

UNIT - IV

VARIABLE INDUCTANCE AND VARIABLE CAPACITANCE TRANSDUCERS

Induction potentiometer - Variable reluctance transducers - EI pick up - principle of operation, construction details, characteristics and applications of LVDT - capacitive transducers and types - capacitor microphone - Frequency response.

Variable Inductance Transducers:

Constructed based on one of the following principles.

- change of self inductance
- change of mutual inductance
- production of eddy currents.

Induction transducers used for measurement of displacement and thickness.

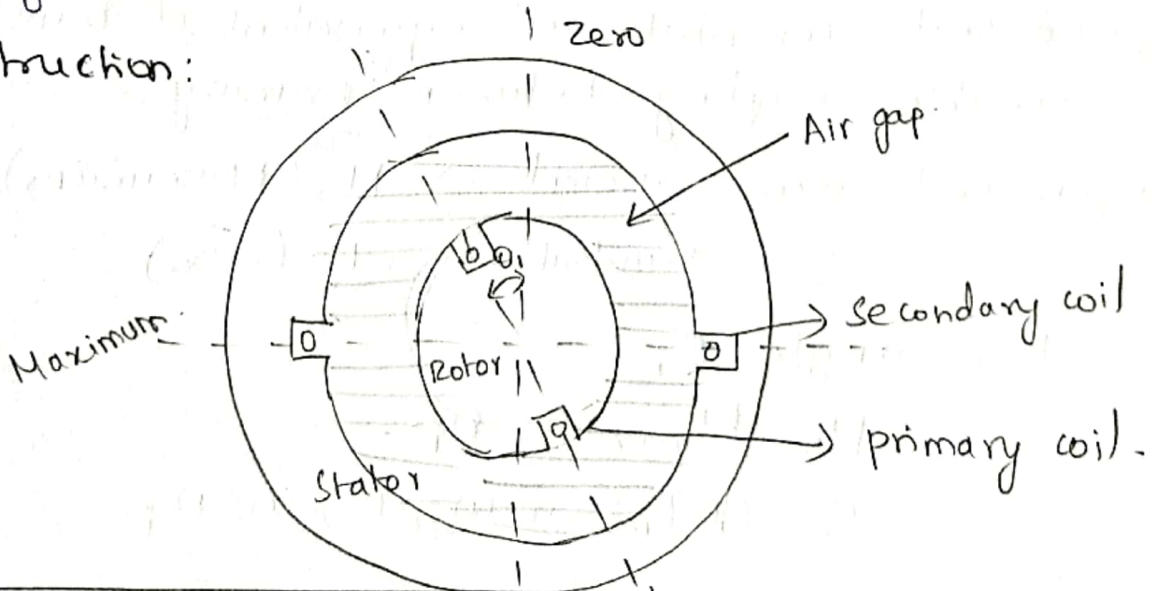
INDUCTIVE POTENTIOMETER:

An induction potentiometer is a linear variable inductor used for the measurement of displacement.

principle:

When there is a displacement, the mutual inductance between two coils changes and voltage is induced in one of the coils.

Construction:



Concentrated or distributed windings are wound on the stator and rotor.

Concentrated coil system output \propto deflection θ ;
Linearity restricted in null position.

Distributed coil system output \rightarrow extended linear range of $\pm 90^\circ$.

Rotor \rightarrow dumbbell shaped

Any other shape which can provide uniform air gap also used.

Rotor winding excited with A.C, which induces voltage in stator depending on position of rotor.

standard commercial

induction pots operates \rightarrow 50 to 400 Hz frequency

Size \rightarrow 1cm to 6cm

Sensitivity \rightarrow 1V/deg rotation

Range of induction pots $< 60^\circ$ of rotation

It is possible to measure displacement (0° to full rotation) by suitable no. of induction pots.

Working:

coils in stator & rotor \rightarrow determine emf induced in one of them
coupled

Two coils constitute an equivalent of transformer with variable coupling between primary & secondary

when coils are coaxial \rightarrow M (Maximum).

quadrature \rightarrow M (Zero).

For angle θ ;

$$M = M_{max} \cos \theta;$$

$$e_o = (k E_m \sin \omega_{ex} t) \cos \theta;$$

③

$k \rightarrow$ constant.

$E_m \sin \omega_e t \rightarrow$ excitation voltage of frequency ω_e .

Hence when displacement to be measured is applied to the rotor, it causes a change in mutual inductance of coils and induces a voltage in stator depending on position of rotor.

Advantages:

Simple in construction.

Smaller in size ranging from 1cm to 6cm.

Good sensitivity of order 1V/deg.

Operate over frequency range 50 to 400 Hz.

Drawbacks:

Output of an induction pot need phase sensitive demodulators and suitable filters making them costly.

Additional dummy coils need to be used to improve linearity and accuracy.

Applications:

Measurement of thickness of rolled sheets or mass produced objects.

Measurement of displacement.

Position Detection

Speed Sensing

Limit Switching

Pulse Generation

Explain in detail about variable Inductance Transducers.

Principle:

- change of self inductance
- change of Mutual inductance
- production of eddy currents.

Transducers working on principle of change of self inductance.

Self inductance of a coil, $L = \frac{N^2}{R}$

$N \rightarrow$ No. of turns

$R \rightarrow$ Reluctance of Magnetic circuit.

$$R = \frac{l}{\mu A}$$

Inductance, $L = N^2 \mu (A/l)$

$$L = N^2 \mu G$$

$\mu \rightarrow$ effective permeability of medium, H/m

$G = A/l$ (geometric form factor)

$A \rightarrow$ Area of cross section of coil, m^2

$l \rightarrow$ length of coil, m

Variation in Inductance caused by,

- i) Change in number of turns, N
- ii) Change in geometric configurations, G
- iii) Change in permeability, μ .

Inductive transducers mainly used for measurement of displacement.

Displacement to be measured is arranged to cause variation of any three variables & thus alter self-inductance L by ΔL .

CHANGE OF INDUCTANCE CAUSED BY CHANGE OF	SINGLE OUTPUT		DIFFERENTIAL OR RATIO OUTPUT	
	N	M or reluctance	N	M or reluctance
AIR				
IRON				
SELF INDUCTANCE				
MUTUAL INDUCTANCE				
PRODUCTION OF EDDY CURRENTS				

5

Differential output of Inductive Transducers :

Change in self inductance \rightarrow Adequate for detection for subsequent stages of instrumentation system.
(ΔL)

If succeeding instrumentation responds to ΔL , rather than to $L + \Delta L \rightarrow$ sensitivity & accuracy higher.

Transducer designed to provide two outputs. one • Increase of self inductance & other decrease in self inductance.

succeeding stages of instrumentation \rightarrow difference between outputs
system measure \swarrow (ie) $2\Delta L$.
known as differential output.

sensitivity & accuracy are increased. output is less affected by external Magnetic fields. Effective variations due to temperature changes, changes in supply voltage & frequency are reduced.

Differential arrangement consists of a coil which is divided into two parts.

