line circuit.

(a) An equal-split Wilkinson power divider in microstrip line form  
Even odd Mode Analysis.



\* This network has been drawn in a form that is symmetric across the midplane; the two source resistors of a normalised value 2 which is combine in parallel to give a resistor of normalized value 1, representing the impedance of a matched source.

\* The quarter wave lines have a normalized characteristic impedance  $Z$ , and the shunt resistor has a normalized value of  $r$ . For the equal-split power divider, ~~these~~ These values should be  $Z = \sqrt{2}$  and  $r = 2$ .

\* Now we define two separate modes of excitation for the circuit.

- (i) Even mode :  $V_{g2} = V_{g3} = 2V_0$
- (ii) Odd mode :  $V_{g2} = -V_{g3} = 2V_0$

\* Superposition of these two modes effectively produces an excitation which can find by using scattering parameters of the network is  $V_{g2} = 4V_0$  and  $V_{g3} = 0$ .

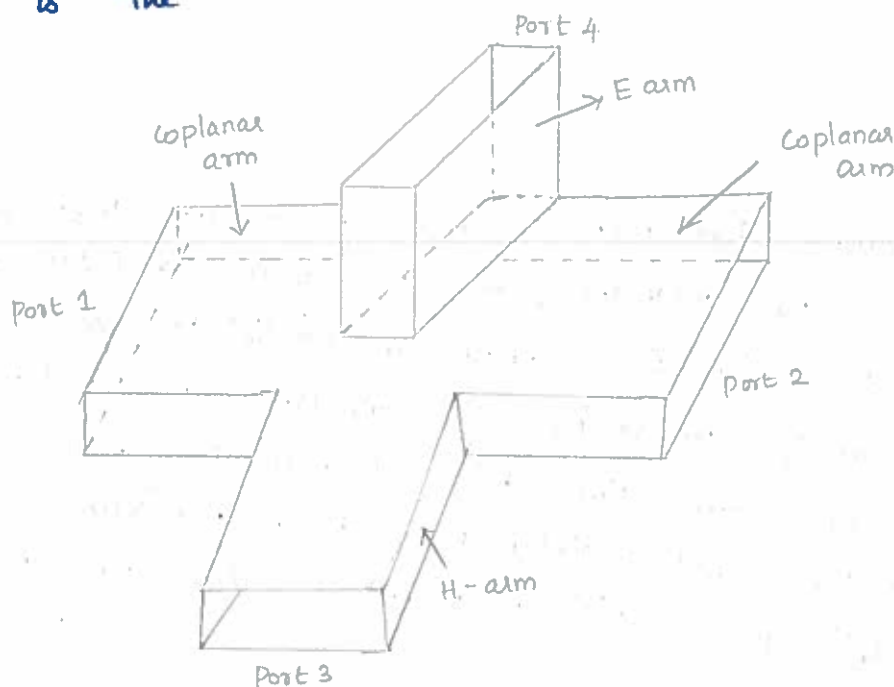
Drawback:-

This divider requires crossovers for these resistors with  $N \geq 3$ , which makes the fabrication difficult in planar form.

### WAVEGUIDE MAGIC-T

\* A hybrid junction is a four-port network in which a signal incident on any one of the port divides between two output ports with the remaining port being isolated.

\* A magic tee is a combination of the E-plane tee and H-plane tee. Ports 1 and 2 are collinear arms, port 3 is the H-arm and port 4 is the E-arm.



### Characteristics of Magic Tee.

\* The magic-T has the following characteristics when all the ports are terminated with matched load.

(i) If two in phase waves of equal magnitude are fed into ports 1 and 2, the output at port 4 is subtractive and

hence zero and the total output will appear additively at port 3. Hence, port 4 is called the difference (or) E-arm and port 3 is the sum (or) H-arm.

\* A wave incident at port 4 divides equally between the ports 1 and 2 but opposite in phase with no coupling to port 3.

\* A wave incident at port 3 divides equally between the ports 1 and 2 and in phase with no coupling to port 4.

$$S_{43} = S_{34} = 0 \longrightarrow \textcircled{1}$$

\* A wave fed into one collinear port 1 or 2 will not appear in the other collinear port 2 or 1. Hence two collinear ports 1 and 2 are isolated from each other

$$S_{12} = S_{21} = 0 \longrightarrow \textcircled{2}$$

\* A magic T can be matched by putting screens suitably in the E and H arms without destroying the symmetry of the junction. For an ideal, lossless magic-T matched at ports 3 and 4.

$$S_{33} = S_{44} = 0 \longrightarrow \textcircled{3}$$

### S-Matrix for magic Tee.

\* [S] is a 4x4 matrix since there are 4 ports,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow \textcircled{4}$$

\* From symmetric property  $S_{ij} = S_{ji}$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{14} = S_{41}, S_{23} = S_{32}, S_{24} = S_{42}, S_{34} = S_{43} \longrightarrow \textcircled{5}$$

\* Port 3 has H-plane tee section,

$$S_{23} = S_{13} \longrightarrow \textcircled{6}$$

By, Port 4 has E-plane tee section

$$S_{24} = -S_{14} \longrightarrow \textcircled{7}$$



\* By sub (1), (3), (5), (6) and (7) in (4),

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \longrightarrow (8)$$

using unitary property on eqn (8),

$$[S][S^*] = I$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (9)$$

$$R_2 C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (10)$$

$$R_3 C_3 = |S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (11)$$

$$R_4 C_4 = |S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow (12)$$

By equating (9) and (10)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2$$

$$|S_{11}| = |S_{22}| \longrightarrow (13)$$

From (11)

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$2|S_{13}|^2 = 1$$

$$|S_{13}| = \frac{1}{\sqrt{2}} \longrightarrow (14)$$

From (12)

$$|S_{14}|^2 + |S_{14}|^2 = 1$$

$$2|S_{14}|^2 = 1$$

$$|S_{14}| = \frac{1}{\sqrt{2}} \longrightarrow (15)$$

\* By sub (14) and (15) in (9)

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

which is valid if,  $S_{11} = S_{12} = 0 \rightarrow (16)$

\* From (13) and (16),

$$S_{22} = 0 \rightarrow (17)$$

\* The  $[S]$  of magic tee is obtained by substituting the scattering parameters from (16) and (17) in (8)

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \rightarrow (18)$$

\* Using (14), (15)

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \rightarrow (19)$$

where

$$|S_{13}| = \frac{1}{\sqrt{2}} = |S_{14}|$$

\* Hence in any four ports junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a magic tee.

Applications:-

- \* Measurement of impedance
- \* As duplexer
- \* As mixer
- \* As an isolator.



## ATTENUATORS :-

\* An attenuator is basically a passive device which controls the amount of microwave power transferred from one point to another without causing a big distortion to its waveform on a microwave transmission system. It results in decreasing the power level of a microwave signal.

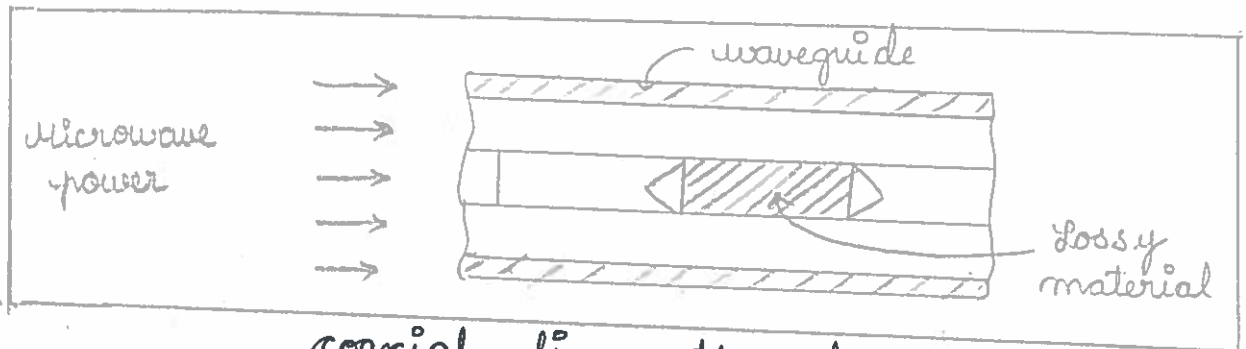
\* Microwave attenuators control the flow of microwave power either reflecting it or absorbing it and it is expressed in decibels of the relative power.

\* Attenuator which attenuates the RF signal in a waveguide system is referred to as a waveguide attenuator.

There are two main types namely,

- \* fixed attenuator
- \* variable attenuator

### Fixed attenuator :-



coaxial line attenuator

\* Fixed attenuators are used where a fixed amount of attenuation is to be provided. If such a fixed attenuator absorbs all the energy entering into it and it is called as waveguide terminator.

\* The coaxial line based fixed type of attenuator, here, resistive film is fixed at the centre of a conductor which absorbs the power and as a result of power loss, that is microwave signal passes through it also gets attenuated.

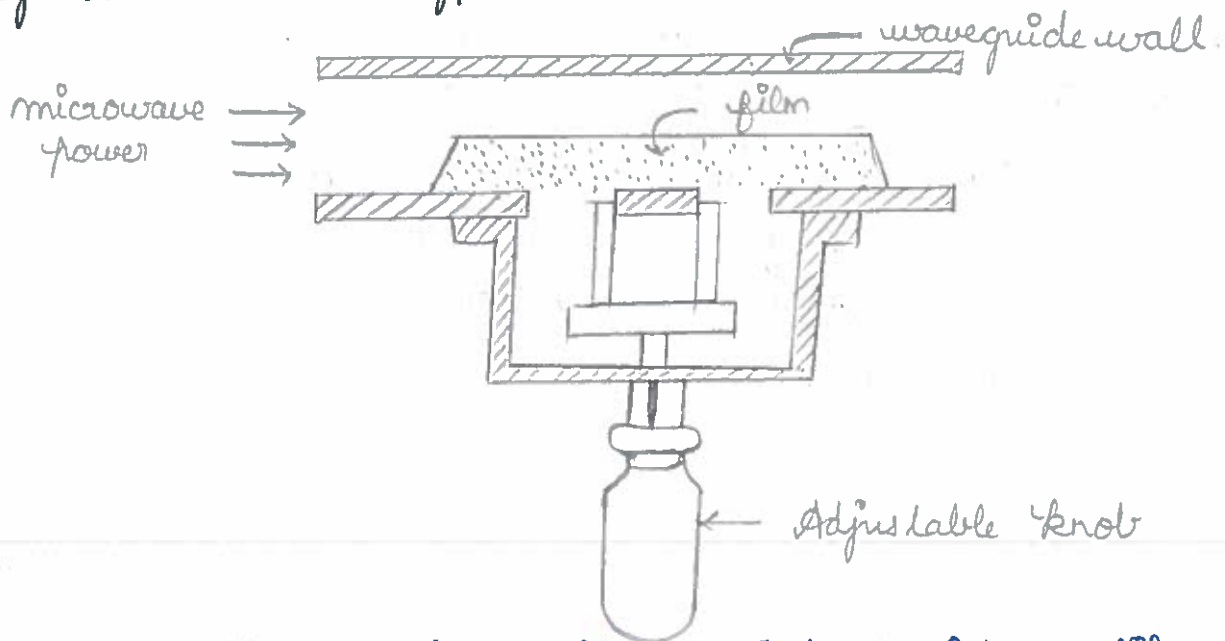
## Variable attenuator :-

\* Variable attenuator provide continuous or step wise variable attenuation. For rectangular waveguides, these attenuators can be flap type or vane type. For circular waveguides, the rotary type is normally used.

\* The most commonly used variable attenuators are,

- \* waveguide variable type
- \* Rotary - vane attenuators

## waveguide variable type :-



\* Thin dielectric strip with coated resistive film is placed at the centre of the waveguide. Thus film is placed in the waveguide which is parallel to the maximum  $E$ -field.

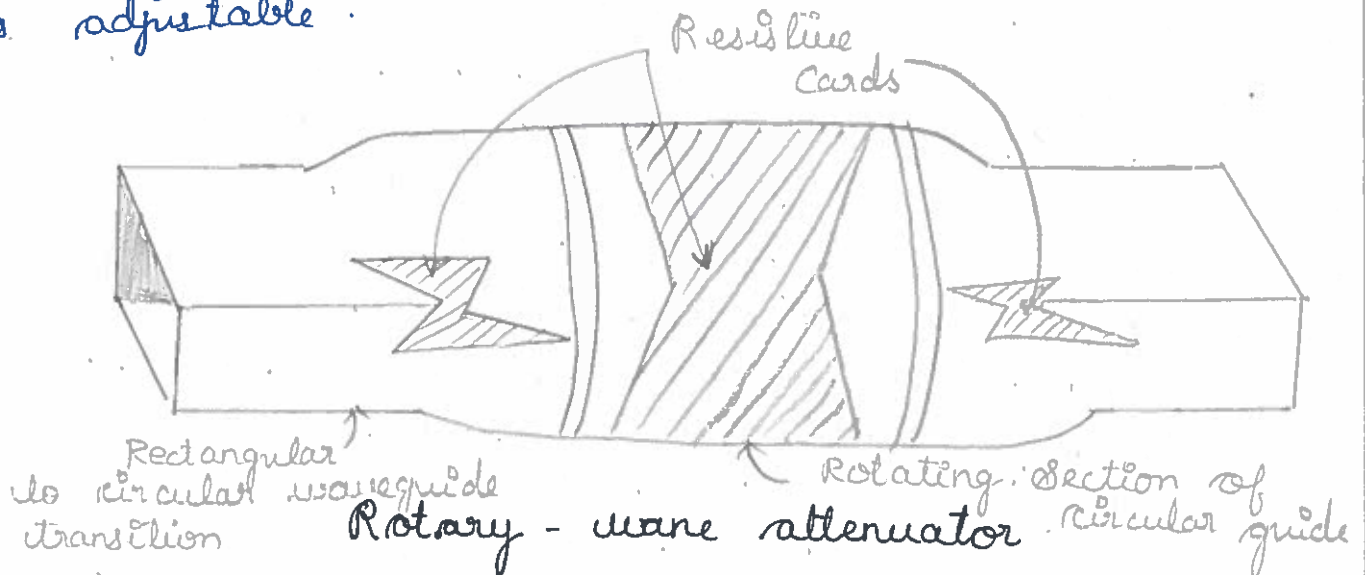
\* Resistive vane is moved from one side of the wall to the centre by using a screw where ' $E$ ' field is considered to be maximum. This resistive film is shaped to give a linear attenuation variation.



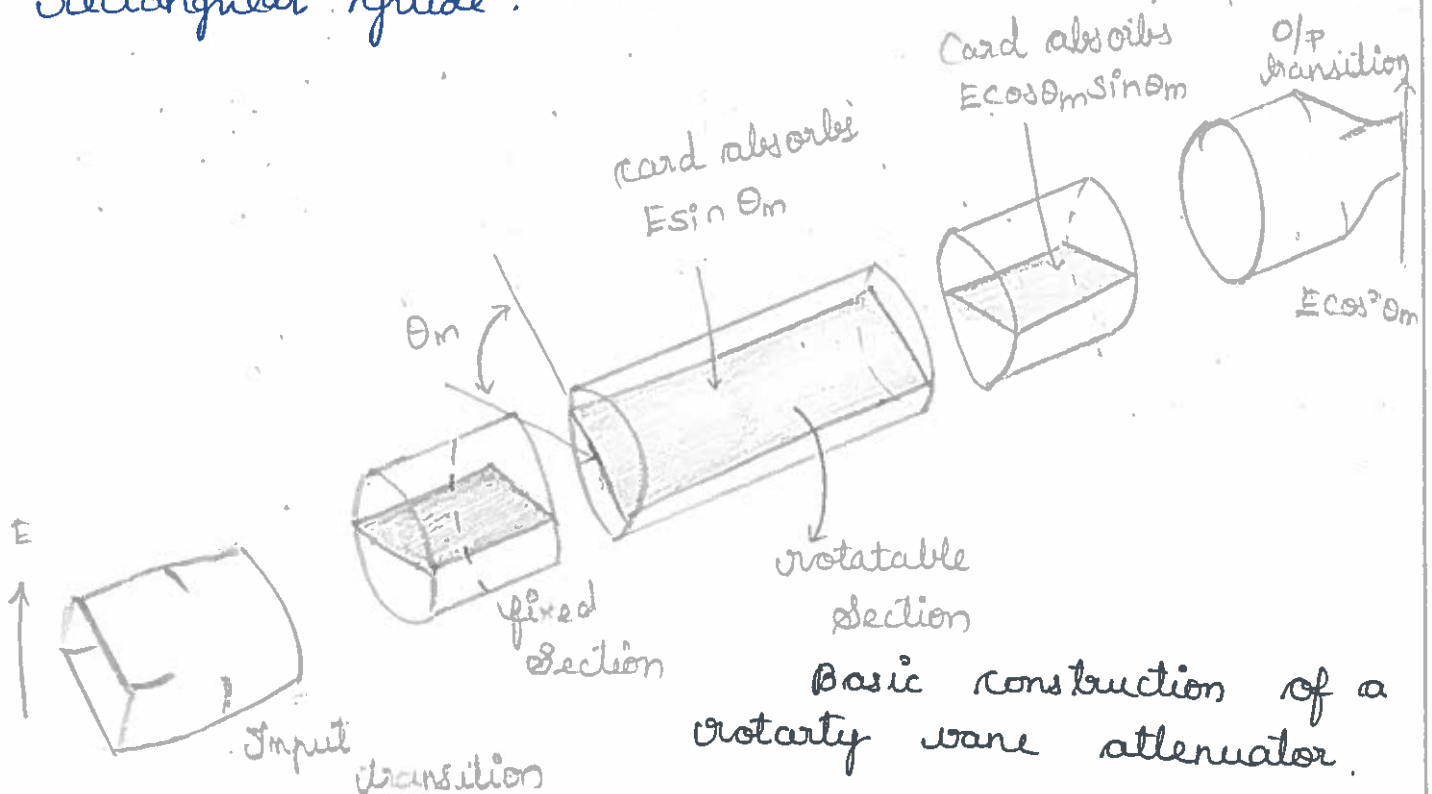
## Rotary - vane attenuator :-

\* Rotary - vane attenuator is so accurate that it is used as a calibration standard in most of the microwave laboratories.

\* Rotary - vane attenuator is a simple form of attenuator consists of a thin tapered resistive card, whose depth of penetration waveguide is adjustable.



\* Rotary vane attenuator has three circular waveguide section, two fixed and one rotatable. It also includes input and output transitions that provides low SWR connections to standard rectangular guide.