In response to displacement, the inductance one part increases from L to $L + \Delta L$ while other part decreases from L to $L - \Delta L$.

change is measured as difference of two resulting in an output. $2\Delta L$ instead ΔL when only a single winding is used.

$$\begin{aligned} &L + \Delta L - (L - \Delta L) \\ &L + \Delta L - L + \Delta L \\ &= 2\Delta L \end{aligned}$$

TRANSDUCERS WORKING ON PRINCIPLE OF CHANGE OF MUTUAL INDUCTANCE

Uses multiple coils.

Mutual inductance between two coils.

$$M = k\sqrt{L_1 L_2}$$

L_1 & $L_2 \rightarrow$ self inductance of two coils

Mutual inductance b/w two coils can be varied by variation of self inductances or coefficient of coupling.

Mutual inductance can be converted self inductance by connecting coils in series.

Varies b/w $L_1 + L_2 - 2M$ to $L_1 + L_2 + 2M$ with

one coils stationary
other is movable.

Self inductance of each coil is constant.
but mutual inductance changes & displacement of movable coil.

In differential arrangement, fixed coil is divided into two parts.

Movement of movable coil increases mutual inductance of one part by ΔM .

Decreases that of other by ΔM .

TRANSDUCERS WORKING ON PRINCIPLE OF PRODUCTION OF EDDY CURRENTS

If conducting plate is placed near a coil carrying alternating current, eddy currents are produced in conducting plate.

conducting plate acts \rightarrow short circuited secondary winding of a transformer.

Eddy current produce Magnetic field against Magnetic field produced by the coil.

Results in reduction of flux & inductance of coil.

Nearest is the plate to the coil

↓
higher eddy currents

↓
higher is reduction in inductance of coil.

↓
Inductance of coil alters with variation of distance between the plate and the coil.

Plate may be at right angle to axis of coil.

Displacement of plate causes a change in inductance of coil.

In other arrangement a conducting sleeve runs in parallel & coaxially over a coil.

If short circuited sleeve is away from the coil, the inductance of coil is high.

If sleeve is covering the coil, its inductance is low.

change in inductance is a measure of displacement.

Types of Inductive transducers:

Air cored coils

Iron cored coils

Air cored coils.

operated at higher carrier frequency.

Absence of eddy current losses.

Inductance independent of current carried by coil

as permeability of air is constant.

Used for measurement of displacement variations occurring at fairly high frequencies.

Iron cored coils

Disadvantages:

Inductance is not constant but depends upon value of current carried by the coil.

At high frequencies, eddy current loss high.

Iron cored coil transducers cannot be used beyond a particular frequency.

Frequency of supply voltage should not exceed 20kHz.

For accurate measurements the frequency of input displacement should not exceed 2kHz.

Advantages:

Their size is much smaller than that of air cored transducers on account of high permeability of iron cores.

Iron cored are less likely to cause external magnetic fields because their magnetic field is confined to iron core of transducer on account of high permeability.

Less affected by stray magnetic fields on account of high magnetic field produced by them.

Most iron cored transducers are variable reluctance type where length of air gap is varied.

VARIABLE RELUCTANCE TRANSDUCER:

Based on change in the reluctance of a magnetic flux path with change in quantity to be measured.

Applications:

Acceleration measurements

sensing force

displacements

velocities & pressure transducers.

Principle

When displacement to be measured is applied, the air gap changes, causing a change in reluctance which in turn causes a change in inductance.

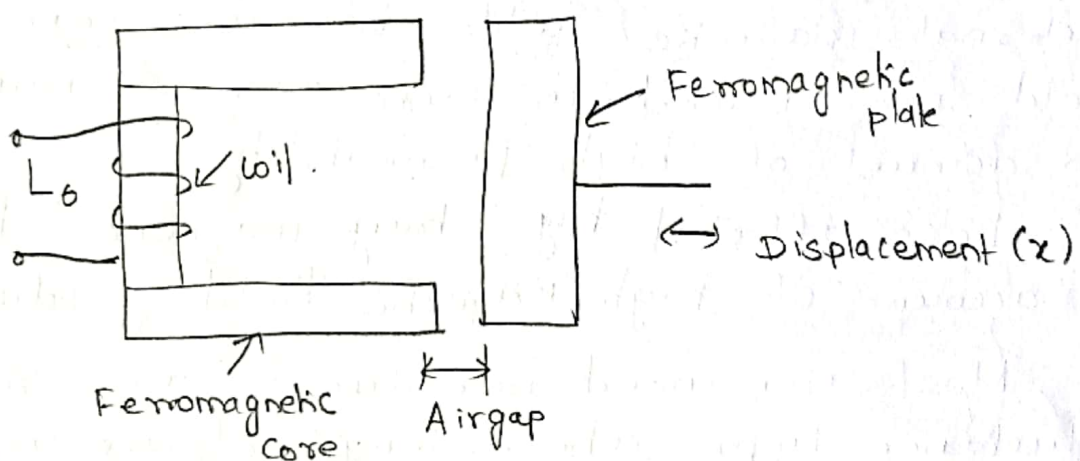
Change in induction = Measure of displacement

Air gap decreases
↓
Reluctance decreases
↓
Inductance increases.

Construction and working:

- Single coil variable reluctance transducer
- Differential variable reluctance transducer (push-pull arrangement).

Single coil variable reluctance transducer.



Three elements.

- ferromagnetic core
- Variable air gap
- Ferromagnetic plate.

coil wound on ferromagnetic core acts as a source of magnetomotive force (mmf)
Used to drive the flux through the magnetic circuit and the air gap

Displacement to be measured is applied to the ferromagnetic plate.

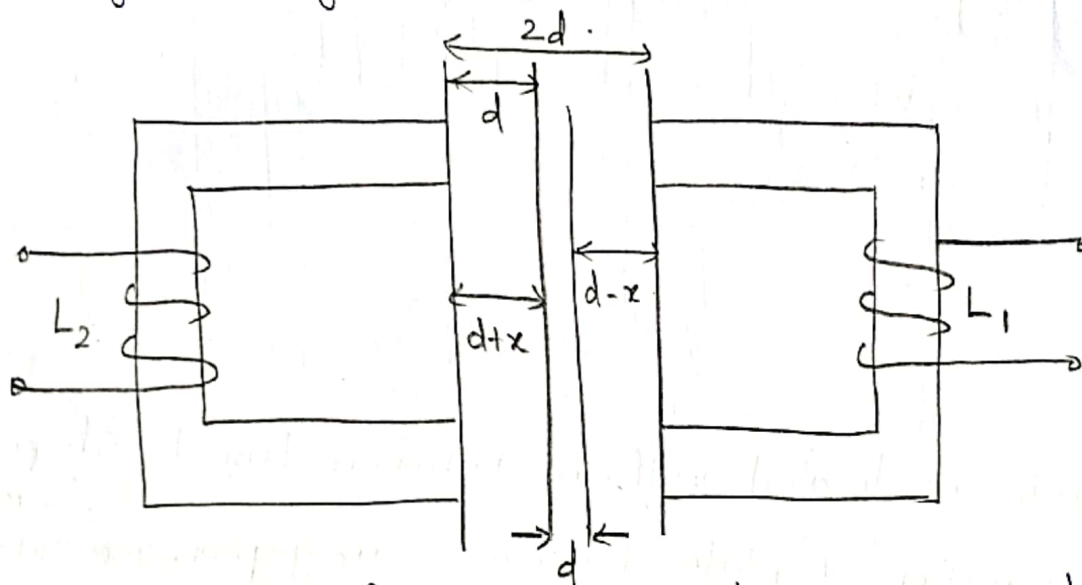
Air gap \downarrow , Reluctance \downarrow & inductance \uparrow .

Hence small variation in the air gap results in a measurable change in inductance.

Single coil system results in a nonlinear relationship.

Differential variable reluctance transducer.

Designed to overcome the problem of nonlinearity occurring in single coil type.



Consists of ferromagnetic plate moving between two identical cores separated by a fixed distance of $2d$.

Motion of plate increases air gap on one side decreases it on another side.

causing reluctance to change, inducing more voltage on one of the coils than on other.

Motion in other direction reverses the action with 180° phase shift occurring at null.

Advantages:

Hysteresis errors are entirely limited.

Both static & dynamic response

continuous resolution & high outputs

Maximum nonlinearity is 0.5%.

Disadvantage:

produce erroneous output in presence of external magnetic fields.

Commercial variable reluctance transducer:

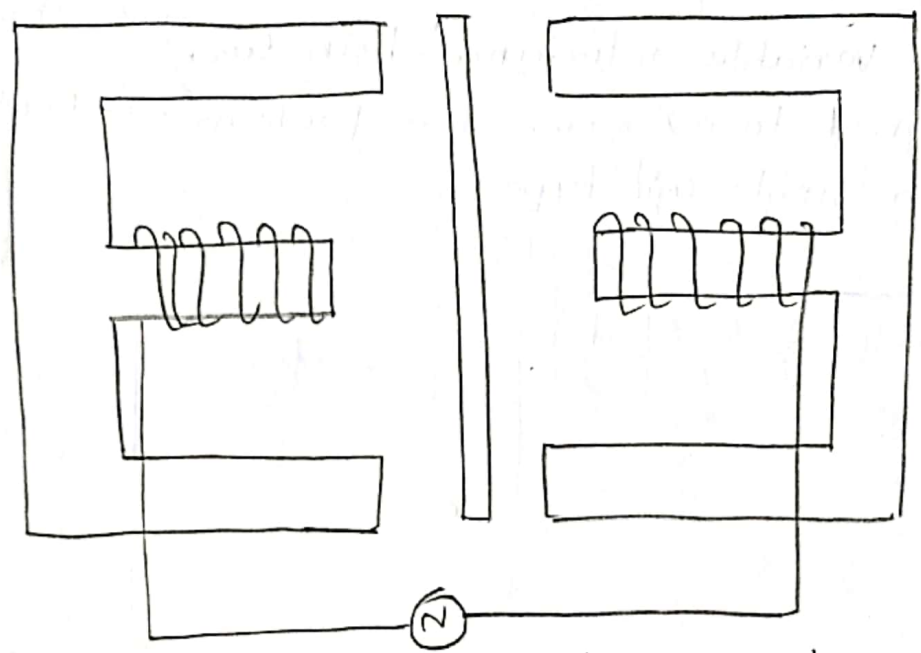


plate is located halfway between two E-shaped frames.

Any motion of plate increases airgap on one side decreases it on other side

Reluctance changes, inducing more voltage on one of the coils than on the other.

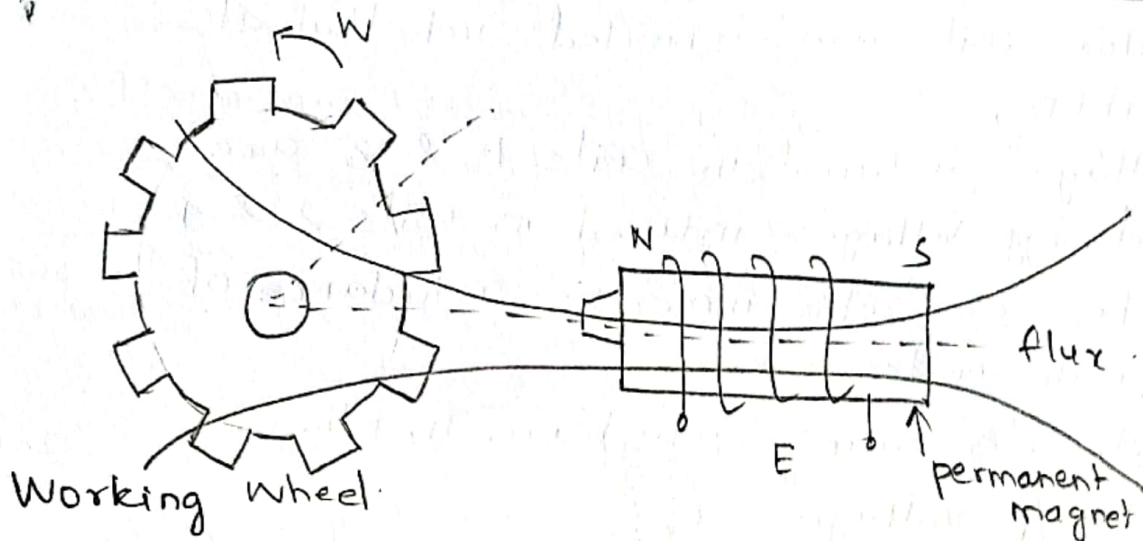
Based on Faraday's law of electromagnetic induction.

Also referred to as electromagnetic sensors.

Induced emf in sensor \propto linear or angular velocity of motion.

Construction:

Consists ferromagnetic toothed wheel attached to rotating shaft. Coil wound onto a permanent magnet extended by a soft iron pole piece.



Wheel moves in close proximity to pole piece causing flux linked by coil to change emf induced in the coil.

Reluctance of circuit \propto width of air gap b/w rotating wheel & pole piece.

when tooth is close to pole piece = reluctance minimum

tooth moves away from pole piece = reluctance increases.

Both amplitude & frequency of generated voltage at the coil \propto angular velocity of wheel.

Advantages:

Suitable for measuring angular velocities.

used in volume flow rate measurements.

total volume flow determination of fluids.

Microsyn:

It is a variable reluctance transducer.

used for angular displacement measurements

Construction

consists of ferromagnetic rotor

stator carrying four coils.

stator coils are connected such that at null position,

voltages induced in coils 1 & 3 are balanced by voltages induced in coils 2 & 4.

Motion of rotor increases reluctance of two opposite coils.

while decreasing reluctance in others.

net o/p voltage = e_o .

Movement in the opposite direction reverses this effect with 180° phase shift.

If direct sensitive ^{dc} output \rightarrow then obtained by using required phase-sensitive demodulators.

Applications:

Microsyn used in gyroscopes.

Advantages:

Sensitivity = 0.2 to 5 V/deg.