\* The attenuation is controlled by rotation of the center section. Here an attenuation is a function of rotation angle  $\theta_m$  only. The minimum loss occurring with  $\theta_m = 0$  and the maximum loss occurs when  $\theta_m = 90^\circ$ . The principle of operation is based upon an interaction between the plane-polarized waves and the thin resistive cards.

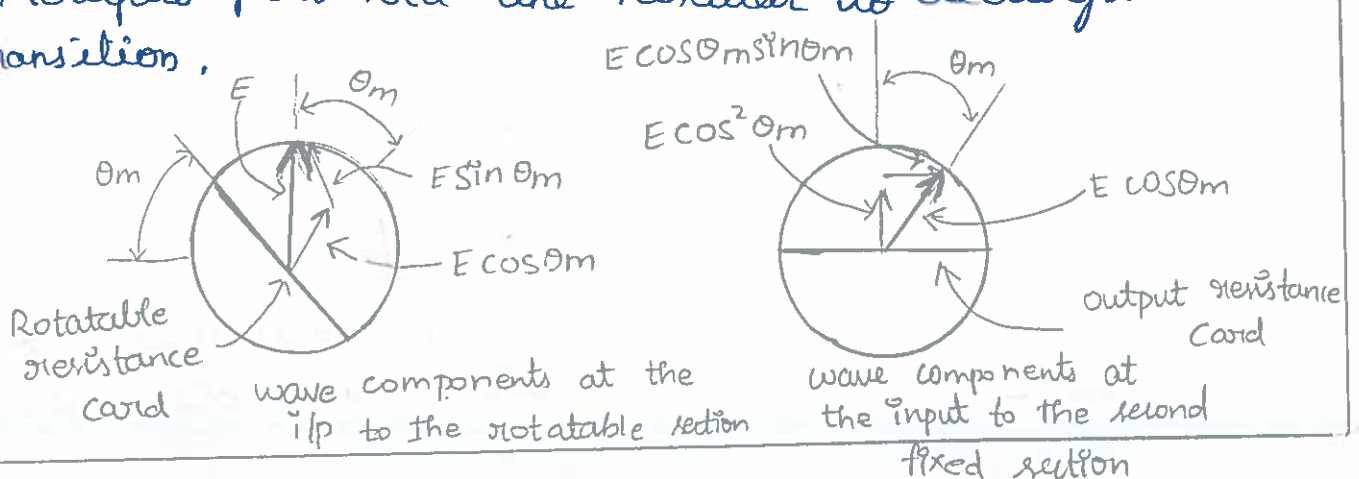
\* The input transition is which converts the  $TE_{10}$  wave into a vertically polarized  $TE_{11}$  wave in a circular guide. The electric field associated with this wave is denoted by  $E$ .

\* In the first fixed section, the resistive card perpendicular to an electric field and the wave propagates without any loss.

\* When the card in the rotatable section is horizontal ( $\theta_m = 0$ ), then the wave passes through it and also to an output fixed section without any loss. Thus for  $\theta_m = 0$ , the total loss is 0 dB.

\* For any other angle, the component which is parallel to the rotatable card ( $E \sin \theta_m$ ) is absorbed and the perpendicular component ( $E \cos \theta_m$ ) arrives at the second fixed section with its polarization at an angle of  $\theta_m$  with respect to the vertical plane.

\* The portion of the wave that is parallel to the output card ( $E \cos \theta_m \sin \theta_m$ ) is absorbed, while the perpendicular components ( $E \cos^2 \theta_m$ ) proceeds to an output port via the circular to rectangular transition.



# MICROWAVE RESONATOR :-

\* Microwave resonators are tunable circuits which are used in microwave oscillators, amplifiers, wavemeters and filters.

\* At the tuned frequency the circuit resonates where the average energies stored in an electric field,  $W_e$  and magnetic field,  $W_m$  are equal and the circuit impedance becomes purely real.

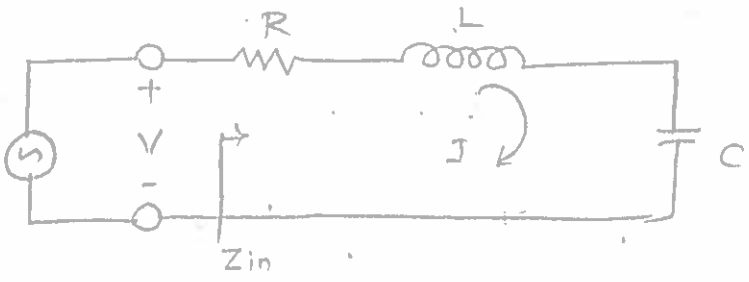
Quality factor  $Q$  which is a measure of the frequency selectivity of a cavity and it is defined as,

$$Q = \frac{2\pi \times \text{Maximum Energy Stored}}{\text{Energy dissipated per cycle.}}$$

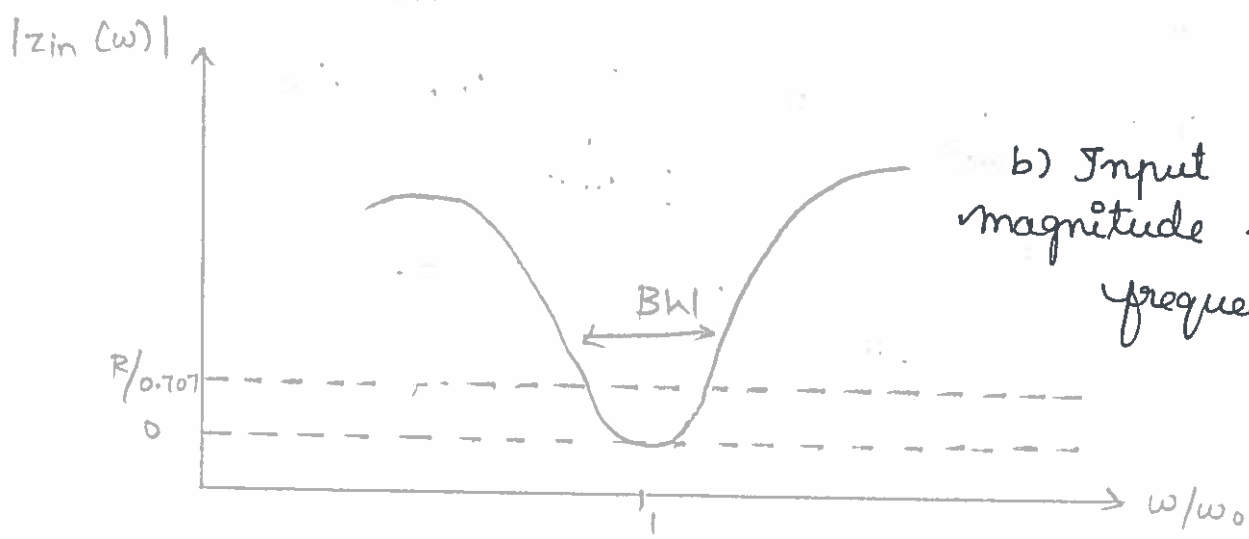
## i) Series resonant circuits :-

At frequencies near resonance, a microwave resonator can usually be modeled by either a series or parallel RLC lumped-element equivalent circuit.

$$Z_{in} = R + j\omega L - j\frac{1}{\omega C} \rightarrow (1)$$



a) A series RLC resonator circuit



b) Input impedance magnitude versus frequency.

\* The complex power delivered to the resonator is given as.

$$P_{in} = \frac{1}{2} VI^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} Z_{in} \left| \frac{V}{Z_{in}} \right|^2 \rightarrow (2)$$

\* By substituting an eqn (1) in eqn. (2) we get

$$P_{in} = \frac{1}{2} |I|^2 \left( R + j\omega L - j\frac{1}{\omega C} \right) \rightarrow (3)$$

\* The power dissipated by the resistor  $R$  is,

$$P_{loss} = \frac{1}{2} |I|^2 R \rightarrow (4a)$$

\* Average magnetic energy stored in the inductor  $L$  is,

$$W_m = \frac{1}{4} |I|^2 L \rightarrow (4b)$$

\* Average electric energy stored in the capacitor  $C$  is,

$$W_e = \frac{1}{4} |V_c|^2 C = \frac{1}{4} |I|^2 \frac{1}{\omega^2 C} \rightarrow (4c)$$

where  $V_c \rightarrow$  voltage across the capacitor

Then the complex power of (3) can be rewritten as,

$$P_w = P_{loss} + 2j\omega (W_m - W_e) \rightarrow (5)$$

and the input impedance of (1) can be rewritten as,

$$Z_{in} = \frac{2 P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega (W_m - W_e)}{\frac{1}{2} |I|^2} \rightarrow (6)$$

$$Z_{in} = \frac{P_{loss}}{\frac{1}{2} |I|^2} = R \rightarrow (7)$$

(4)

\* From equation 4b & 4c,  $W_m = W_e$  implies that the resonant frequency,  $\omega_0$  can be defined as,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \rightarrow (8)$$

\* Quality factor of a resonant circuit is defined as,

$$Q = \omega \frac{\text{Average energy stored}}{\text{Energy loss / second}} = \omega \frac{W_m + W_e}{P_{\text{loss}}} \quad \rightarrow (9)$$

\* Thus  $Q$  is a measure of the loss of a resonant circuit that is, lower loss implies a higher  $Q$ .

\* The  $Q$  of the resonator itself, disregarding external loading effects, is called as unloaded  $Q$  and it is denoted as  $Q_0$ .

$$Q_0 = \omega_0 \frac{2W_m}{P_{\text{loss}}} = \omega_0 \frac{2 \times \frac{1}{4} |I|^2 L}{\frac{1}{2} |I|^2 R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$\rightarrow$  (10)

Average energy stored =  $W_m + W_e$   
At resonance,  $W_m = W_e$   
So,  $2W_m$  or  $2W_e$

\* Eqn (10) show that  $Q$  increases as  $R$  decreases, At resonance, bandwidth is

$$B.W = \frac{1}{Q_0} \quad \rightarrow (11)$$



## ii) Parallel Resonant circuits :

The parallel RLC resonant circuit which is the dual of the series RLC circuit and an input impedance is expressed as

$$Z_{in} = \left( \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \longrightarrow (12)$$

The complex power delivered to the resonator is given as

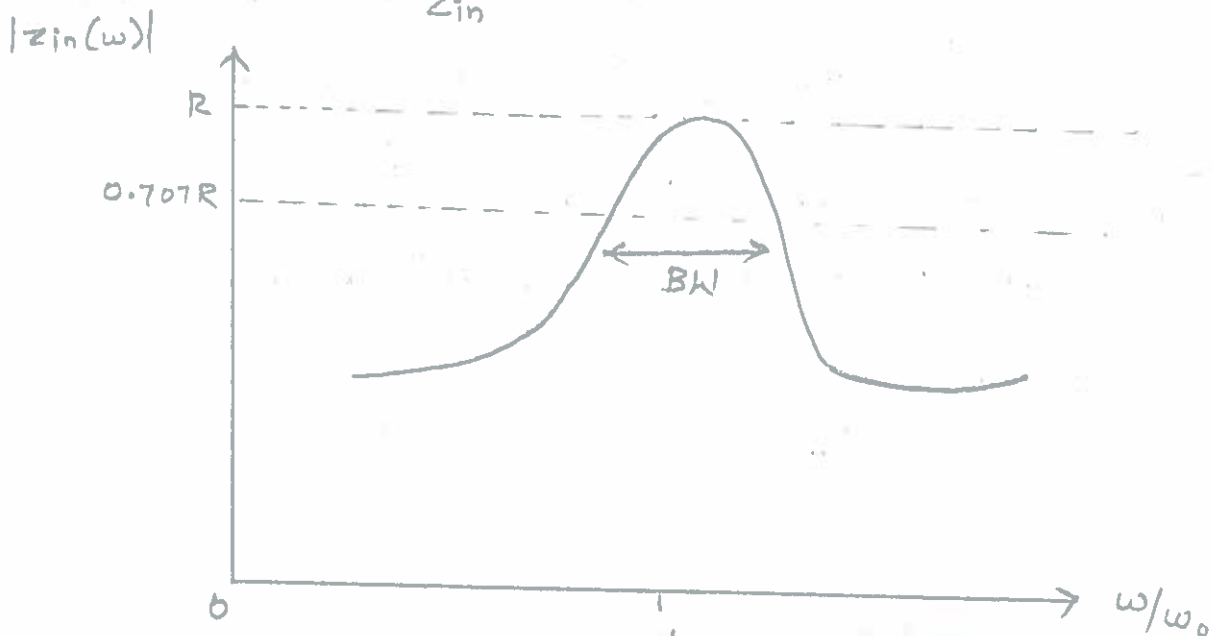
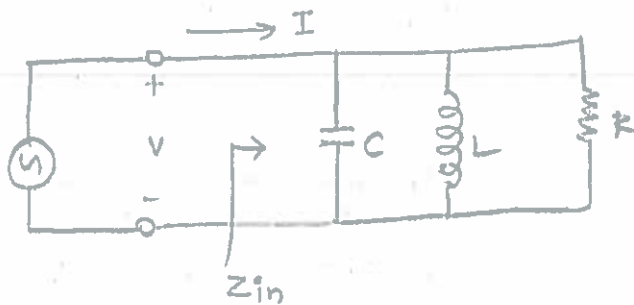
$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} |V|^2 \frac{1}{Z_{in}} \longrightarrow (13)$$

By substituting eqn. (12) in eqn (13) we get

$$P_{in} = \frac{1}{2} |V|^2 \left( \frac{1}{R} + \frac{j}{\omega L} - j\omega C \right) \longrightarrow (14)$$

The power dissipated by the resistor R is,

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R} \longrightarrow (15a)$$



\* The average electric energy stored in the capacitor C is

$$W_e = \frac{1}{4} |V|^2 C \longrightarrow (15b)$$

\* The average magnetic energy stored in the inductor L is,

$$W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L} \longrightarrow (15c)$$

\* where  $I_L$  is the current through the inductor. Then, the complex power from eqn (14) can be rewritten as,

$$P_{in} = P_{loss} + 2j\omega (W_m - W_e) \longrightarrow (16)$$

\* Similarly, the input impedance can be expressed as,

$$Z_{in} = \frac{2 P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega (W_m - W_e)}{\frac{1}{2} |I|^2} \longrightarrow (17)$$

Resonance occurs when  $W_m = W_e$ . Then from eqn (17) and (15a), the input impedance at a resonance is expressed as,

$$\begin{aligned} Z_{in} &= \frac{P_{loss}}{\frac{1}{2} |I|^2} && [\because |V|^2 = |I|^2 R^2] \\ &= \frac{\frac{1}{2} \frac{|V|^2}{R}}{\frac{1}{2} |I|^2} = R \longrightarrow (18) \end{aligned}$$

This is a purely real impedance. From eqn. (15b) and eqn. (15c), we get  $W_m = W_e$

implies that the resonant frequency  $\omega_0$  can be defined as,

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow (19)$$

$$Q_0 = \omega_0 \frac{W_m}{P_{loss}}$$

By using (15a) and (15c) in the above eqn. we obtained,

$$= \omega_0 \frac{2 \times \frac{1}{4} |V|^2 \frac{1}{\omega_0^2 L}}{\frac{1}{2} \frac{|V|^2}{R}}$$

$$= \frac{R}{\omega_0 L}$$

$$= \omega_0 R C \longrightarrow (20)$$

Eqn. 20 show that the  $Q$  of the parallel resonant circuit increases as  $R$  increases.

The behaviour of the magnitude of the input impedance versus frequency. The half-power bandwidth edges occur at frequencies  $\left( \frac{\Delta \omega}{\omega_0} = \frac{B.W}{2} \right)$  such that

$$|Z_{in}|^2 = \frac{R^2}{2}$$

which implies that

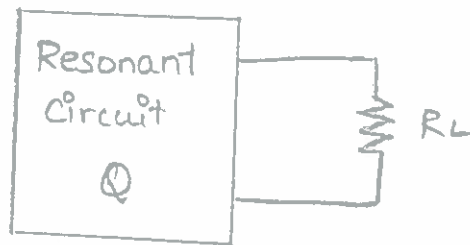
$$B.W = \frac{1}{Q_0} \longrightarrow (21)$$

Quantity	Series Resonator	Parallel Resonator
Input impedance/ admittance	$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$ $\approx R + j\frac{2RQ_0\Delta\omega}{\omega_0}$	$Y_{in} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$ $\approx \frac{1}{R} + j\frac{2Q_0\Delta\omega}{R\omega_0}$
Power Loss	$P_{loss} = \frac{1}{2}  I ^2 R$	$P_{loss} = \frac{1}{2} \frac{ V ^2}{R}$
Stored magnetic energy	$W_m = \frac{1}{4}  I ^2 L$	$W_m = \frac{1}{4}  V ^2 \frac{1}{\omega^2 L}$
Stored electric energy	$W_e = \frac{1}{4}  I ^2 \frac{1}{\omega^2 C}$	$W_e = \frac{1}{4}  V ^2 C$
Resonant frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
unloaded Q	$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$	$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$
External Q	$Q_e = \frac{\omega_0 L}{R_L}$	$Q_e = \frac{R_L}{\omega_0 L}$

iii) Loaded and unloaded  $Q$  :-

\* The unloaded  $Q$  that is  $Q_0$  is defined as a characteristic of the resonator itself in the absence of any loading effects caused by an external activity.

\* A resonator is invariably coupled to an other circuitry, which will have the effect of lowering the overall or loaded  $Q$ , that is  $Q_L$  of the circuit.



\* A resonator coupled to an external load resistor  $R_L$ . If the resonator is a series RLC circuit, the load resistor  $R_L$  adds in the series with  $R$  so that an effective resistance  $R + R_L$ .

If the resonator is a parallel RLC circuit the load resistor  $R_L$  combines in parallel with  $R$ , so an effective resistance is  $RR_L / (R + R_L)$ .  
A external  $Q$  is defined as

$$Q_e = \begin{cases} \frac{\omega_0 L}{R_L} & \text{for series circuits} \\ \frac{R_L}{\omega_0 L} & \text{for parallel circuits} \end{cases}$$

then the loaded  $Q$  can be expressed as,

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_0}$$



## Transmission Line Resonators:-

\* Ideal lumped circuit elements are often unattainable the at microwave frequencies, so distributed elements are frequently used

\* The different sections of transmission line are used with various lengths and terminations to form resonators.