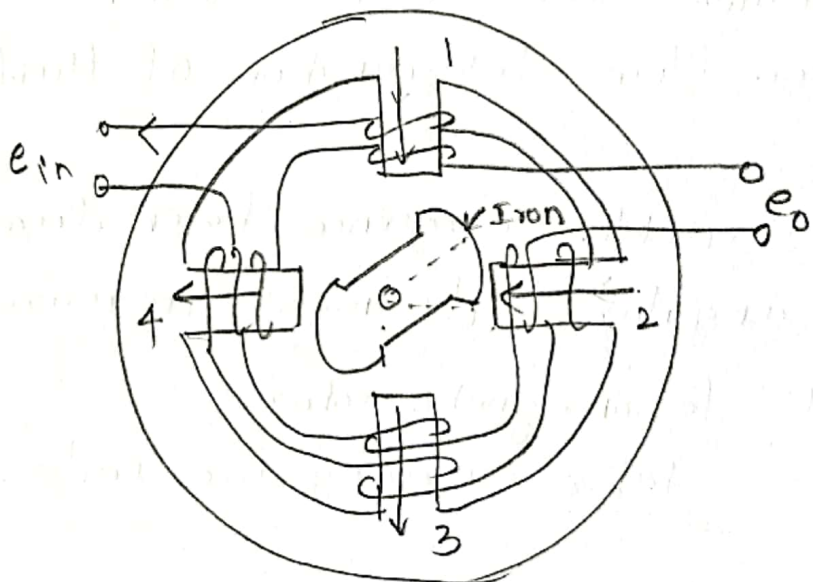
Excitation voltage = 5 to 50 Volts at 50 to 5000 Hz

Non linearity = 0.5% full scale, $\pm 7^\circ$ rotation

= 1.0% full scale, $\pm 10^\circ$ rotation

Does not have windings & slip rings.

Magnetic reaction torque is also negligible.



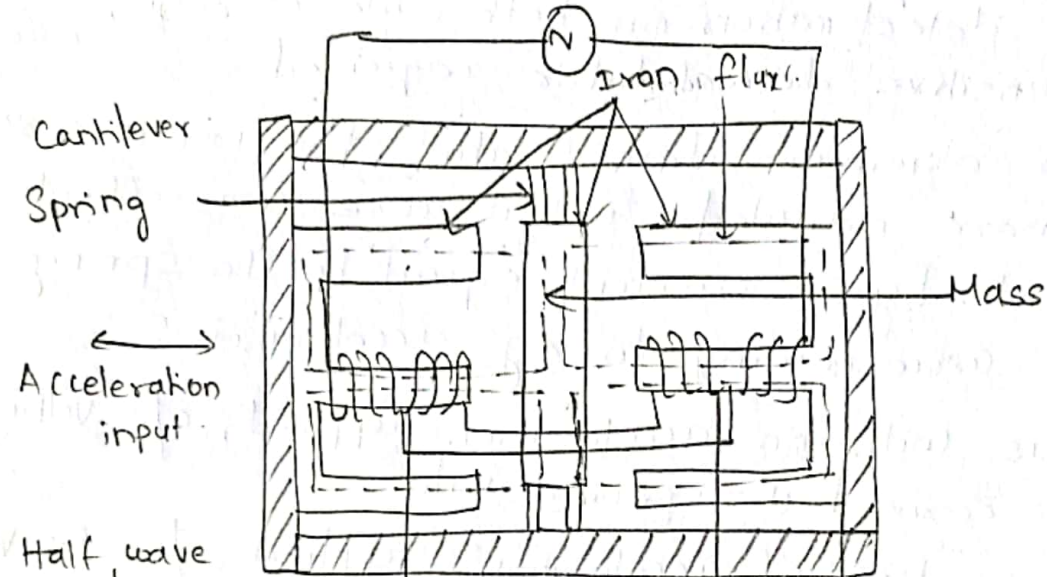
VARIABLE RELUCTANCE ACCELEROMETER

Variable reluctance principle

Measurement of acceleration in range $\pm 4g$.

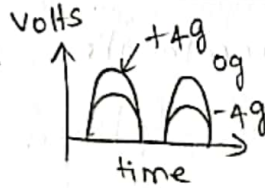
Construction:

Excitation 16 volts, 16000cps

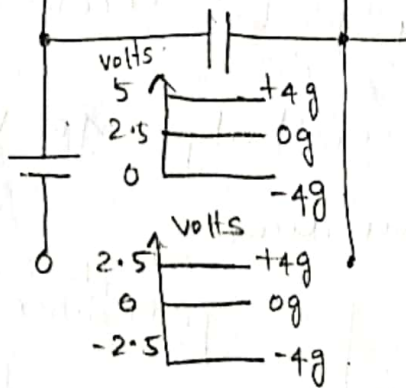


Half wave non phase sensitive demodulator

low pass filter



Bucking battery 2.5V



Force required to accelerate a mass of acceleration

Springs supporting mass of acceleration deflect

Mass is of iron serves as both

Inertial element for transducing acceleration to force

Magnetic circuit element transducing motion to reluctance.

Instrument would be constructed so that iron core would be half way between two E frames when acceleration was zero.

Thus zero output voltage for zero acceleration. To detect motion on both sides of zero, phase-sensitive demodulator required.

To eliminate demodulator, the iron core & springs were adjusted so that core was offset to one side by an amount equal to the spring deflection corresponding to 4g acceleration.

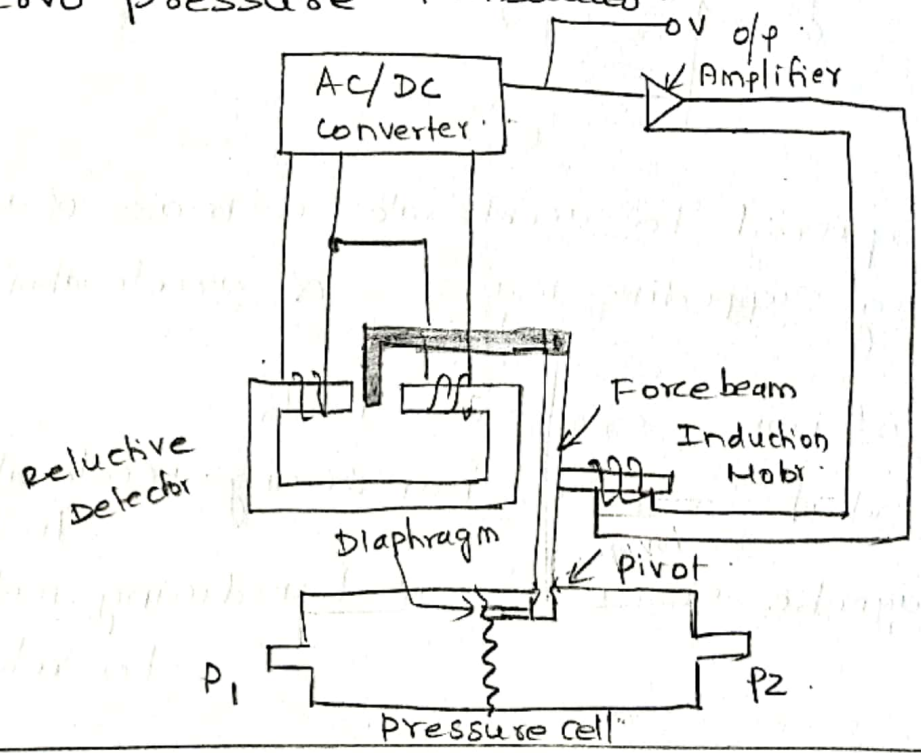
Thus with no acceleration applied, o/p voltage was not zero but specific value.

when +4g of acceleration applied o/p = 5.0 volts
-4g of acceleration applied o/p = 0V

Bucking battery, } → Phase sensitive output
Simple diode }
& } acceleration is obtained.
filtering circuit }

Displacement sensitivity = 400 V/cm.