### 1) Short-circuited $\lambda/2$ line:-

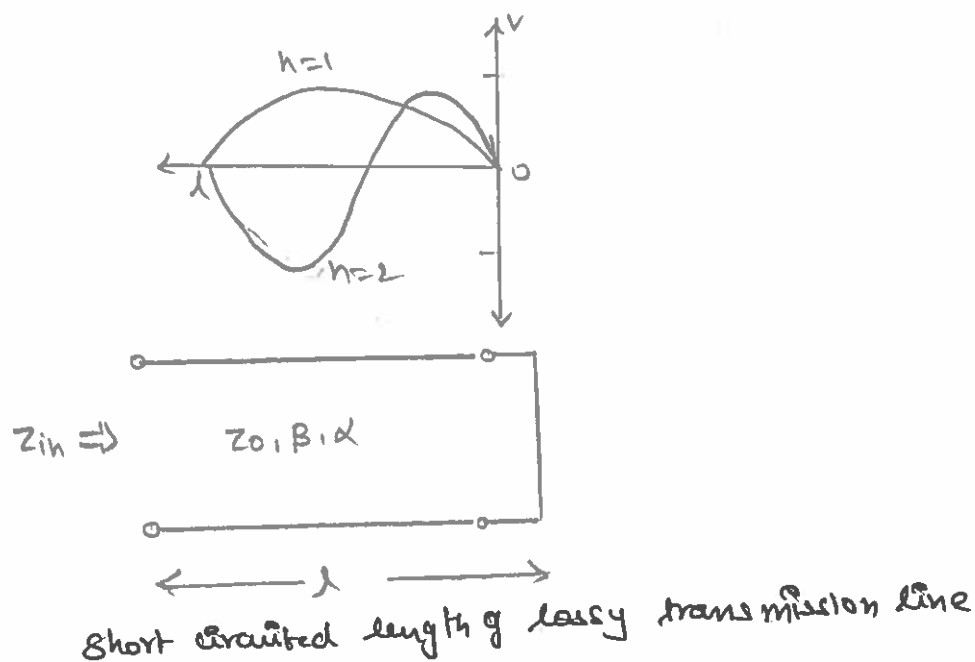
\* A length of lossy transmission line, short circuit at ~~the~~ <sup>one</sup> end is shown in Fig 6-31. The line has a characteristic impedance,  $Z_0$ , propagation constant,  $\beta$ , and attenuation constant,  $\alpha$ ,

\* At the resonant frequency  $\omega = \omega_0$ , the length of the line is  $l = \lambda/2$  and an input impedance  $Z_{in}$  is expressed as

$$Z_{in} = Z_0 \tanh(\alpha + j\beta)l \quad \text{--- (1)}$$

using an identity for the hyperbolic tangent and it gives

$$Z_{in} = Z_0 \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tan \beta l \tanh \alpha l} \quad \text{--- (2)}$$



\* For a lossless line  $\alpha = 0$  and  $Z_{in} = jZ_0 \tan \beta l$ . In practice it is usually desirable to use a low-loss transmission line, so we assume that  $\alpha l \ll 1$  and then  $\tanh \alpha l \approx \alpha l$ . For  $\beta l \ll \pi/2$  ( $\omega \ll \omega_0$ ), the input impedance becomes

$$Z_{in} = R + 2jL\Delta\omega \longrightarrow \textcircled{2}$$

when

$$R = Z_0\alpha l \longrightarrow \textcircled{3}$$

and inductance of the equivalent circuit as

$$L = \frac{Z_0\pi}{2\omega_0} \longrightarrow \textcircled{4}$$

\* The capacitance of the equivalent circuit is given as

$$C = \frac{1}{\omega_0^2 L} \longrightarrow \textcircled{5}$$

\* The resonator of this resonates for  $\Delta\omega = 0$  ( $l = \lambda/2$ ) and its input impedance at resonance is  $Z_{in} = R = Z_0\alpha l$

\* Resonance also occurs for  $l = n\lambda/2$ ,  $n = 1, 2, 3, \dots$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha l} = \frac{\beta}{2\alpha} \longrightarrow \textcircled{6}$$

2) Short-circuited  $\lambda/4$  line:-

\* The input impedance of a shorted line of length  $l$  is expressed as

$$Z_{in} = Z_0 \tanh(\alpha + j\beta)l = Z_0 \frac{\tanh\alpha l + j\tan\beta l}{1 + j\tan\beta l \tanh\alpha l}$$

$$Z_{in} = Z_0 \frac{1 - j\tanh\alpha l \cot\beta l}{\tanh\alpha l - j\cot\beta l} \longrightarrow \textcircled{7}$$

\* For  $l = \lambda/4$  at  $\omega = \omega_0$ , for small loss  $\tanh\alpha l \approx \alpha l$  and for  $\alpha l \pi \Delta\omega / 2\omega_0 \ll 1$ , then an input impedance is expressed as

$$Z_{in} = \frac{1}{(1/R) + 2j\Delta\omega C} \longrightarrow \textcircled{8}$$

\* The resistance of the equivalent circuit is expressed as

$$R = \frac{Z_0}{\alpha l} \longrightarrow \textcircled{9}$$

capacitance

$$C = \frac{\pi}{4\omega_0 Z_0} \longrightarrow \textcircled{10}$$

Inductance

$$L = \frac{1}{\omega_0^2 C} \longrightarrow \textcircled{11}$$

\* The Resonator has a parallel-type resonance for  $l = \lambda/4$ , with an input impedance at resonance of  $Z_{in} = R = \frac{Z_0}{\alpha l}$ .

Using equation (9) and (10) we get

$$= \omega_0 \times \frac{Z_0}{\alpha l} \times \frac{\pi}{4\omega_0 Z_0} = \frac{\pi}{4\alpha l} \longrightarrow (12)$$

At resonance,

$$l = \frac{\pi}{2\beta} = \frac{\pi}{4\alpha \times \frac{\pi}{2\beta}} = \frac{\beta}{2\alpha} \longrightarrow (13)$$

3) open circuit  $\lambda/2$  line:—

\* The input impedance of an open-circuited lossy transmission line of length  $l$

$$Z_{in} = Z_0 \coth(\alpha + j\beta)l = Z_0 \frac{1 + j \tanh \beta l \tanh \alpha l}{\tanh \alpha l + j \tanh \beta l} \longrightarrow (14)$$

when  $l = \lambda/2$  at  $\omega = \omega_0$ , at resonance and  $\tanh \alpha l \approx \alpha l$

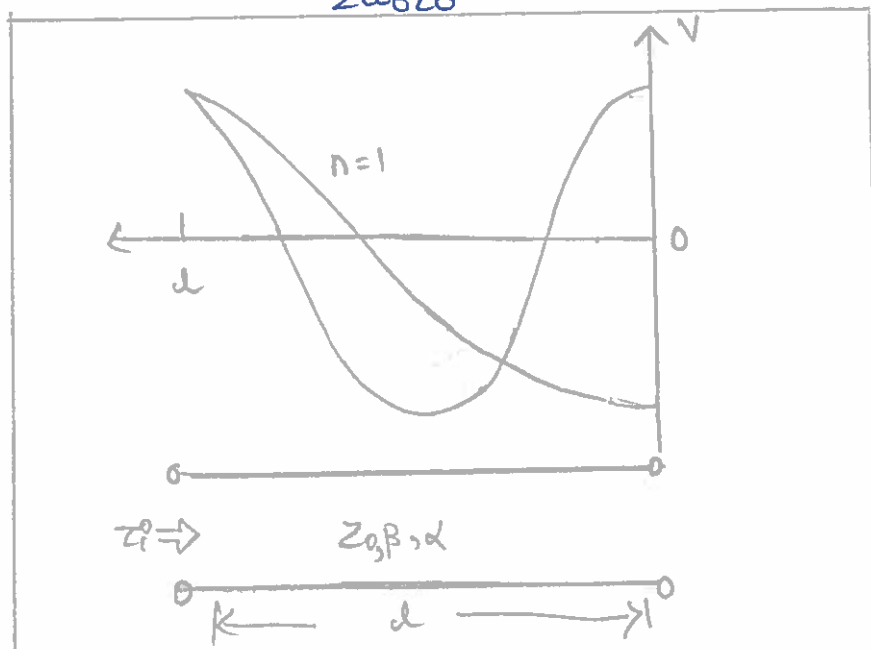
$$Z_{in} = \frac{Z_0}{\alpha l + j(\Delta \omega \pi / \omega_0)} \longrightarrow (15)$$

\* The resistance of the equivalent PLC circuit is

$$R = \frac{Z_0}{\alpha l} \longrightarrow (16)$$

and capacitance

$$C = \frac{\pi}{2\omega_0 Z_0} \longrightarrow (17)$$



Inductance of the equivalent

$$L = \frac{1}{\omega_0^2 C} \quad \text{--- (18)}$$

\* From equation (16) and (17), the unloaded  $Q$  is obtained as

$$Q_0 = \omega_0 R_c = \omega_0 \times \frac{Z_0}{2\alpha} \times \frac{\pi}{2\omega_0 Z_0} = \frac{\pi}{2\alpha l} \quad \text{--- (19)}$$

At resonance  $\alpha = \pi/l$  then equation (18) becomes

$$Q_0 = \frac{R}{2\alpha} \quad [\beta = \pi/l] \quad \text{--- (20)}$$

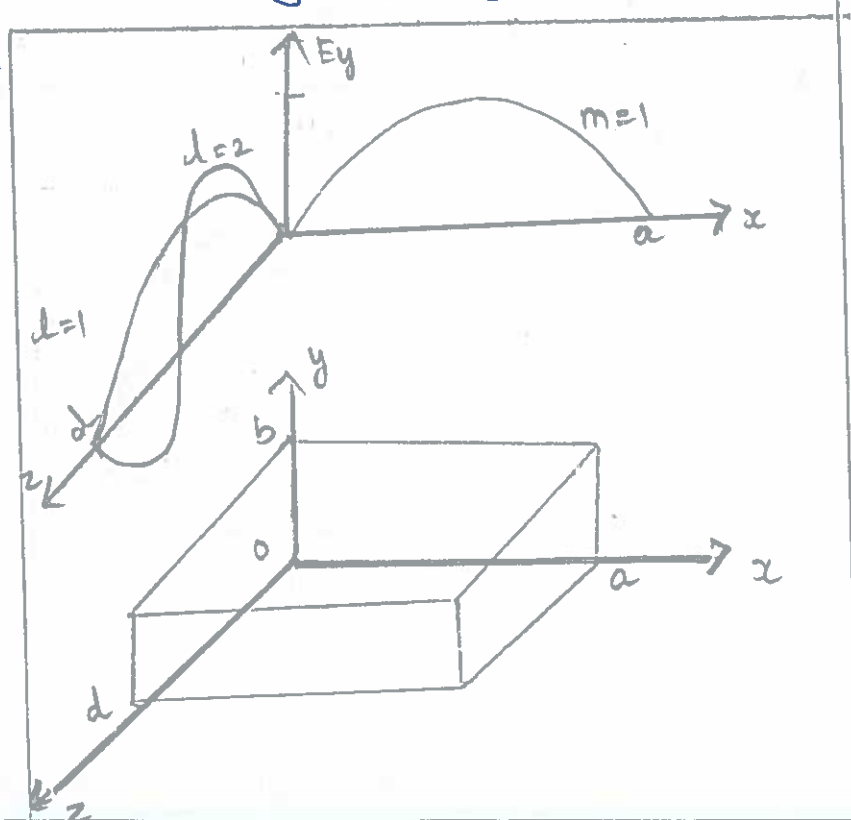
Rectangular waveguide - cavity Resonator:-

\* Microwave resonators can also be constructed from the closed sections of waveguide. It is because the radiation loss from an open-ended waveguide can be significant.

\* Coupling to a cavity resonator may be by a small aperture, or a small probe or loop. The mode having the lowest resonant frequency is known as the dominant mode.

Resonant Frequencies:-

\* To find the fields of the TE or TM waveguide mode that satisfy the necessary boundary conditions on the walls of the cavity.



\* The transverse electric fields ( $E_x, E_y$ ) of the  $TE_{mn}$  or  $TM_{mn}$ , rectangular waveguide mode can be written as

$$\vec{E}_t(x, y, z) = \vec{E}(x, y) (A^+ e^{-j\beta_{mn}z} + A^- e^{j\beta_{mn}z}) \rightarrow \textcircled{1}$$

\* where  $\vec{E}(x, y)$  is the transverse variation of the mode, and  $A^+, A^-$  are arbitrary amplitude of the forward and backward traveling waves

\* The propagation constant of the  $m, n$ th TE or TM mode is obtained as

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \rightarrow \textcircled{2}$$

where  $k = \omega\sqrt{\mu\epsilon}$ , and  $\mu$  and  $\epsilon$  are the permeability and permittivity of the material filling the cavity.

\* A resonance wave number for the rectangular cavity can be defined as

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \rightarrow \textcircled{3}$$

In the  $TM_{mnl}$  or  $TE_{mnl}$  resonant mode of the cavity  $m, n, l$  indicate the number of variations in the standing wave pattern in the  $x, y, z$  directions respectively.

\* The resonant frequency of the  $TE_{mnl}$  and  $TM_{mnl}$  mode is given by

$$f_{mnl} = \frac{ck_{mnl}}{2\pi\sqrt{\mu_r\epsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \rightarrow \textcircled{4}$$

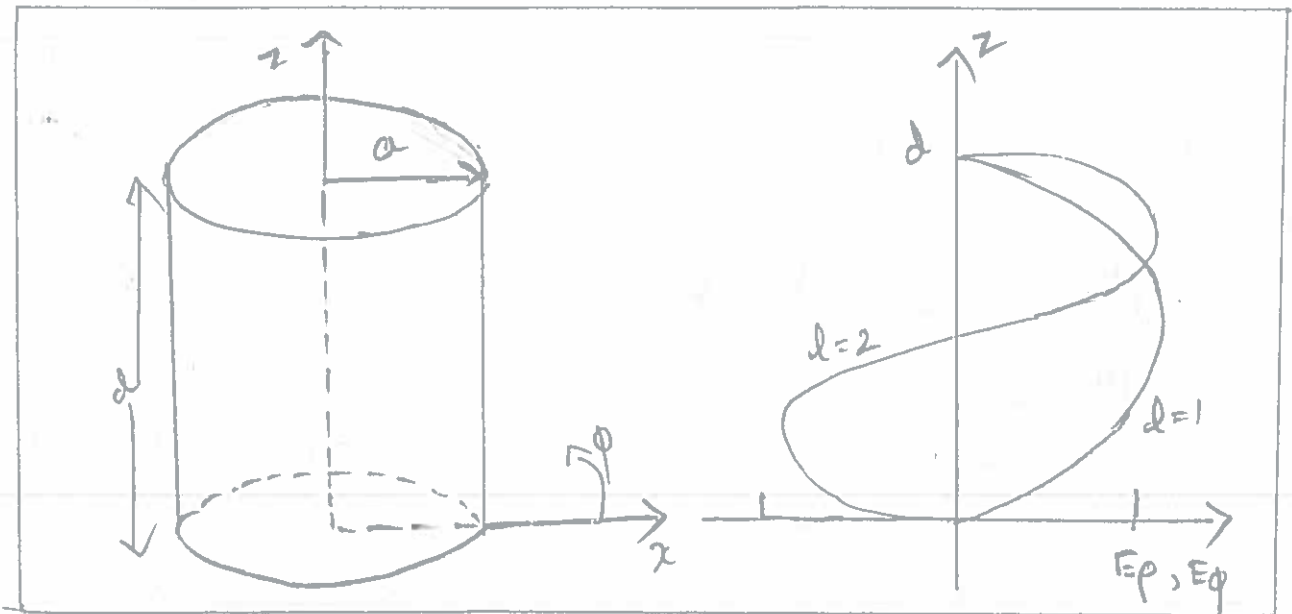
If  $b < a < d$ , the dominant resonant mode will be the  $TE_{101}$  mode, corresponding to the  $TE_{10}$  dominant waveguide mode in a shorted guide of the length  $d/2$ , and it is similar to the short-circuited  $\lambda/2$  transmission line resonator. The dominant TM resonant mode is the  $TM_{110}$  mode



## Circular waveguide - cavity Resonator:-

\* A cylindrical cavity resonator can be constructed from a section of a circular waveguide which is shorted at both ends, similar to rectangular cavities. Because the dominant circular waveguide mode is the  $TE_{11}$  mode, the dominant cylindrical cavity mode is the  $TE_{111}$  mode.

\* In operation, power will be absorbed by the cavity as it is tuned to an operating frequency of the system and this absorption can be monitored with a power meter elsewhere in the system.



### 1) Resonant frequencies

\* The geometry of a cylindrical cavity is shown. The transverse electric fields ( $E_p, E_\phi$ ) of the  $TE_{nm}$  or  $TM_{nm}$  circular waveguide mode can be written as

$$\vec{E}_t(\rho, \phi, z) = \vec{e}(\rho, \phi) (A^+ e^{-j\beta_{nm}z} + A^- e^{j\beta_{nm}z}) \quad \text{--- (5)}$$

where  $\vec{e}(\rho, \phi)$  represents the transverse variation of the mode, and  $A^+$  and  $A^-$  are arbitrary amplitudes of the forward and backward traveling waves.