Servo pressure Transducers.



An increase in pressure P_1 over P_2 flexes diaphragm & moves short end of force beam.

force beam pivots & long end moves a magnetic material in reluctance detector.

Signal from reluctance detector is converted from a.c to d.c power & sent to amplifier.

Amplifier responds by activating inductive motor that moves the force beam back towards its original position.

Very little flexing ever occurs in diaphragm even over entire range of instrument.

As a result, diaphragm lasts a long time.

Servo pressure transducers are available in multitude of pressure ranges.

Used for measurement of pressure below 500 psi.

Do not respond to high frequency pressure oscillations.

Other servo pressure instruments use capacitive detectors & some use Bourdon tube as the sensing element.

EI Pick up

Principle

Eddy currents are produced in a conducting plate placed near a coil carrying alternating current.

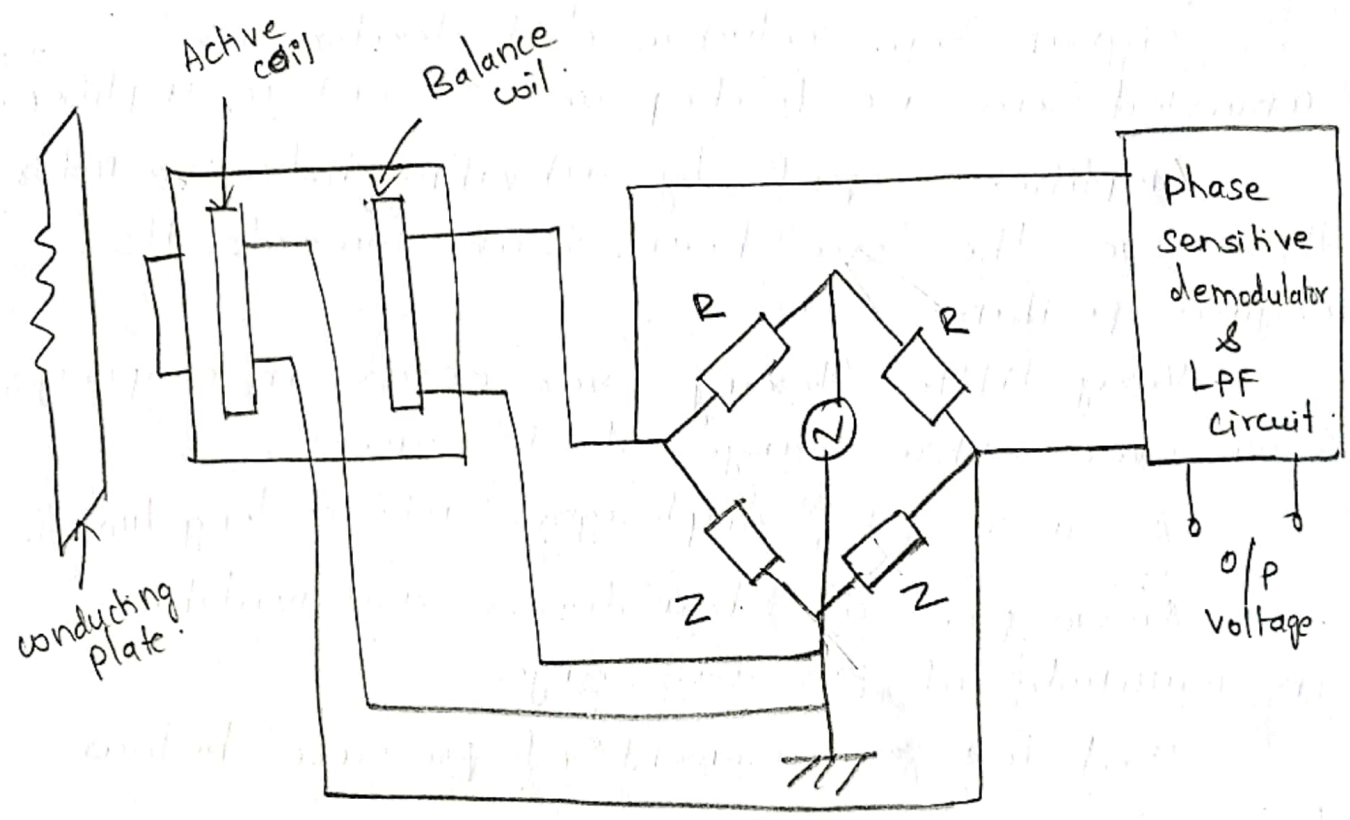
Eddy current produced would produce magnetic field of their own that act against magnetic field produced by the coil.

Results in reduction of flux & inductance in coil.

Nearer is the plate more is the reduction in inductance of the coil.

Distance b/w plate & coil \rightarrow inductance of coil varies

Construction :



probe types containing two coils.
 one coil \rightarrow active coil (influenced by presence of conducting plate).
 second coil \rightarrow balance coil (serves to complete the bridge circuit & provides to compensation)

Magnetic flux from active coil passes into conductive plate by means of a probe plate close to probe, flux from probe links with plate.

\downarrow
 producing eddy currents within the plate.

eddy current density greatest at plate surface negligible. Small below surface (three skin depth).

Skin depth \propto type of material used & excitation frequency.

Thinner plates (minimum 3 skin depth) to minimize the T₀ effects.

plate comes closer to probe \rightarrow eddy currents stronger.

\downarrow
Inductance of active coil \downarrow .

\downarrow
altering the balance of bridge related to plate position

\downarrow
Unbalance voltage demodulated, filtered & linearized to produce d.c o/p \propto plate displacement.

Bridge oscillation = 1 kHz (good frequency response)

probe = 0.25 to 30mm (diameter).

Nonlinearity = 0.5%.

Maximum resolution = 0.0001mm.

LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

passive inductive transducer based on variation of mutual inductance with displacement.

Translate the linear motion into electrical signal.

principle

Depending on position of the core, flux linking with coils & hence voltages induced in them varies.

construction:

Sonists of single primary winding (P).
two secondary windings (S₁ & S₂)

wound on cylindrical former.

secondary windings have equal no. of turns. identically placed on either side of primary winding.

movable soft iron core is placed inside the former.