\* The propagation constant of the TE<sub>nm</sub> mode is

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{P'_{nm}}{a}\right)^2} \longrightarrow \textcircled{6}$$

\* The propagation constant of the TM<sub>nm</sub> mode is

$$\beta_{nm} = \sqrt{k^2 - \left(\frac{P_{nm}}{a}\right)^2} \longrightarrow \textcircled{7}$$

$$\text{where } k = \omega \sqrt{\mu \epsilon}$$

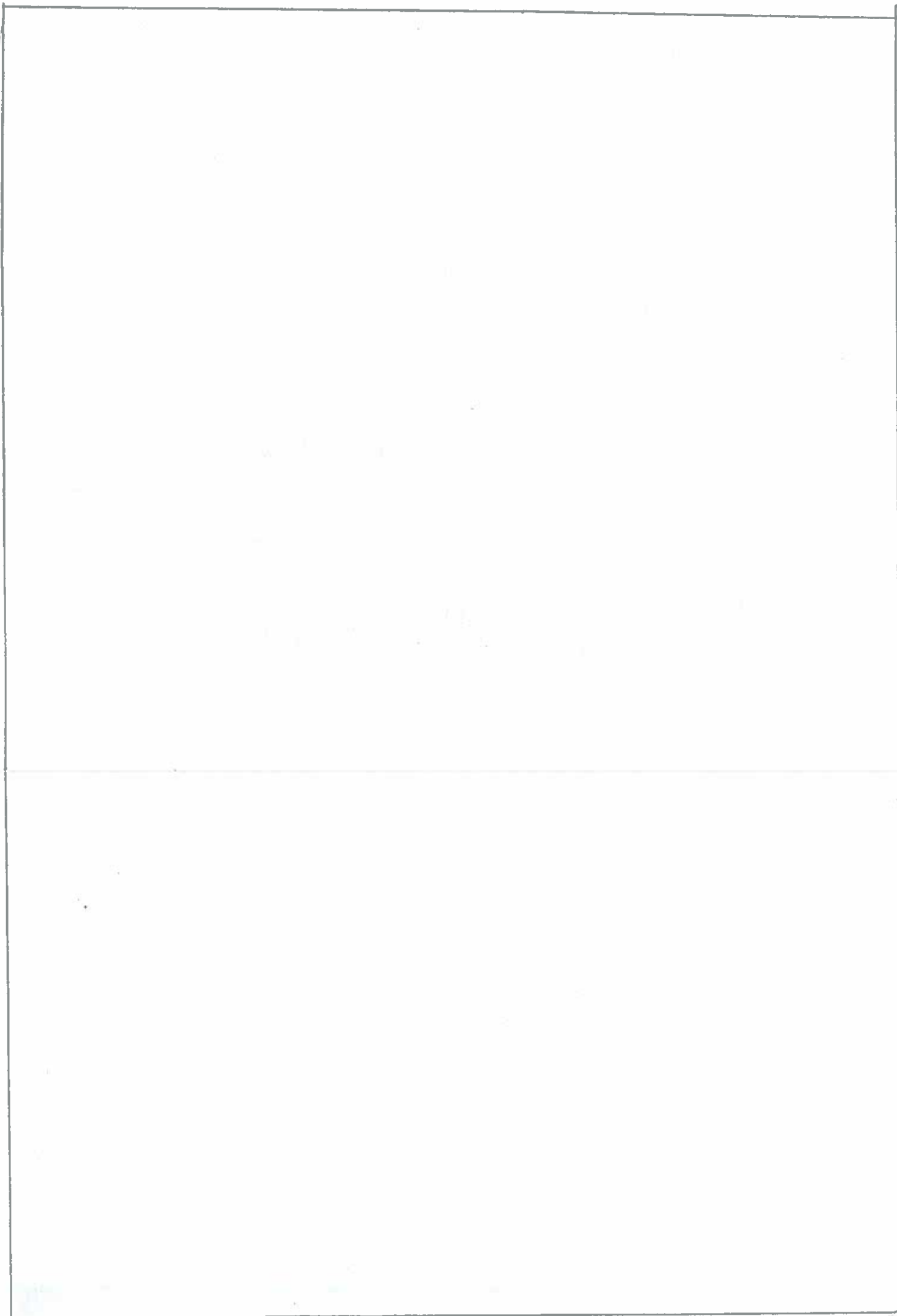
\* The resonant frequency of the TE<sub>nm</sub> mode is

$$f_{nml} = \frac{c}{2\pi \sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \longrightarrow \textcircled{8}$$

$$\text{for } l = 0, 1, 2, 3, \dots$$

and the resonant frequency of the TM<sub>nm</sub> mode is

$$f_{nml} = \frac{c}{2\pi \sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \longrightarrow \textcircled{9}$$



## PRINCIPLES OF MICROWAVE SEMICONDUCTOR DEVICES:

### GUNN DIODE OSCILLATOR:

\* Transferred electron oscillator or Gunn diode oscillator makes use of two terminal devices based on the phenomenon known as "transferred electron effect."

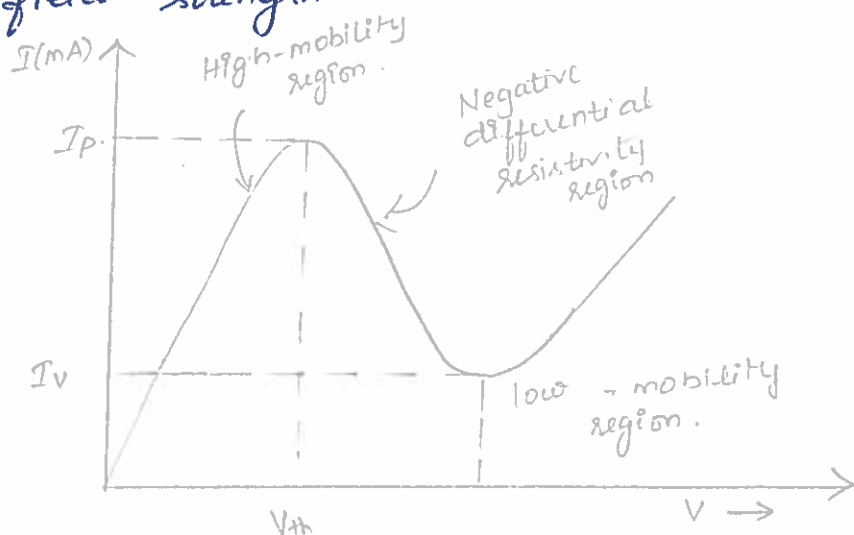
\* Gunn diodes are negative resistance devices which are normally used as a low power oscillator at microwave transmitter and also as local oscillator in receivers.

### Transferred Electron Effect:

→ Some materials like gallium arsenide (GaAs) exhibits a negative differential mobility (ie a decrease in the carrier velocity with an increase in the electric field) when biased above a threshold value of an electric field.

→ The electrons in the low-energy band will be transferred into the higher energy band. The behaviour is called Transferred electron effect (or) Gunn effect and the device is called Transferred electron device (TED) or transferred electron oscillator or Gunn diode or Gunn oscillator.

\* The conductivity is directly proportional to the mobility and hence the current decreases with an increase in electric field strength.



## Applications of Gunn Diode:

(i) Gunn diodes are negative resistance devices which are normally used as a low power oscillator at the microwave frequencies in transmitter and also as local oscillator in receiver front ends.

(ii) Used in parametric amplifiers as pump source

(iii) Used in radar transmitters (police radar, CW Doppler radar).

(iv) In broadband microwave amplifiers.

(v) Pulsed Gunn diode oscillators used in transponders for air traffic control (ATC) and in industry telemetry systems.

(vi) Fast combinational and sequential logic circuits.

(vii) Low and medium power oscillator in microwave receivers.

\* TED's are fabricated from compound semiconductors such as Gallium arsenide (GaAs), Indium phosphide (InP) or Cadmium telluride (CdTe).

\* The positive resistances absorb power (passive devices) whereas negative resistances generate power (active devices).

## Features of TED's:

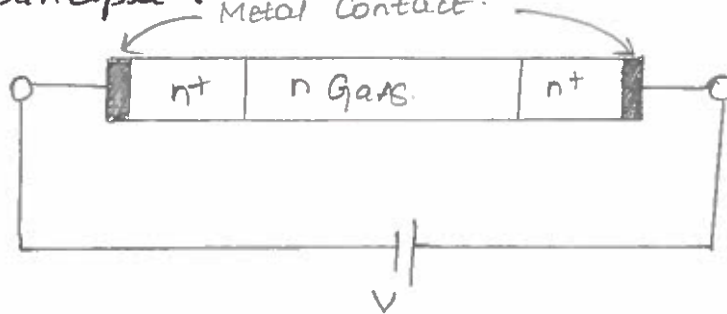
(i) TED's are bulk devices without junctions.

(ii) TED's are operate with hot electrons having more thermal energy and

(iii) TED's are tunable over a wide frequency range with low a noise characteristics.



Working Principle :- Metal Contact.

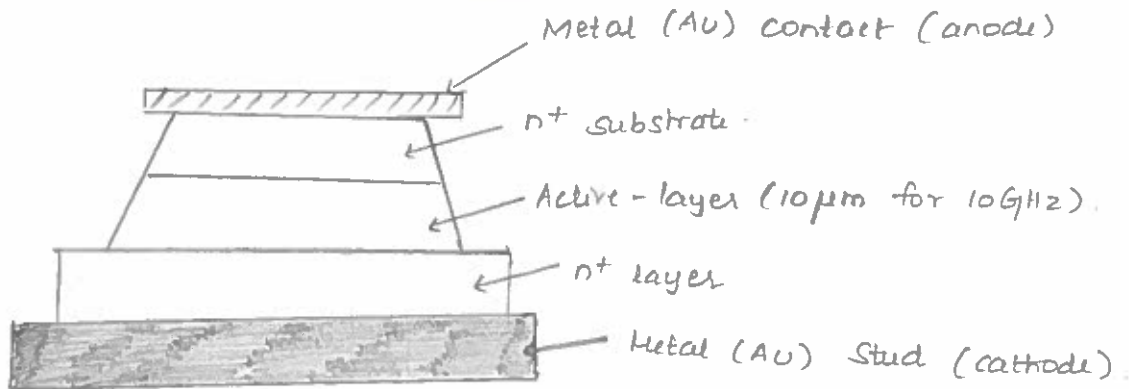


A simple Gunn Oscillator.

\* The basic structure of a Gunn diode as shown in figure which consists of n-type GaAs Semiconductor with regions of high doping ( $n^+$ ).

\* Even though there is no junction this is called a diode with reference to the positive end (anode) and negative end (cathode) of the dc voltage ( $V$ ) which is applied across the device.

\* If a dc (or) diode voltage (or) an electric field at low level is applied to the GaAs an electric field is established across it. Initially, the current will increase with a rise in the voltage.



Basic Construction of Gunn diode.

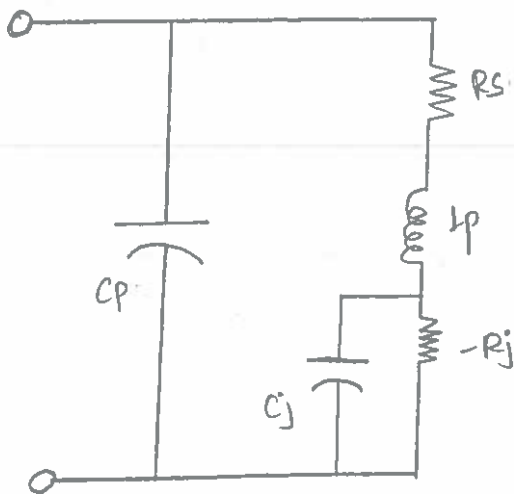
\* At low  $E$ -field in the material, most of the electrons will be located in the lower energy band.

\* When the diode voltage exceeds a certain threshold value ( $V_{th}$ ) a high electric field (3.2 kV/cm for GaAs) is produced across the active regions and thus electrons are excited from their initial lower valley to the higher valley where they become virtually immobile.

\* If the rate at which the electrons transferred is very high, the current will decrease with an increase in voltage, resulting in an equivalent negative resistance effect.

### Negative Resistance:

The carrier drift velocity is linearly increased from zero to a maximum when an electric field is varied from zero to a threshold value. When the electric field is beyond the threshold value of 3000V/cm, the drift velocity is decreased and the diode exhibits negative resistance.



- $C_j$  - Diode capacitance
- $-R_j$  - Diode resistance
- $R_s$  - Total resistance of leads, Ohmic contact, bulk resistance of diode.
- $L_p$  - Package inductance and
- $C_p$  - Package capacitance.

\* GaAs is a poor conductor, considerable heat is generated in the diode. The diode should be well bonded into a heat sink. The negative resistance has a value that typically lies in the range -5 to -20 ohm.

## AVALANCHE TRANSIT - TIME DEVICES:

\* Avalanche transit-time devices (W.T. Read, 1958) are p-n junction diode with the highly doped p and n regions. They could produce a negative resistance at microwave frequencies by using a carrier impact ionization avalanche breakdown and carriers drift in the high field intensity region under the reverse biased condition.

### Modes of Avalanche Device:

There are three distinct modes of avalanche oscillators.

- (i) IMPATT: Impact Ionisation Avalanche Transit Time Device.
- (ii) TRAPATT: Trapped Plasma Avalanche Triggered Transit Device.
- (iii) BARITT: Barrier Injected Transit - Time Device

\* It has long drift regions similar to those of IMPATT diodes. The carriers traversing the drift regions of BARITT diodes. However they are generated by minority carrier injection from forward-biased junctions rather than being extracted from the plasma of an avalanche region.

\* BARITT diodes have low noise figures of 15dB, but their bandwidth is relatively narrow with low output power.

## IMPATT DIODE OSCILLATOR AND AMPLIFIER:

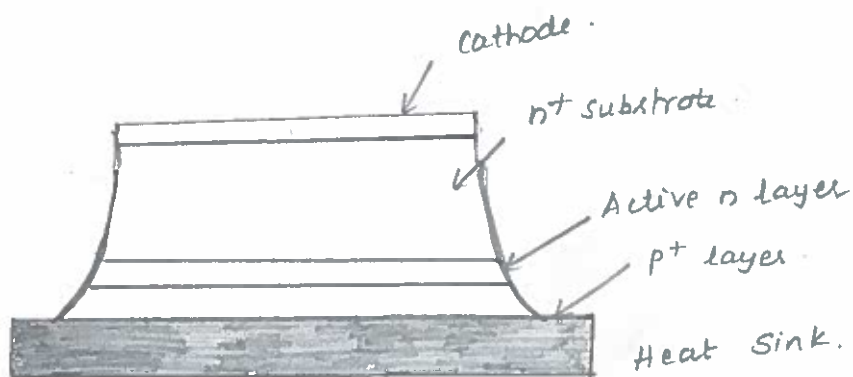
\* The word IMPATT is an acronym for Impact Ionization Avalanche Transit Time. These diodes employ impact ionization and transit time properties of semiconductor structure to produce negative resistance at microwave frequencies.

\* IMPATT diodes have many forms,  $n^+p^+p^+$  or  $p^+n^+n^+$  read device,  $p^+n^+n^+$  abrupt junction and  $p^+i^+n^+$  diode.

\* Many IMPATT diodes consist of a high doping avalanche region followed by a drift region where the field is low enough that the carriers can transverse through it without avalanching.

\* IMPATT diodes can be manufactured from Ge, Si, GaAs or InP. Among these, GaAs provides the highest efficiency, the highest operating frequency and least noise figure. But the fabrication process is more difficult and it is more expensive than Si.

\* A typical construction and package of a simple IMPATT diode is shown in figure. An  $n$ -type epitaxial layer is formed over the  $n^+$  substrate. On top of this is the diffused  $p^+$  layer. A metallised cathode and plated heat sink as anode are also included.



Construction and package of  $p^+n^+n^+$  IMPATT diode.

## Principles of Operation:

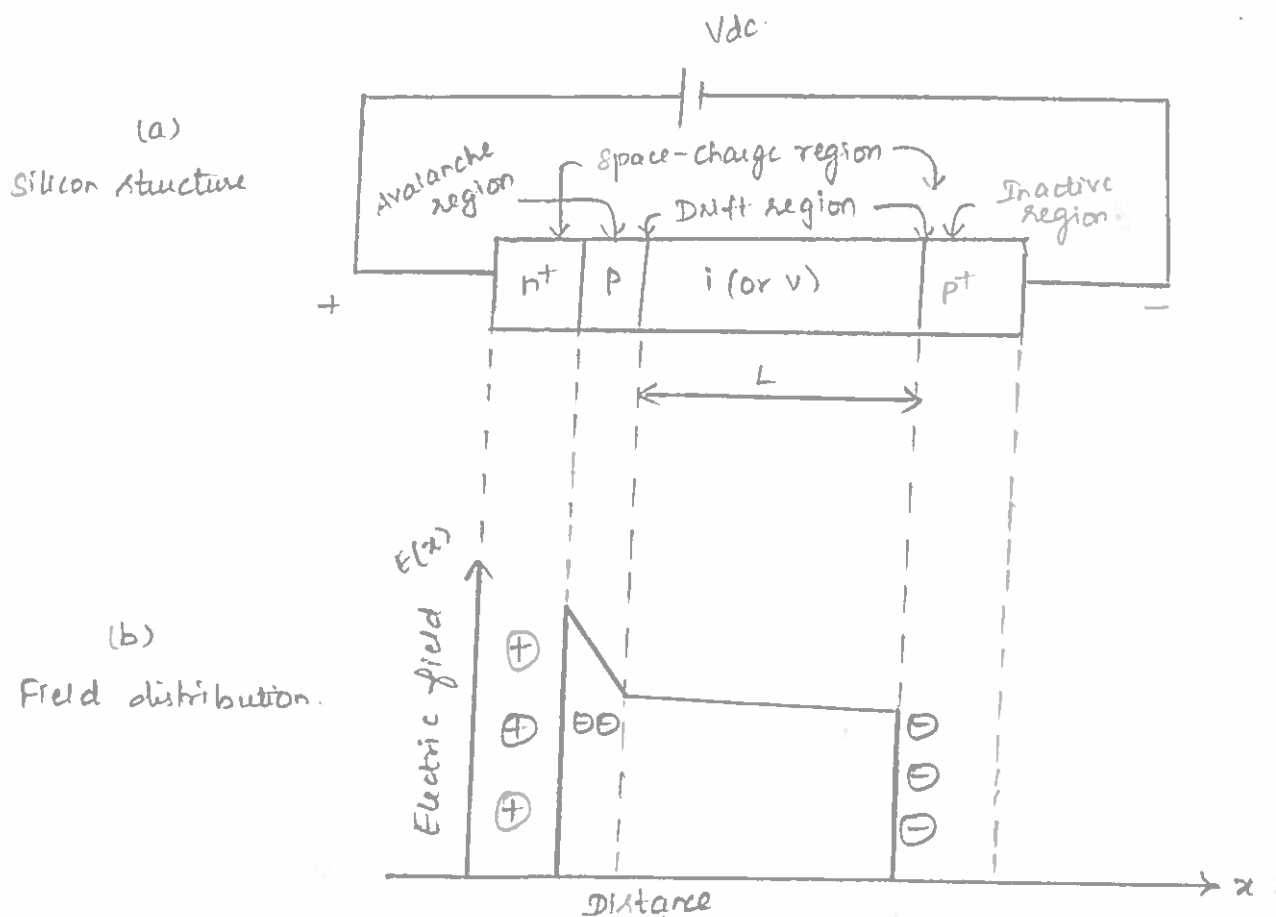
\* From the figure, IMPATT diode is an  $n^+p-i-p^+$  structure where the superscript plus sign denotes very high doping and 'i' or 'v' refers to intrinsic material.

\* The device consists essentially two regions.

(i) The thin 'p' region at which avalanche multiplication occurs. This region is also called the high-field region or the avalanche region.