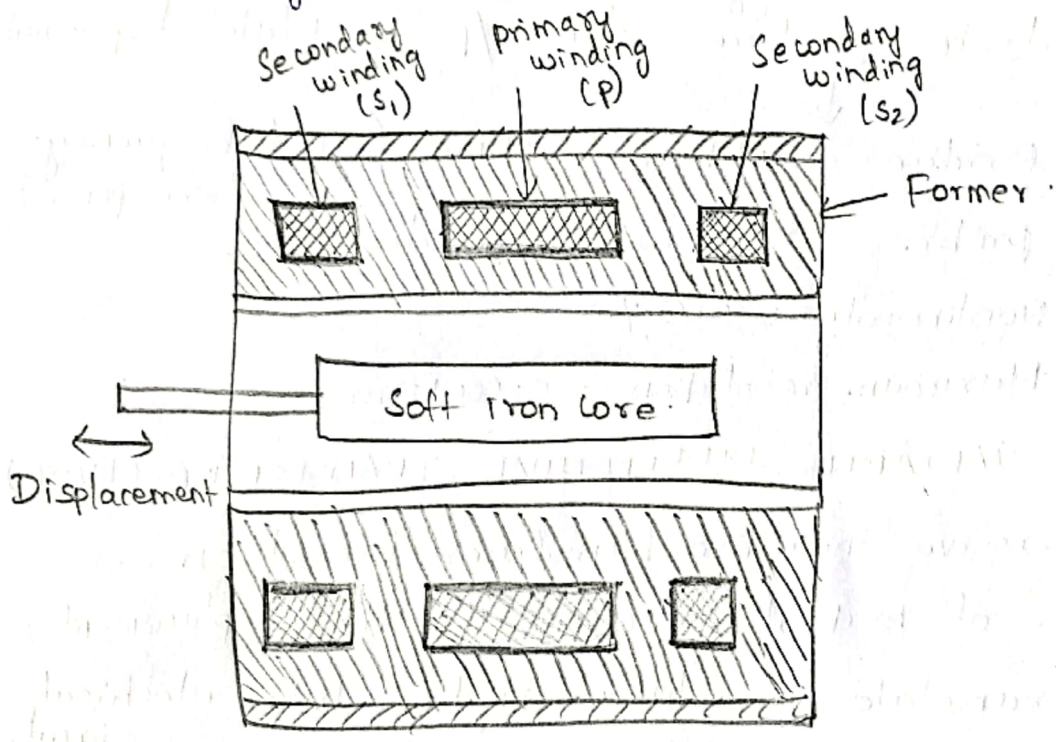
core made of hydrogen, high permeability nickel iron

↓
low harmonics, low null voltage & high sensitivity.

↓
to reduce eddy current losses.

placed in stainless steel & end lids provide shielding from electrostatic & electromagnetic fields.

Primary winding → Ac voltage, 50Hz & 20kHz (frequency)

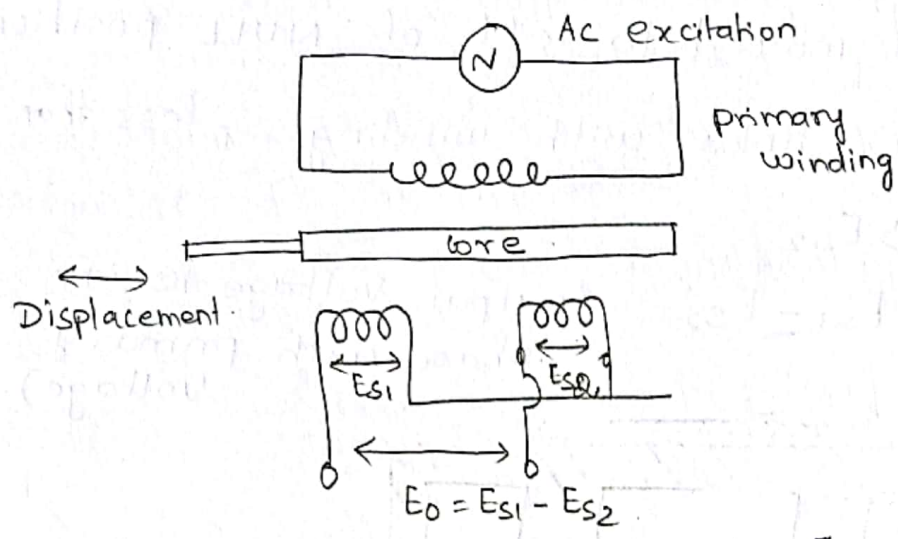


Working

primary winding is excited by Ac source

↓
produces alternating Magnetic field

↓
induces Ac voltages in two secondary windings.



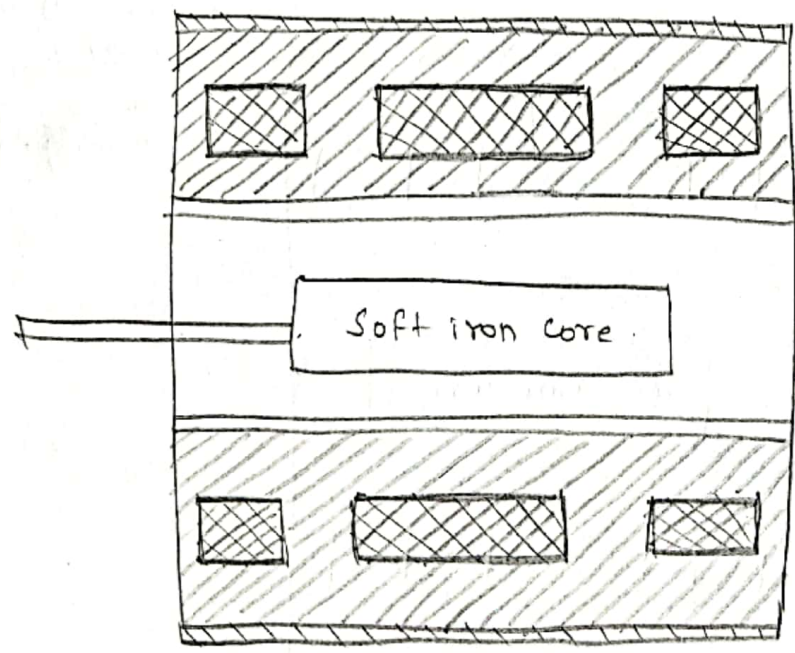
Output voltage of S_1 is E_{s1}
 S_2 is E_{s2}

To convert output from S_1 & S_2 into single voltage signal, S_1 & $S_2 \rightarrow$ connected in series opposition.

output voltage, $E_0 = E_{s1} - E_{s2}$.

Case 1: when core is at its NULL position.

flux linking with both secondary winding is equal and emfs induced in them
 At null position $E_{s1} = E_{s2}$
 $E_0 = 0$.



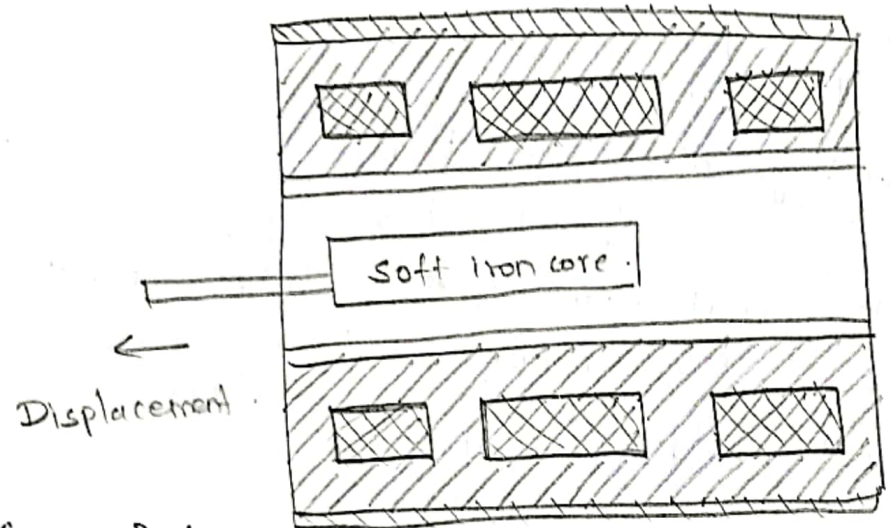
Case 2 :