(ii) The 'i' or 'v' region through which the generated holes must drift in moving to the  $p^+$  contact. This region is also called as the intrinsic region or the drift region.

\* The space between the  $n^+p$  junction and  $i-p^+$  junction is called the Space charge region.





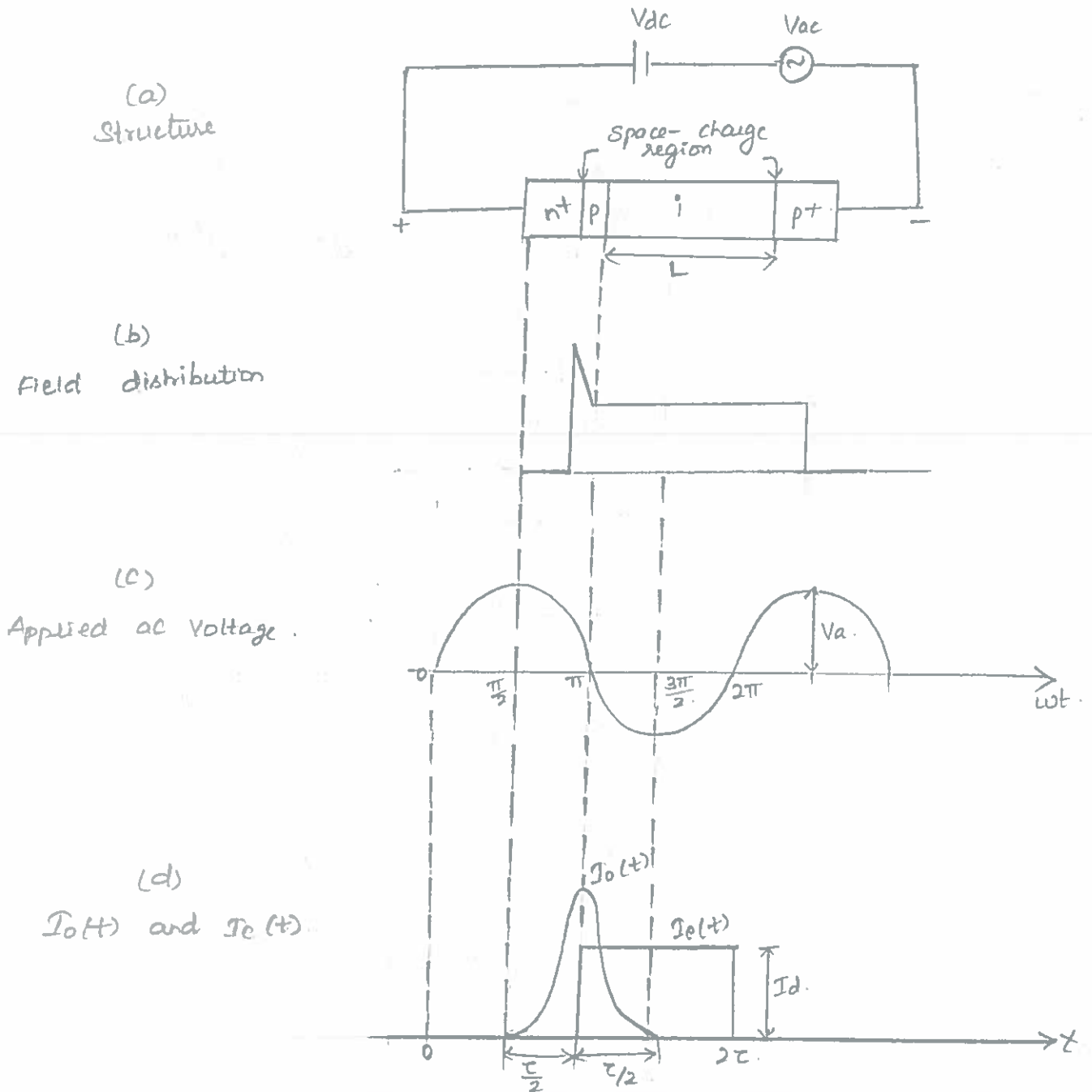
## Avalanche multiplication

The avalanche multiplication factor  $M$  is expressed as

$$M = \frac{1}{1 - \left(\frac{V_{dc}}{V_b}\right)^n} \rightarrow \textcircled{1}$$

## Mechanism of Oscillations:-

When the IMPATT diode is mounted in a microwave resonant circuit an ac voltage can be maintained at a given frequency in the circuit which is shown below.



Field, voltage and currents in IMPATT diode.

Carrier Current  $I_0(t)$  and External Current  $I_e(t)$ :-

\* The total field across the diode is the sum of the dc and ac fields. This total field causes break down at the  $n^+ - p$  junction during the positive half of the ac voltage cycle.

\* If the field is above the breakdown voltage and the carrier current  $I_0(t)$  generated at the  $n^+ - p$  junction by an avalanche multiplication grows exponentially with time while the field is above the critical value.

\* During the negative half cycle when the field is below the breakdown voltage, the carrier current  $I_0(t)$  decays exponentially to a small steady-state value.

\* The carrier current  $I_0(t)$  reaches its maximum in the middle of the ac voltage cycle. Under the influence of an electric field the generated holes are injected into the space-charge region towards the negative terminal.

\* The injected holes traverse the drift space and they can induce a current  $I_e(t)$  in the external circuit.

$$I_e(t) = \frac{Q}{T} = \frac{V_d Q}{L} \rightarrow (2)$$

where  $Q$  - Total charge of the moving holes.

$V_d$  - Hole drift velocity

$L$  - length of the drift region

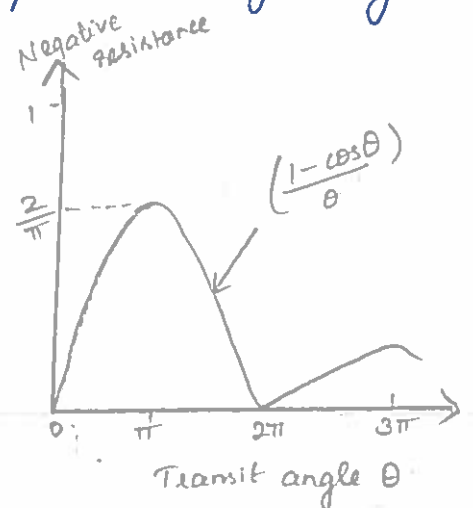
\* When the pulse of hole current  $I_0(t)$  is suddenly generated in the  $n^+p$  junction, a constant current  $I_e(t)$  starts flowing in an external circuit and continues to flow during the time ' $\tau$ ' in which the holes are moving across the space-charge region.

**Negative Resistance:**

These diodes exhibit a differential negative resistance mainly by two effects.

(i) The impact ionization avalanche effect, which causes the carrier current  $I_0(t)$  and the ac voltage to be out of phase by  $90^\circ$ .

(ii) The transit-time effect, which further delays the external current  $I_e(t)$  relative to the ac voltage by  $90^\circ$ .



Negative resistance versus transit angle.

\*  $\theta$  is the transit angle and it is given as

$$\theta = \omega \tau = \omega \frac{L}{V_d} \rightarrow (3)$$

\* The peak value of the negative resistance occurs near  $\theta = \pi$ . For transit angles larger than  $\pi$ , the negative resistance of the diode decreases rapidly.

## Resonant Frequency:

\* The resonant frequency of the cavity is given by

$$f = \frac{1}{2\tau} = \frac{V_d}{2L} \rightarrow \textcircled{A}$$

\* If the resonator is tuned to this frequency, IMPATT diodes provide a high power CW and pulsed microwave source.

## Efficiency:

The efficiency of the IMPATT diode is given by

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{\text{RF power output}}{\text{dc input power}} = \left(\frac{V_a}{V_d}\right) \left(\frac{I_a}{I_d}\right) \rightarrow \textcircled{B}$$

where  $V_a$  &  $I_a$  — ac voltage and current

$V_d$  &  $I_d$  — dc voltage and current

$$\text{Output power} = P_{out} = P_{ac} = \eta P_{dc}$$

## Performance Characteristics:

Theoretically,  $\eta = 30\%$  ( $< 30\%$  in practical) and

15% for Si;

23% for GaAs.

\* GaAs IMPATTs have higher power and efficiency in the 40- to -60 GHz region whereas Si IMPATTs are produced with higher reliability and yield in the same frequency region.

## IMPATT Diode Power Amplifier:

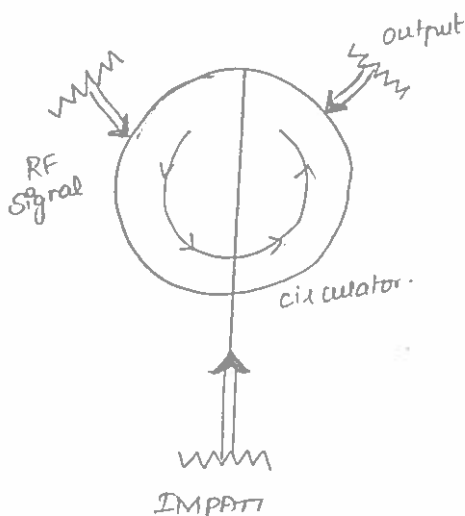
\* The IMPATT diode can be used as an amplifier with the same basic circuit arrangement as oscillator, provided  $R_L > |R_d|$  where  $R_L$  is the load resistance results and  $R_d$  is the diode negative resistance.

\* The circulator is incorporated with IMPATT diode as shown in figure. The negative resistance is used to terminate one port of the circulator and an actual load is connected to the other port.

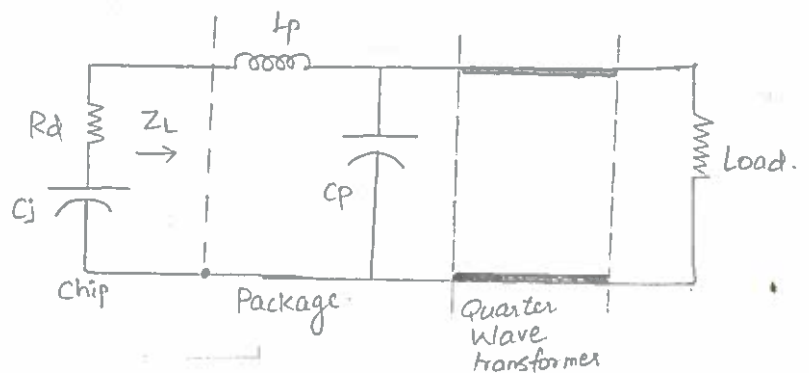
\* The input RF power is fed from the remaining port. The negative resistance results in voltage reflection coefficient at the port which is greater than unity.

\* Thus the average power from the source  $P_{av}$  circulates to the negative resistance port and the reflected power is greater than an incident power. The reflected power is delivered to the load.

IMPATT circulator type amplifier



Equivalent circuit of IMPATT diode.



\* Here,  $R_d$  is the diode negative resistance and  $L_p$ ,  $C_p$  are the package lead inductance and capacitance, respectively.



### Advantages:

IMPATT diodes provide potentially reliable, compact, inexpensive and moderately efficiency microwave power sources.

### Disadvantages:

- (i) IMPATT diodes have low efficiency.
- (ii) It tend to be noisy due to an avalanche process and requires the high level of operating current.
- (iii) A typical noise figure is 30 dB which is worse than that of the Gunn diodes.

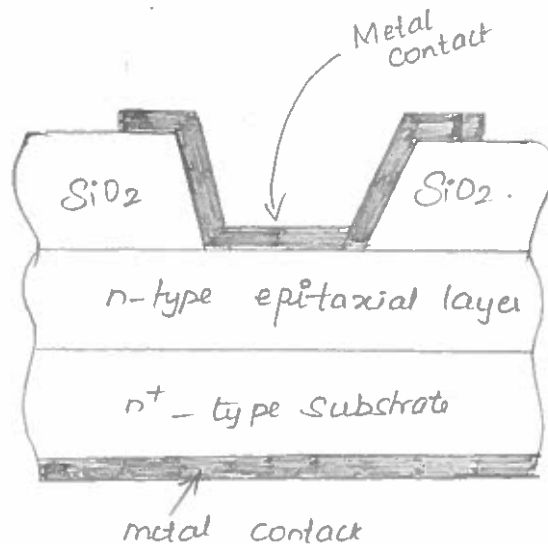
### Applications:

- (i) Used in microwave generators
- (ii) Used in modulated output oscillators.
- (iii) Used in receiver local oscillators.
- (iv) Used in parametric amplifier as pumps.
- (v) IMPATT diodes are suitable for negative resistance amplification.

### SCHOTTKY BARRIER DIODE (SBD)

SBD is a simple metal semiconductor barrier diode that exhibiting a non-linear impedance and it is basically an extension of the point contact diode.

Schottky diode.

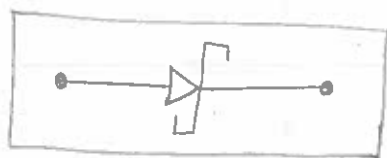


### Construction:

\* The diode is constructed on a thin silicon ( $n^+$ -type) substrate by growing epitaxially on  $n$ -type active layer of about 2-micron thickness. A thin  $\text{SiO}_2$  layer is grown thermally over this active layer.

\* Metal-Semiconductor junction is formed by depositing metal over  $\text{SiO}_2$  - Schottky diodes also exhibit a square-law characteristic and have a higher burn out rating low  $1/f$  noise and better reliability than point contact diodes.

### Operation:

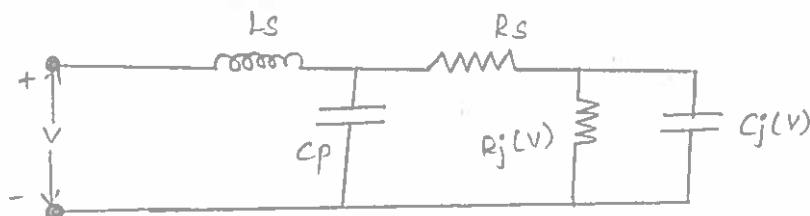


Symbol of SBD

\* When the device is forward biased, the barrier height gets reduced. The major carriers (electrons) can be easily injected from the highly doped  $n$ -semiconductor material into the metal with an approximately  $v$ - $i$  characteristic.

\* When it is reverse-biased, the barrier height becomes too high for the electrons to cross and thus no conduction takes place.

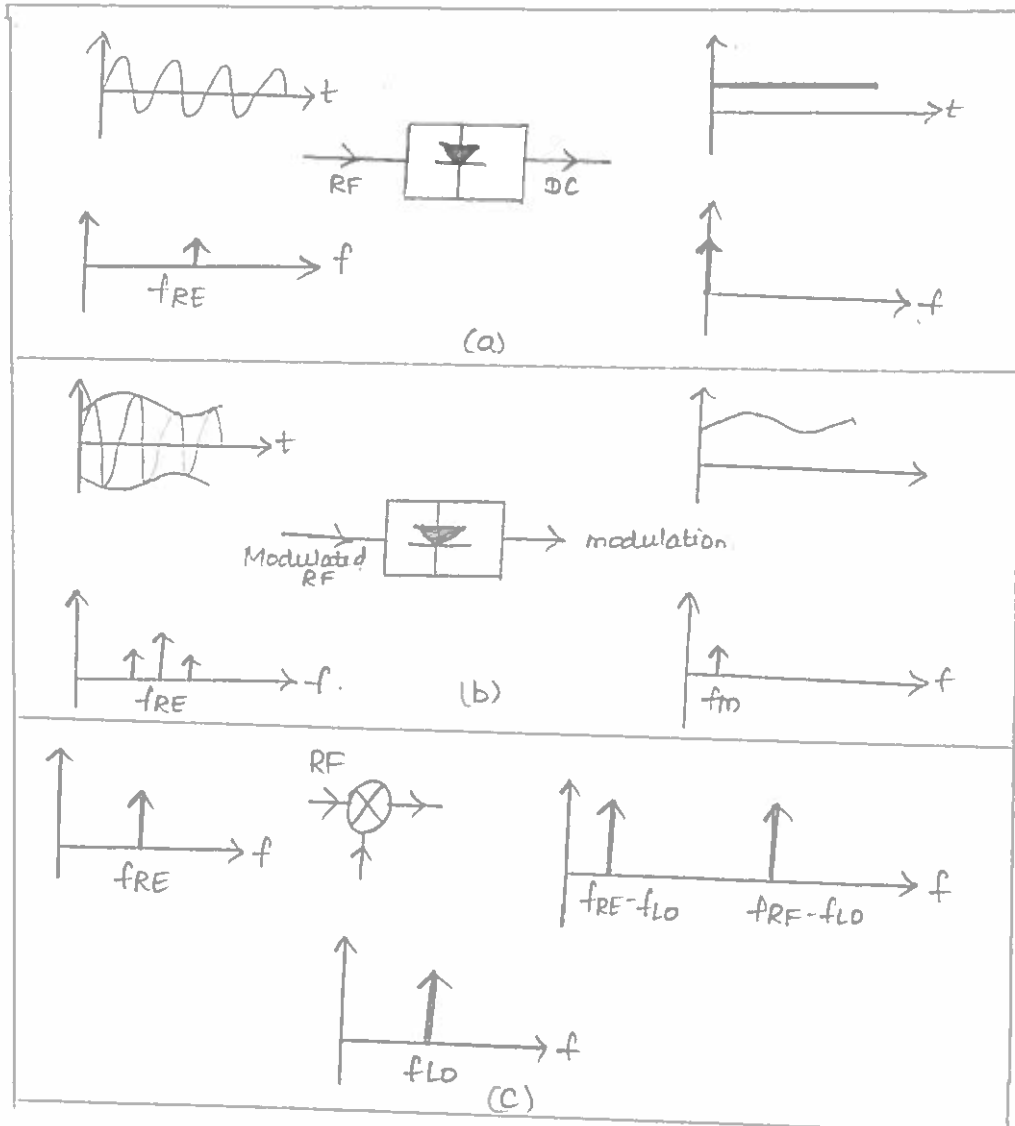
### Equivalent Circuit:



- where
- $R_j$  - Junction resistance of the diode.
  - $C_j$  - Junction capacitance.
  - $R_s$  - Series resistor.
  - $L_s$  - Series inductance.
  - $C_p$  - Shunt Capacitance.

## Applications:

The primary applications of Schottky diodes is in frequency conversion of an input signal. The below figure illustrates the three basic frequency conversion operations of rectification (conversion to DC), detection (demodulation of an amplitude-modulated signal) and mixing (frequency shifting).