If core is moved to left of NULL position

More flux links with winding S_1 less than S_2

$$E_{s1} > E_{s2}$$

$$E_0 = E_{s1} - E_{s2} \quad (\text{output voltage is in phase with primary voltage}).$$



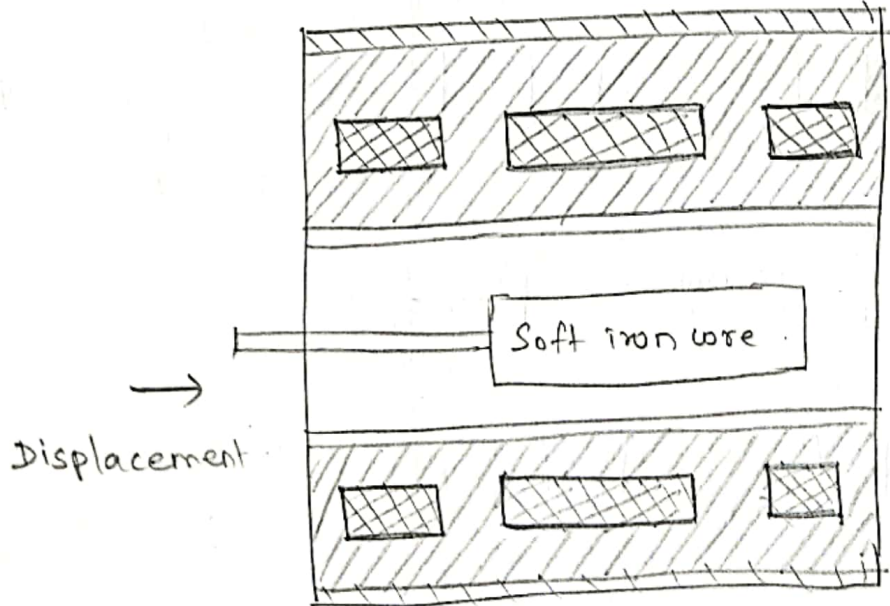
Case 3 :

If core is moved to right of NULL position

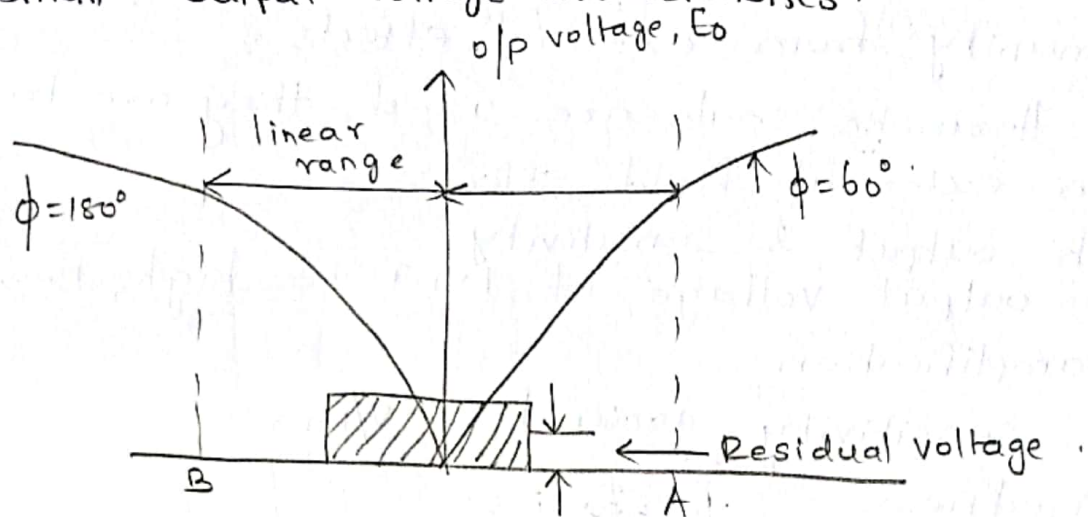
flux links with S_2 is greater than S_1

$$E_{s2} > E_{s1}$$

$$E_0 = E_{s2} - E_{s1} \quad (180^\circ \text{ out of phase with primary voltage}).$$



Displacement - output voltage characteristics.



output signal may also be applied to a recorder or to a controller that can restore moving system to its normal position.

Current is practically linear for a limited range of displacement from null position.

Beyond this range of displacement curve starts to deviate from st. line.

For zero displacement, there is small voltage at output called residual voltage.

Due to presence of harmonics in power supply & also due to use of iron core.

Also be caused by stray Magnetic fields & Temperature effects.

By use of better AC sources, residual voltage can be reduced to almost a negligible value.

Advantages

High Range :

1.25mm to 250mm → displacement measurement.

Friction & electric isolation:

No physical contact between movable core and coil structure.

Immunity from external effects:

Hermetic seals are used, they are free from external field effects.

High output & sensitivity
output voltage of LVDT is high, needs no amplification.

sensitivity around 40V/mm

Ruggedness

Tolerate a high degree of shock & vibrations.

Light weight, easy to align & maintain.

Low hysteresis

Repeatability is excellent under all conditions.

Low power consumption

consume power of less than 1W

Drawbacks:

Large displacements are required for getting the output voltage.

Dynamic response is limited.

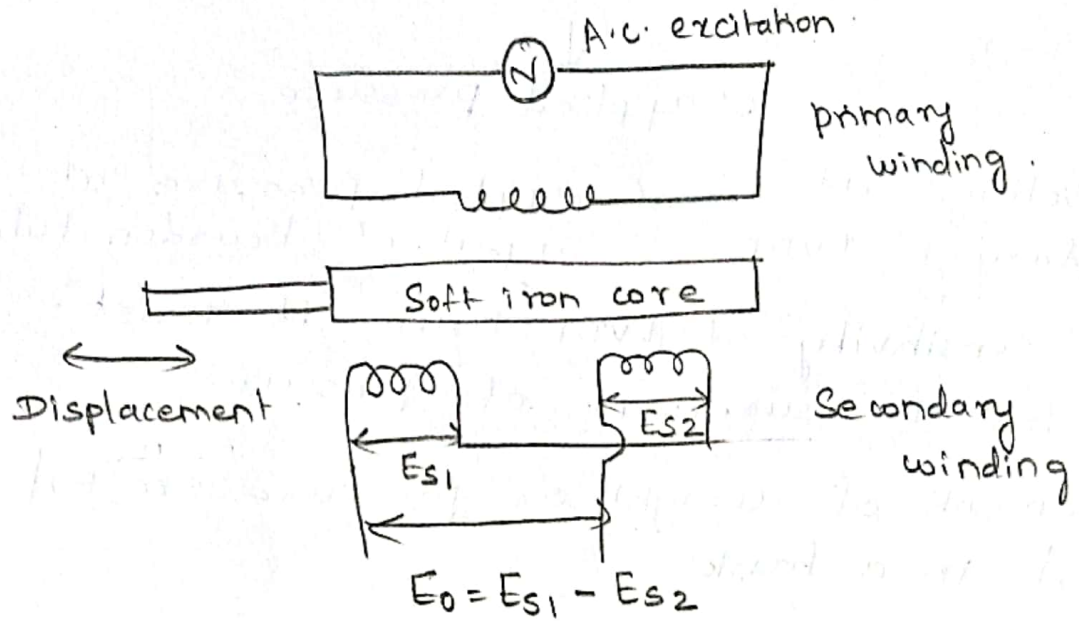
To affects their performance, which can be overcome by using manganin wire instead of copper wire.