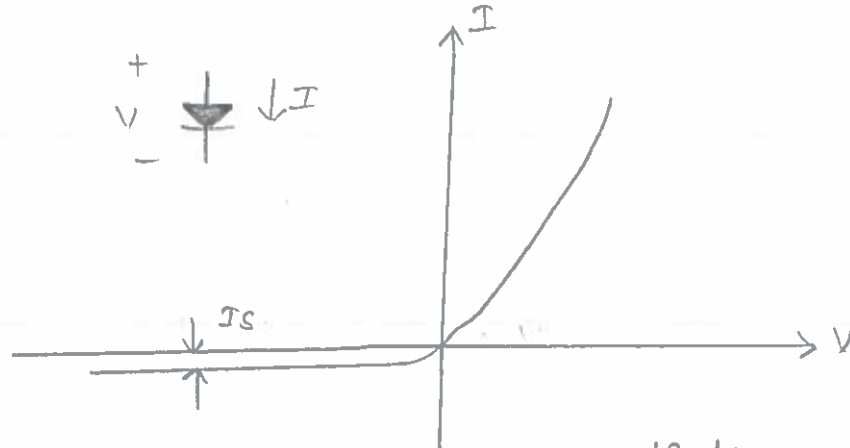
Basic frequency conversion operations of rectification, detection and mixing.

(a) Diode rectifier (b) Diode detector (c) Mixer.

\* A junction diode can be modeled as a nonlinear resistor with a small-signal  $V$ - $I$  relationship expressed as

$$I(V) = I_s (e^{\alpha V} - 1)$$

where  $\alpha = q/nkT$  and  $q$  is the charge of an electron,  $k$  is Boltzmann's constant,  $T$  is temperature,  $n$  is the ideality factor and  $I_s$  is the saturation current.



$V$ - $I$  characteristics of a Schottky diode.

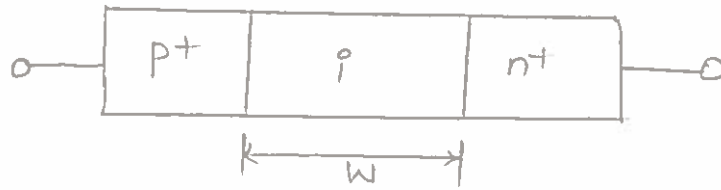
### PIN DIODES:

\* PIN diodes can be used to construct an electronic switching element easily integrated with planar circuitry and it is capable of high-speed operation.

\* Switching speeds typically ranges from 1 to 10ps, although speeds as fast as 20ns are possible with careful designed of the diode driving circuit. PIN diodes can also be used as power limiters, modulators and variable attenuators.

### PIN diode characteristics:

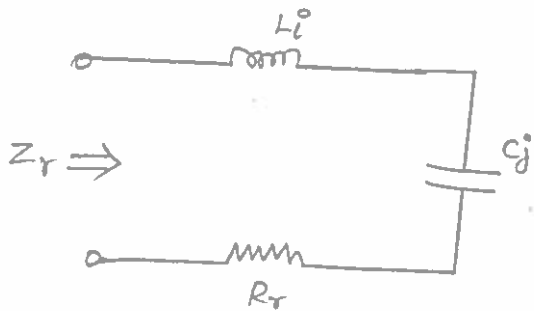
A PIN diode consist of a high-resistivity intrinsic (i) semiconductor layer between two highly doped p<sup>+</sup> and n<sup>+</sup> Si layers as shown in figure. The device acts as electrically variable resistor which is related to the 'i' layer thickness.



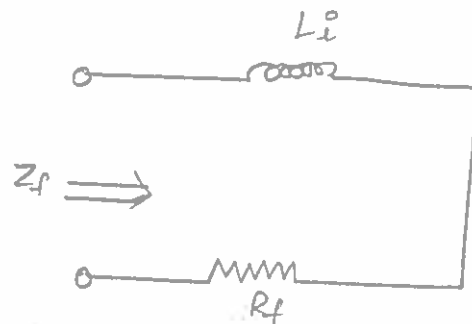
\* The intrinsic layer has a very large resistance in reverse bias and it decreases in forward bias. When mobile carriers from p and n regions are injected into i layer, carriers take time such that the diode ceases to act as a rectifier at the microwave frequency and appears as a linear resistance.

\* The property makes it suitable for being a variable attenuator at the microwave frequencies.

Equivalent Circuit:-



(a) Reverse bias state



(b) Forward bias state

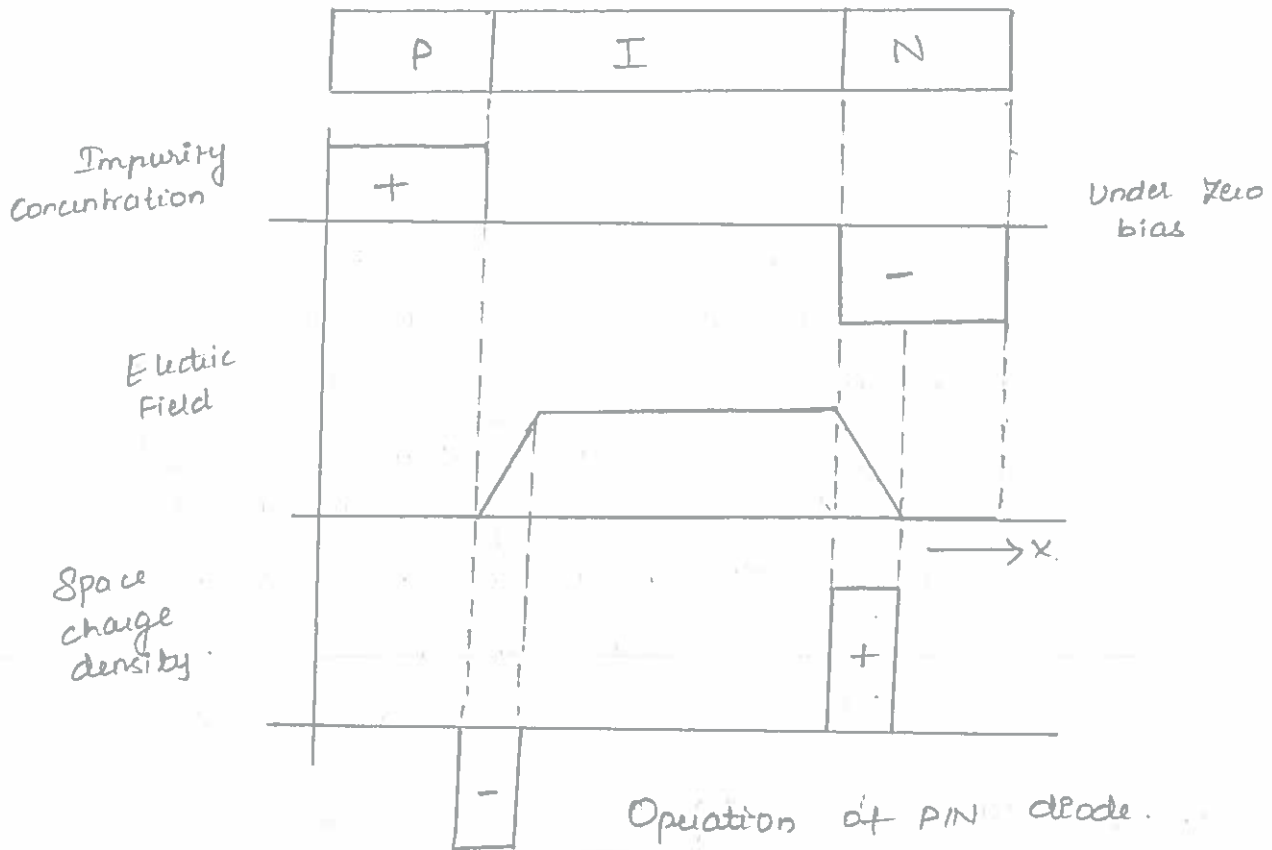
\* The parasitic inductance,  $L_i$ , is typically less than  $1 \text{ nH}$ . The reverse resistance,  $R_r$ , is usually small relative to the series reactance due to the junction capacitance and is often ignored.

Operation of PIN diode:-

The operation can be explained by considering

- Zero bias
- reverse bias
- forward bias





### (i) Zero bias:

\* At the zero bias, the diffusion of the holes and electrons across the junction causes space charge (density) region of thickness which is inversely proportional to the impurity concentration.

\* An ideal  $i$  layer has no depletion region (i.e.)  $p$  layer has a fixed negative charge and  $n$  layer has a fixed positive charge under zero bias.

### (ii) Reverse bias:

\* As reverse bias is applied, the space charge regions in the  $p$  and  $n$  layers will become thicker. The reverse resistance will be very high and almost constant.

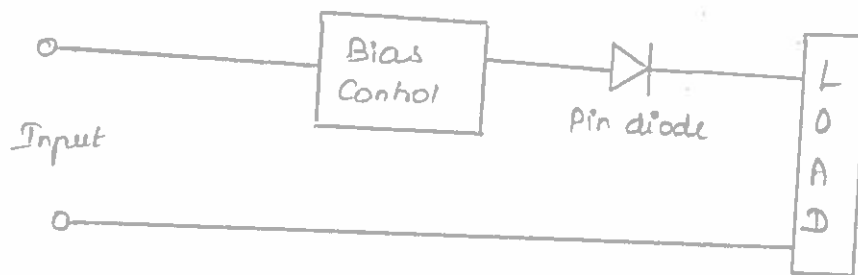
(ii) forward bias:

\* With the forward bias, carriers will be into the  $i$  layer and the  $p$  and  $n$  space charge regions will become thinner (i.e) electrons and holes are injected into the  $p$  layer from  $p$  and  $n$  layers respectively.

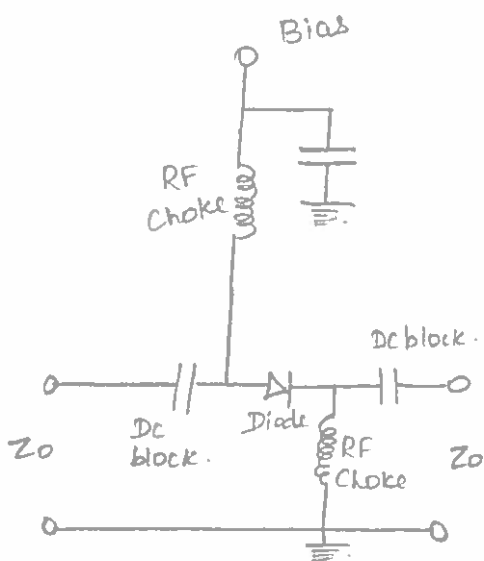
\* This results in the carrier concentration in the  $i$  layer becoming raised above an equilibrium levels and the resistivity drops as the forward bias is increased. Thus, the low resistance is offered in the forward direction.

### Applications of PIN Diodes:

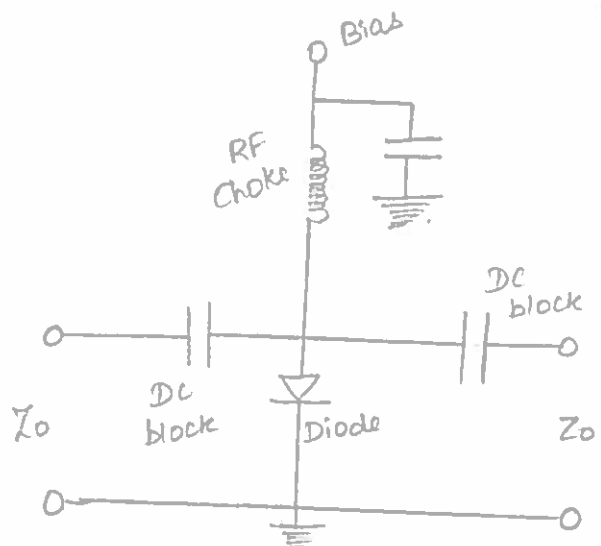
(1) Single - Pole PIN diode Switches:-



\* A PIN diode can be used in either a series or a shunt configuration to form a single-pole, single-throw RF Switch as shown below.

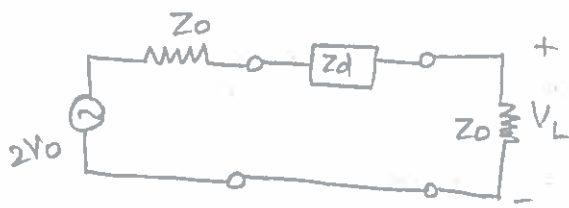


(a) Series Configuration

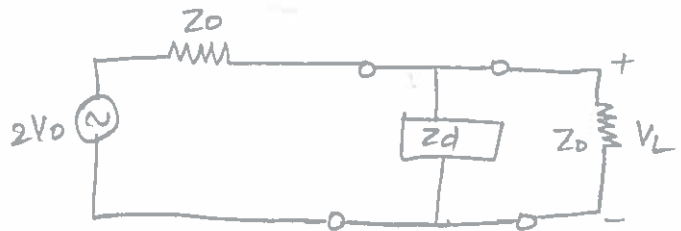


(b) Shunt Configuration

\* In the series configuration of (a) the switch is in 'ON' when the diode is forward biased, while in the shunt configuration of (b) the switch is in 'ON' when the diode is reverse biased.



(a) Series Switch



(b) Shunt Switch

\* In both the above cases, input power is reflected when the switch is in the OFF state. The DC blocking capacitors should have a relatively low impedance at the RF operating frequency, while the RF choke inductors should have a relatively high RF impedance.

\* In some designs, high-impedance quarter-wavelength lines can be used in the place of the chokes to provide RF blocking.

\* The insertion loss in terms of the actual load voltage,  $V_L$  and  $V_0$  which is the load voltage that would appear if the switch ( $Z_d$ ) were absent:

$$I_L = -20 \log \left| \frac{V_L}{V_0} \right| \rightarrow \textcircled{1}$$

\* Simple circuit analysis applied to the two cases from the above figure gives the following results

$$I_L = -20 \log \left| \frac{2Z_0}{2Z_0 + Z_d} \right| \text{ (series switch)} \rightarrow \textcircled{2a}$$

$$I_L = -20 \log \left| \frac{2Z_d}{2Z_d + Z_0} \right| \text{ (shunt switch)} \rightarrow \textcircled{2b}$$

\* In both cases,  $Z_d$  is the diode impedance for either the reverse or forward bias state. Thus,

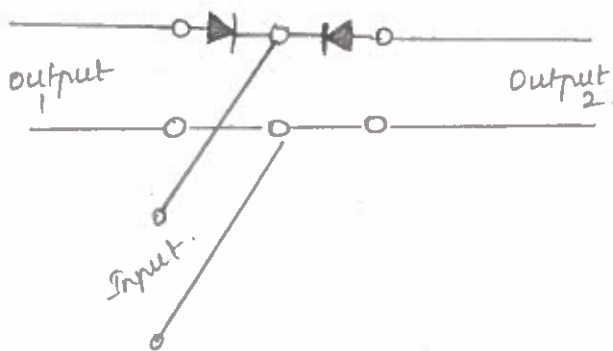
$$Z_d = \begin{cases} Z_r = R_r + j(\omega L_i - \frac{1}{\omega C_j}) & \text{for reverse bias} \\ Z_f = R_f + j\omega L_i & \text{for forward bias} \end{cases} \rightarrow \textcircled{3}$$

\* The ON-state or OFF state insertion loss of a switch can usually be improved by adding an external reactance in series or in parallel with the diode, to compensate for the diode reactance. This technique is usually used reduces the bandwidth.

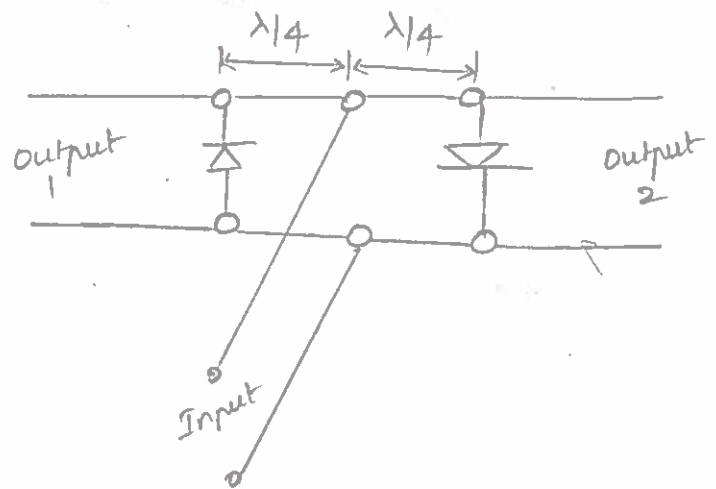
### (2) Double Switches:

\* Several single-throw switches can be combined to form a variety of multiple pole and/or multiple-throw configurations.

\* The PIN diode double switch circuit uses two diodes called as Single-pole Double-Throw (SPDT). The below figure shows a series and shunt circuits for a single-pole, double-throw switch; such a switch requires at least two switching elements.



(a) Series



(b) Shunt.

\* In an operation, one diode is forward biased in the low-impedance state, with the other diode of reverse biased in the high-impedance state. The input signal is switched from one output to the other by reversing the diode bias states.

\* The Quarter-wave lines of the shunt circuit limits the bandwidth of this configuration.

### (3) PIN Diode Phase Shifters:

Several types of microwave phase shifters can be constructed with the PIN diode switching elements.

#### Advantages:

Compared with ferrite phase shifters, diode phase shifters have the advantages of small size, integrability with planar circuitary and high speed.

#### Drawbacks:

\* The power requirements for diode phase shifters are generally greater than those for a latching ferrite phase shifter because diodes require continuous bias current, while a latching ferrite device requires only a pulsed current to change its magnetic state.

\* There are basically three types of PIN diode phase shifters:

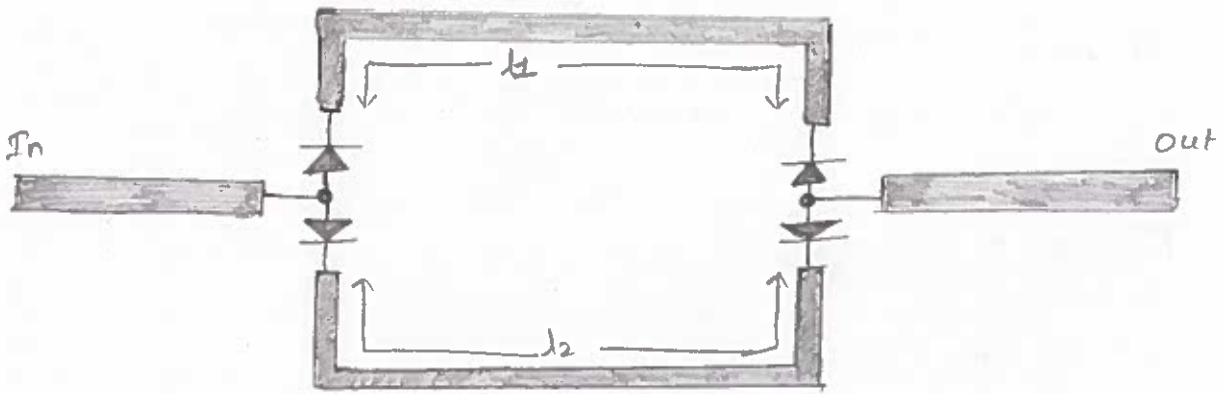
(i) Switched line

(ii) Loaded line

(iii) Reflection.



(i) Switched-line Phase shifter:



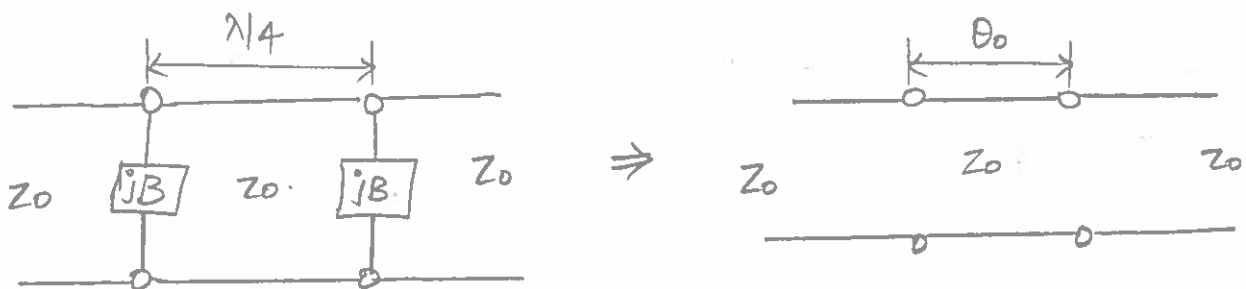
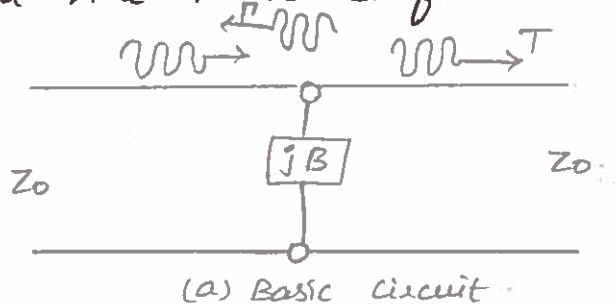
\* The switched-line phase shifter is the most straightforward type using two single-pole, double-throw switches to route the signal flow between one of two transmission lines of the different lengths.

\* The different phase shift between the two paths is expressed as,

$$\Delta\phi = \beta(l_2 - l_1) \longrightarrow \textcircled{4}$$

where  $\beta$  is the propagation constant of the line.

(ii) Loaded-Line Phase Shifter:



(b) Practical loaded-line phase shifter and its equivalent circuit

\* A design that is useful for small amounts of phase shift (generally  $45^\circ$  or less) is the loaded-line phase shifter which is illustrated above figure.

\* The reflection and transmission coefficients can be written as

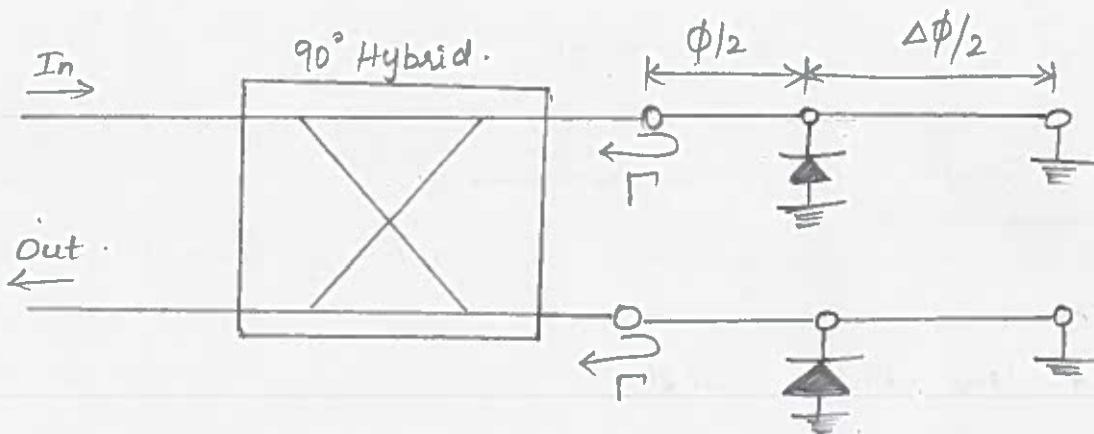
$$\Gamma = \frac{1 - (1 + jb)}{1 + (1 + jb)} = \frac{-jb}{2 + jb} \rightarrow \textcircled{5a}$$

$$T = 1 + \Gamma = \frac{2}{2 + jb} \rightarrow \textcircled{5b}$$

The phase shift in the transmitted wave introduced by the load is

$$\Delta\phi = \tan^{-1}\left(\frac{b}{2}\right) \rightarrow \textcircled{6}$$

(iii) Reflection Phase shifter:



\* The reflection phase shifter which uses an SPST switch to control the path length of a reflected signal.

\* Ideally the diodes would look like short circuits in their ON state and open circuits in their OFF state, so that the reflection coefficients at the right side of hybrid written as

$$\Gamma = \begin{cases} e^{-j(\phi + \pi)} & \text{— Diodes 'ON' state} \\ e^{-j(\phi + \Delta\phi)} & \text{— Diodes 'OFF' state} \end{cases} \rightarrow \textcircled{7}$$

# MICROWAVE TUBES

Microwave tubes are connected to overcome the limitations of conventional electronic vacuum tubes such as triodes, tetrodes and pentodes.

These conventional electronic vacuum tubes fails to operate above the 1GHz.

In microwave tubes the electron transit time is utilized for microwave oscillation or amplification.

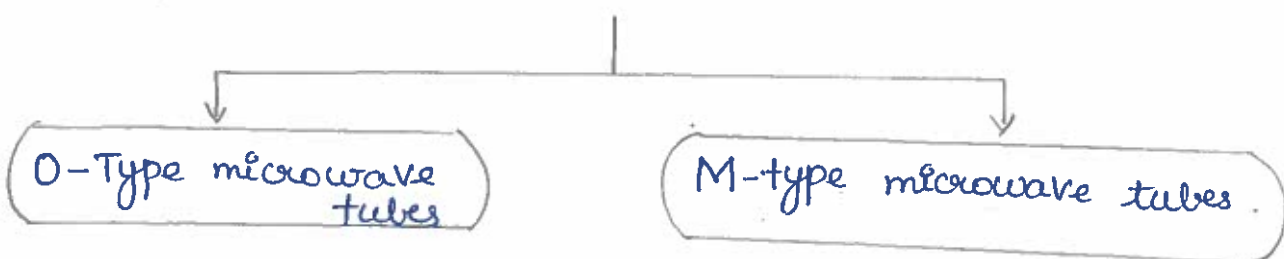
Transit time:-

The transit time is the time taken for an electron to travel from Cathode to anode.

The principle used in the microwave tubes are an electron beam on which Space-charge waves interact with an electromagnetic fields in the microwave cavities to transfer energy to an output circuit of the cavity (klystrons and Magnetrons) (or)

interact with an electromagnetic fields in a Slow-wave structure to give an amplification through the transfer of energy (traveling wave tubes)

Microwave tubes.



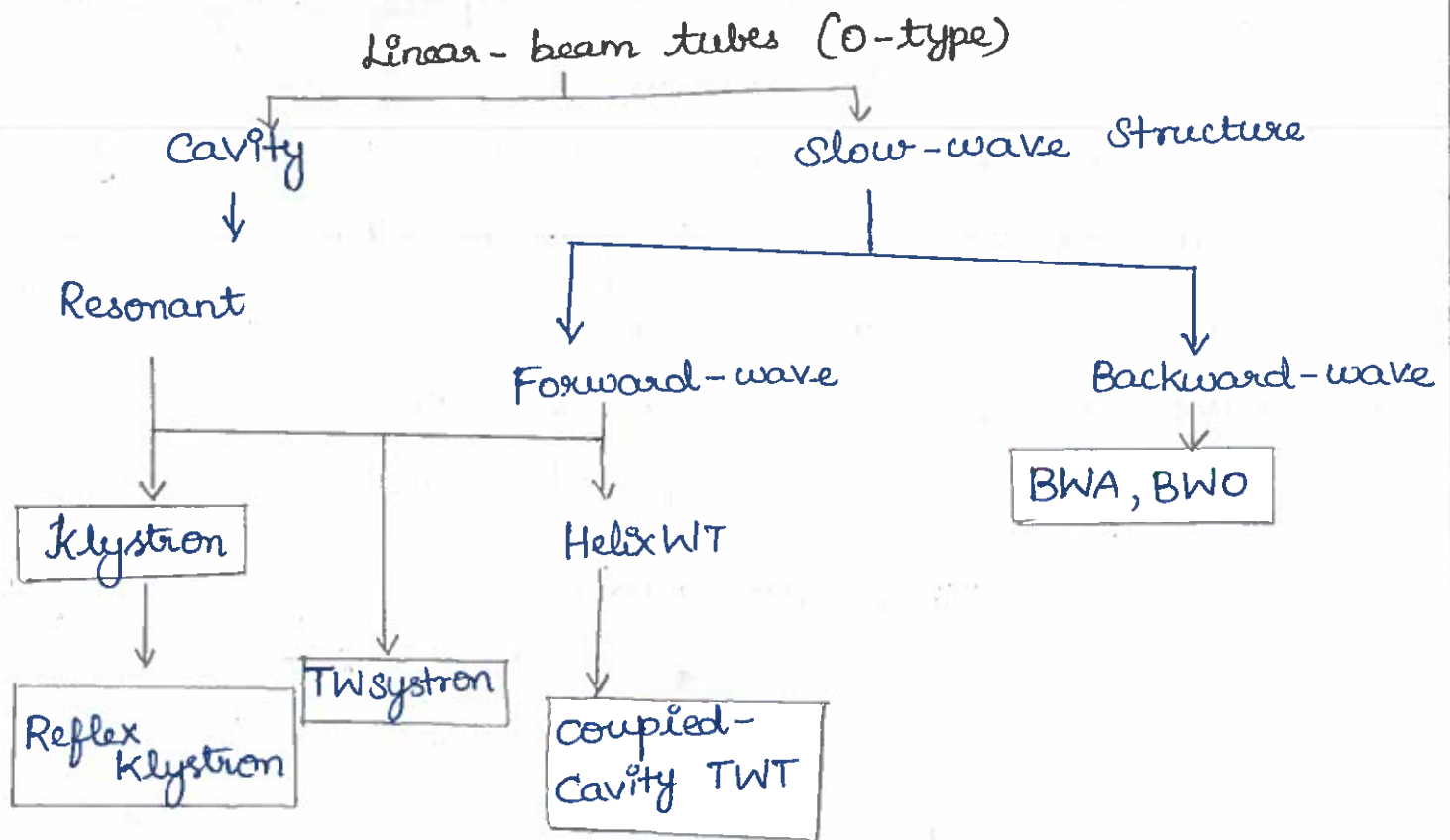
## M-type Microwave tube:-

Magnetrons are crossed field devices (M-type) where the static magnetic field is perpendicular to an electric field. In this tube, the electrons can travel in a curved path.

## O-Type microwave Tube:-

The most important microwave tubes are linear beam or 'O'-type tubes in recognition of the straight path taken by an electron beam.

Klystrons and TWTs are linear beam tubes in which an accelerating electric field in the same direction as the static magnetic field used to focus an electron beam.



# KLYSTRONS

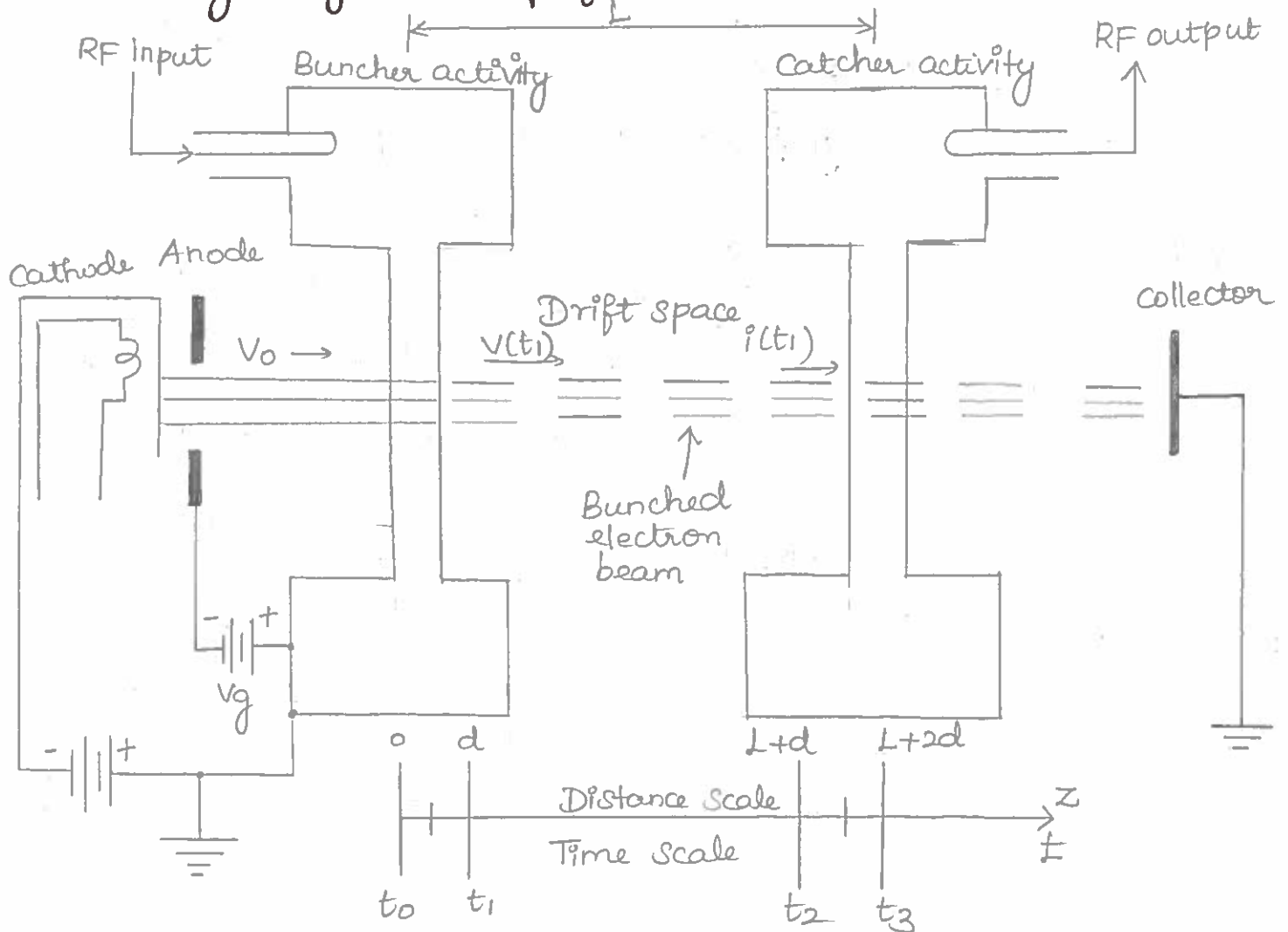
## Definition:

A klystron is a vacuum tube that can be used either as a generator or as an amplifier of power at the microwave frequencies operated by the principles of Velocity and Current modulation.

There are 2 basic configuration of klystron tubes,

- (i) Reflex klystron - It is used as power microwave oscillator and
- (ii) Two cavity (or) Multicavity klystron - It is used as low power microwave amplifier.

## Two cavity klystron amplifier:-



## Introduction:

A two-cavity klystron amplifier is a velocity modulated tube in which the velocity modulation process produces a density modulated stream of electrons. It consists of two cavities namely, buncher (input) cavity and catcher (output) cavity.

## Drift space:

The separation between the buncher and catcher grids is called as drift space.

## Operation:-

→ Cathode emits an electrons beam. This electron beam first reaches the anode. The accelerating anode produces a high velocity electrons beam.

→ The input RF signal to be amplified excites the bunch activity with a coupling loop.

## Bunching:-

The electrons beam passing the buncher activity gap at zeros of the gap voltage  $V_g$  (voltage between buncher grids) passes through an unchanged velocity.

The electrons beam passing through the positive half cycles of the gap voltage undergoes an increase in velocity, those passing through the negative swings of the gap voltage undergoes a decrease in velocity.



As a result of these actions, the electrons gradually bunch together as they travel down the drift space. This is called bunching.

The first cavity act as the buncher and Velocity - modulates the beam. Thus the electron beam is velocity modulated to form bunches or undergoes density modulation in accordance with the input RF signal cycle.

Velocity modulation:

→ The Variation in electron velocity in the drift space is known as the Velocity modulation.

When this density modulated electron beam passing through the catcher cavity grid, it induces RF current (ac current) and thereby excite the RF field in an output activity at an input signal cycle.

→ The ac current on the beam is such that the level of excitation of the second cavity is much greater than that in the buncher activity and hence the amplification takes place.

→ If desired, a portion of an amplified output can be fed back to the buncher cavity in a regenerative manner to obtain a self-sustained oscillations.

The maximum bunching should occur approximately at a midway between the second activity grids during its retarding phase, thus the kinetic energy is transferred from the electrons to field of the second cavity.

The electrons then emerge from the second cavity with reduced velocity and terminate at the collector.

Catcher activity:

The output cavity catches energy from the bunched electron beam. Therefore, it is also called as catcher activity.

Characteristics and applications:

Characteristics:

(i). Efficiency = 40%.

(ii) Power output:

(a) continuous wave average power = 500kW

(b). pulsed power 30MW at 10GHz.

(iii). Power gain  $\approx 30$ dB

Applications:

Used in Troposphere scatter transmitters

Satellite communication ground stations.

Used in UHF TV transmitters

Radar transmitters.

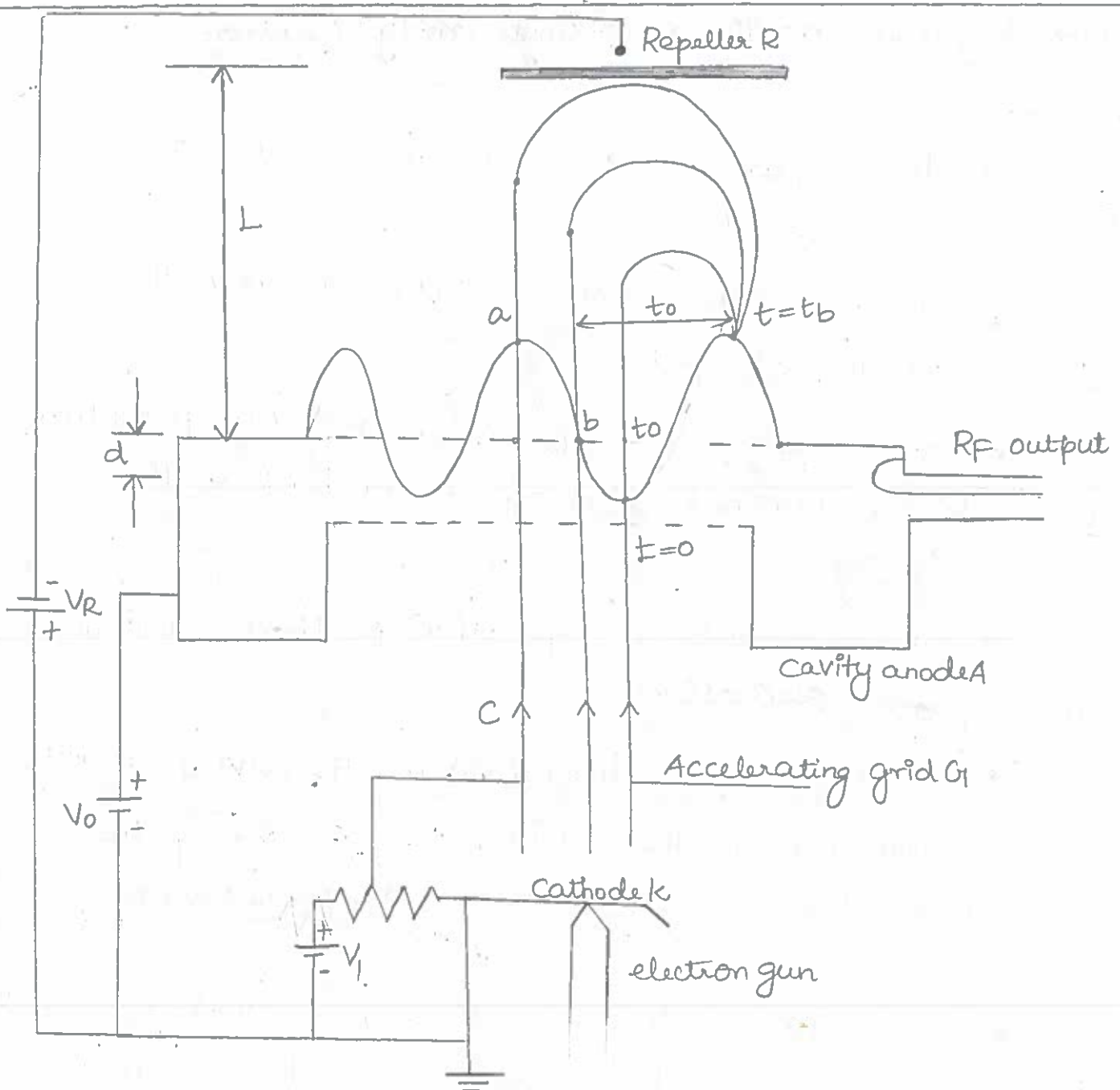
## Reflex klystron oscillator :- single cavity klystron.

### Introduction:

- \* The reflex klystron is an oscillator with a built in feedback mechanism.
- \* It uses the same activity for both the bunching and the output.
- \* The repeller electrode is a negative potential and sends the bunched electron beam back to the resonator cavity.
- \* This provides a positive feedback mechanism which supports oscillations.
- \* Due to dc voltage ( $V_0$ ) in the cavity circuit, RF noise is generated in the cavity. The electromagnetic noise field in the cavity act as a cavity resonant frequency.
- \* when the oscillation frequency is varied, the resonant frequency of activity and the feedback path phase shift must be readjusted for a positive feedback.

### Mechanism of oscillation:

- \* The electron beam injected from the cathode is first velocity - modulated by cavity - gap Voltage.
- \* The electron which encountered the positive half cycle of the RF field, where in the cavity gap velocity will be accelerated.



The electrons which encountered zero RF field will pass with unchanged original velocity, and the electrons which encountered the negative half cycle velocity will be decelerated.

All these velocity modulated electrons will be repelled back to the cavity by the repeller due to its negative potential.

(4)

The repeller distance  $L'$  and the voltages (beam & repeller voltage) can be adjusted to receive all the velocity modulated electrons at a same time on the positive peak of the cavity RF field.

The velocity modulated electrons are bunched together and lose their kinetic energy when they encounter the positive cycle of the RF field. This loss of energy is transferred to the cavity in order to conserve the total power.

If the power delivered by the bunched electrons to the cavity is greater than the power loss in the cavity, an electromagnetic field amplitude at the resonant frequency of the cavity will increase to produce the microwave oscillation.

The electrons passing through the buncher grids are accelerated / passed through which an unchanged initial dc velocity depending upon whether they encounter the RF signal field at the buncher cavity gap at positive / negative / zero crossing phase of the cycle respectively as shown by distance-time plot. This is called the applegate diagram.

**Explanation:**

When the gap voltage is at a positive peak, an electron passing at this moment is called early electron. This electron is accelerated towards

the repeller and travels at a distance, which is longer comparatively.

The electron at a neutral zero of gap voltage is called the reference electron. When the gap voltage is at a negative peak the corresponding electron is called the late electron. This electron decelerated and travels at a less distance.

These electrons have different velocities cover a different distances and forms a bunch at the cavity gap.

Modes of oscillation:

The condition for oscillation of reflex klystron is

$$t_0 = \left(n + \frac{3}{4}\right) T = NT$$

$$\text{where } N = n + \frac{3}{4}$$

Mode of oscillation =  $n = 0, 1, 2, 3, \dots$

$T$  is the time period at the resonant frequency  
 $t_0$  is the time taken by the reference electron to travel in the repeller space.

Characteristics:

Frequency range: 1 to 25 GHz

Power output: It is a low-power generator of 10 to 500mW.

Efficiency: About 20 to 30%



## Applications:

The main applications of reflex klystrons are, this type is widely used in the laboratory for microwave measurements.

In microwave receivers, as local oscillators in commercial and military applications.

Also plays a role in airborne Doppler radars as well as missiles.

## Drawbacks of klystrons:

The klystrons are having the following drawbacks:

(i) Klystrons are essentially narrow band devices as they utilize cavity resonators to velocity modulate the electron beam over a narrow gap.

(ii) In klystrons and magnetrons, the microwave circuit consists of a resonant structure which limits the bandwidth (or the operating frequency range) of the tube.

## Helix - Traveling Wave tube (or) Travelling wave tube amplifier:

### Definition:

A Traveling wave Tube amplifier (TWT) circuit uses a helix slow wave non resonant microwave guiding structure and thus a broad band microwave amplifier.

Two main constituents of TWT are,

(i) An electron beam and

(ii) A structure supporting a slow electromagnetic wave (slow-wave structure).

In the case of TWT, the microwave circuit is non-resonant and the wave propagate with the same speed as the electron in the beam.

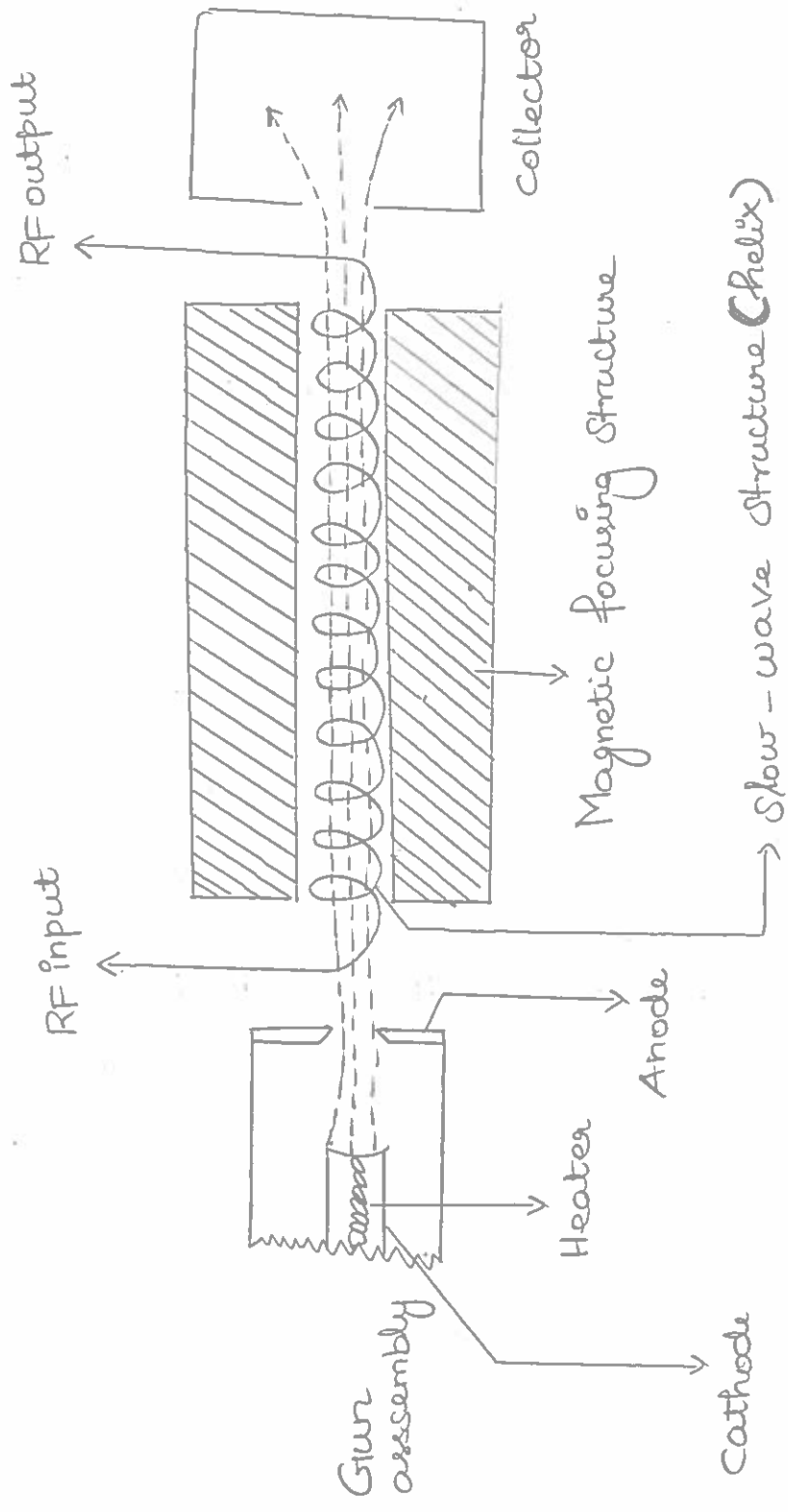
The initial effect of on the beam is a small amount of velocity modulation caused by the weak electric fields associated with the traveling wave.

This velocity modulation later translates to current modulation, which then induces an RF current in circuit, causes an amplification.

Operation:-

The electron beam is focused axially by a static magnetic field and collected in a collector circuit.

The microwave input signal is injected on the helix slow-wave circuit surrounding an electron beam, which produces an axial electric field of the signal at the centre of helix and it can interact with the electron beam.



Simplified TWTA circuit.

The dc beam voltage is adjusted so that the beam velocity is slightly greater than the axial component of a field on the slow-wave structure.

During transit along the axis, an electron beam transfers great amount of energy to the traveling signal wave and thus signal field amplitude increases.

**Attenuator! -**

An attenuator is placed over a part of the helix near an output end to attenuate any reflected waves due to impedance mismatch that can be feedback to an input to cause the oscillations.

**Magnet :-**

The magnet produces an axial magnetic field to prevent spreading of an electron beam as it travels down the tube.

**Need of slow-wave structures (Helix Tube):**

Slow-wave structures are special circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that an electron beam and the signal wave can interact.

## Characteristics of TWTA:-

Frequency range : 3GHz and higher

Bandwidth : about 0.8GHz

Efficiency : 20 to 40%

Power output : upto 10kW average

power gain : upto 60dB.

## Applications of TWTA:-

The main applications of TWTA are,

- Used in medium power satellite
- used in high power satellite transponder output,
- used in radar transmitters, and
- used in broadband microwave amplifier.

## Microwave crossed-field Tubes (M-Type)

### Introduction

A crossed field microwave tube is a device that converts dc into microwave energy using an electronic energy - conversion process.

M-type devices or crossed field tubes in which the dc magnetic field and dc electric field are perpendicular to each other. The principal tube in this type is called Magnetron.



In all crossed-field tubes, the dc magnetic field plays a direct role in the RF interaction process. A magnetron oscillator is used to generate high microwave power.

### Power output and efficiency

A magnetron can deliver a peak power output of up to 40 MW with the dc voltages of 50 kV at 10 GHz. The average power output is 800 kW.

The magnetron possesses a very high efficiency ranging from 40 to 70%. Magnetrons are commercially available for peak power output from 3 kW and higher.

### Applications

Radar transmitter with high output power  
Satellite and missiles of telemetry  
Industrial heating and  
Microwave ovens.

### Cylindrical Magnetron: Magnetron:

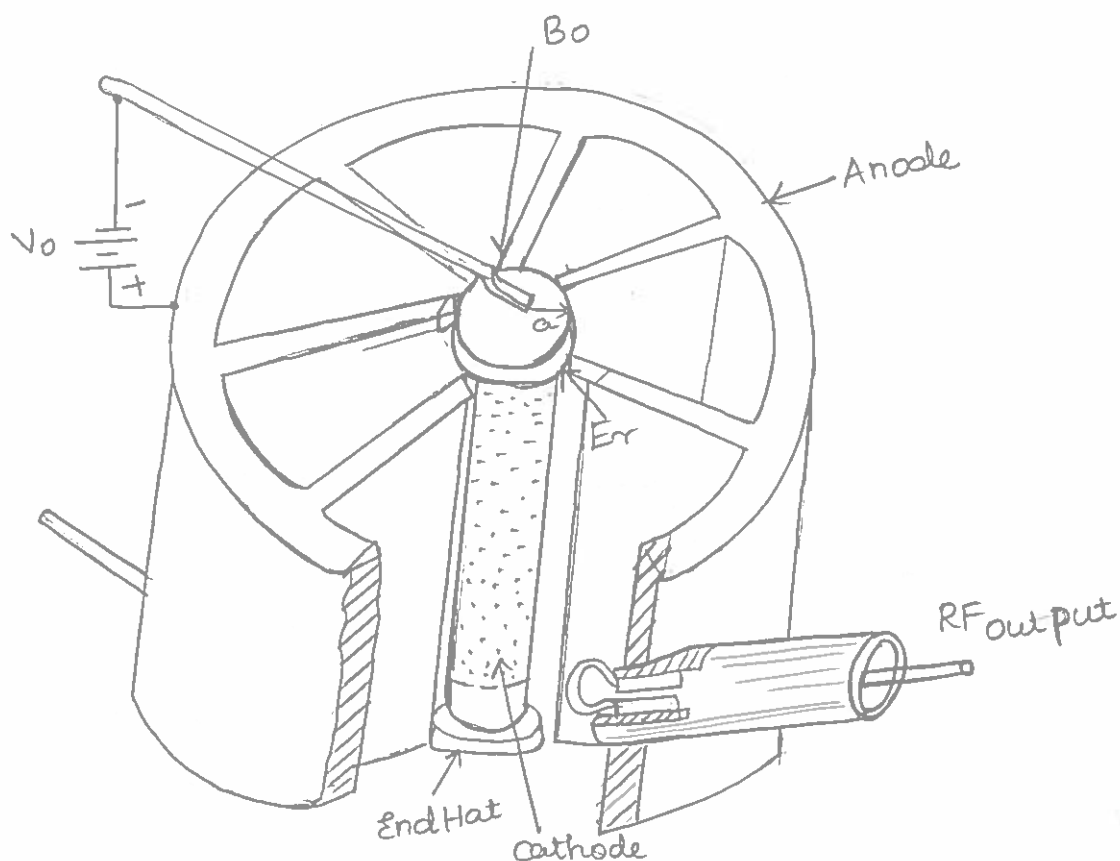
This type of magnetron is also called as a conventional magnetron. It consists of an cylindrical cathode of finite length and radius  $a$  at the centre surrounded by a cylindrical anode of radius  $b$ .



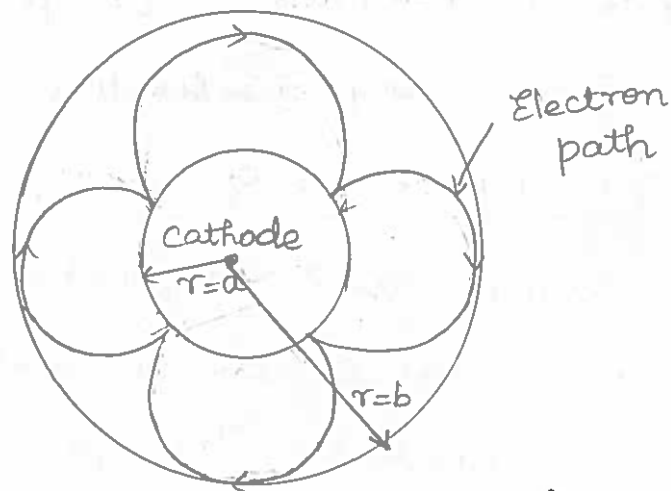
The anode is a slow wave structure consisting of several reentrant cavities equi-spaced around the circumference and coupled together through the anode cathode space by means of slots.

The dc voltage  $V_0$  is applied between the cathode and the anode and a dc magnetic flux density  $B_0$  is maintained in the positive  $z$  direction by means of a permanent magnet or an electromagnet.

The accelerated electrons in the curved trajectory, when retarded by the RF field, the transfer energy from an electron to the cavities to grow RF oscillations till the system RF losses balances the RF oscillations for stability.



Schematic diagram of a cylindrical magnetron



Electron path in a cylindrical magnetron