To also causes phase shifting effects minimized by using a capacitor.

Also affected by stray fields & vibrations but are easily eliminated by use of hermetic seals & springs respectively.

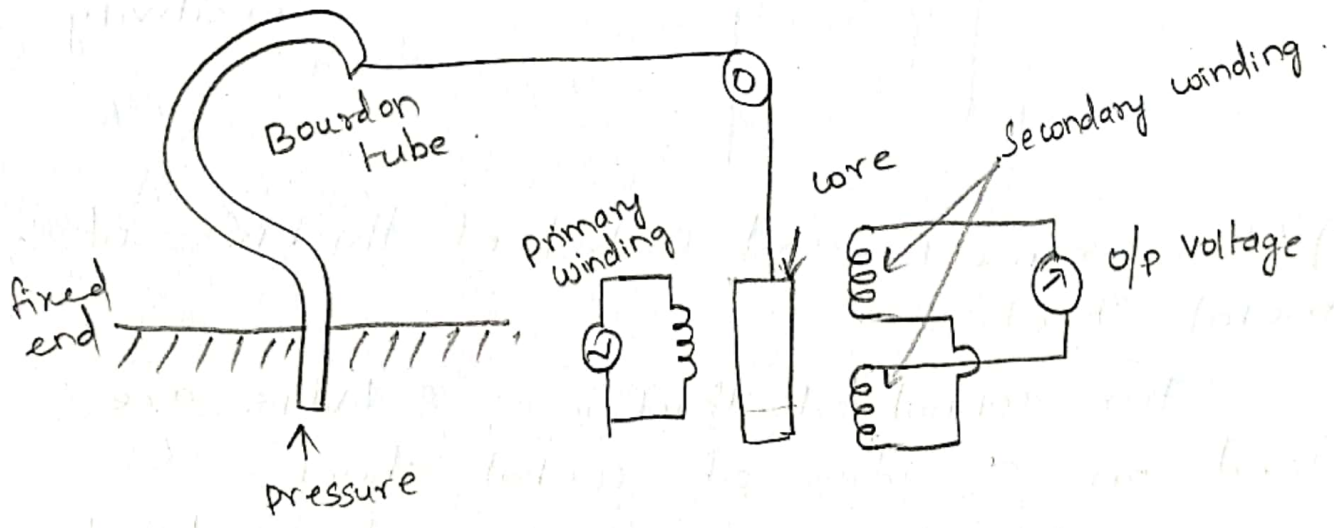
Applications:

1) Measurement of Displacement.



Measuring displacements ranging from a fraction of mm to a few cm.
 Convert input displacement directly into output voltage in one stage.

2) Measurement of pressure:



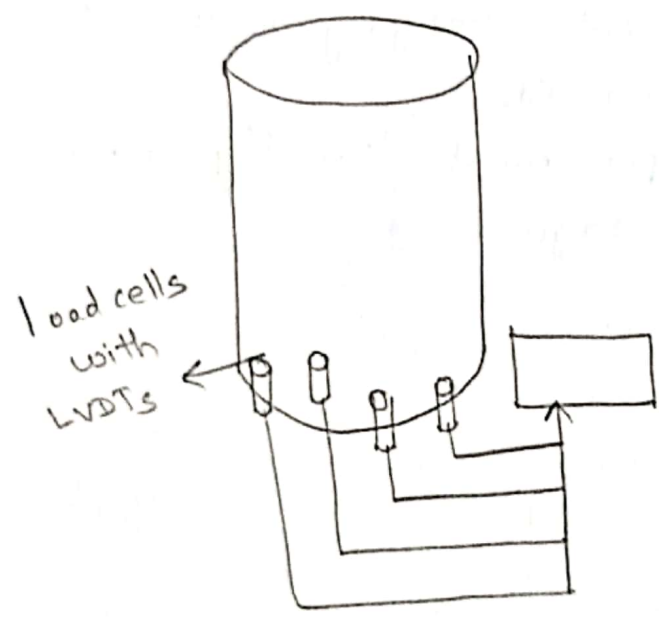
LVDT acts as Secondary transducer.
 Bourdon tube → primary transducer.
 converts pressure into displacement at its free end

moves core of LVDT produces output.
 \propto movement of core
 \downarrow
 \propto applied pressure.

output voltage at secondary of LVDT \propto applied pressure at input of Bourdon tube.

High sensitivity of LVDT makes it most suitable for measurement of pressure.

3) Measurement of weight or pressure exerted by liquid in a tank.



4 LVDTs are employed for measurement of pressure or weight exerted by liquid in tank.

4 LVDTs are excited in parallel to increase sensitivity.

4) Measurement and control of thickness of metal sheet.

For control of thickness, 2 LVDTs are used on 2 sides of metal sheet.

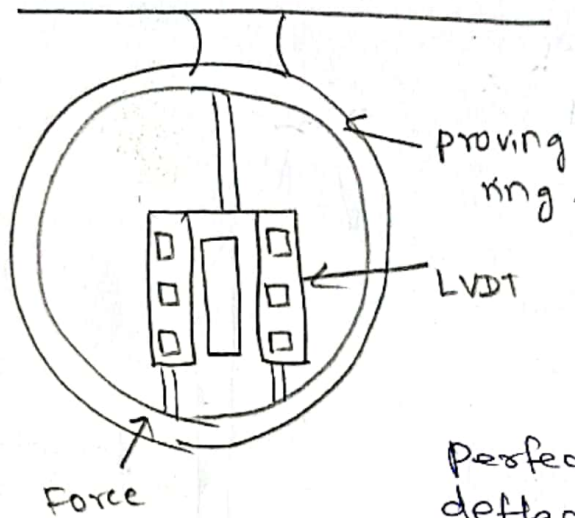
When thickness of sheet = desired value then output of 2 LVDTs are equal & net output = 0.

Whenever sheet thickness deviates from desired value, net o/p = difference of 2 LVDTs voltages.

This voltage can be further used for control of thickness of metal sheet.

used in processes for machining jobs

5) LVDT load cells



Many elastic members can be loaded in either tension or compression leading to deflection in either direction

Bidirectional nature of displacement Characteristics of LVDT perfectly complement bidirectional deflection of elastic member.

CAPACITIVE TRANSDUCER:

Capacitance between two conducting plates is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

C → capacitance between conducting plates

ϵ_0 → absolute permittivity

ϵ_r → relative permittivity

A → area of cross section of plate.

d → distance between plates in m.

Change in capacitance with change in physical quantity under measurement.

- 1) change in area
 / linear
 \ angular
- 2) change in distance between plates
- 3) change in dielectric constant.

Capacitive linear displacement transducer based on change in area of plates.

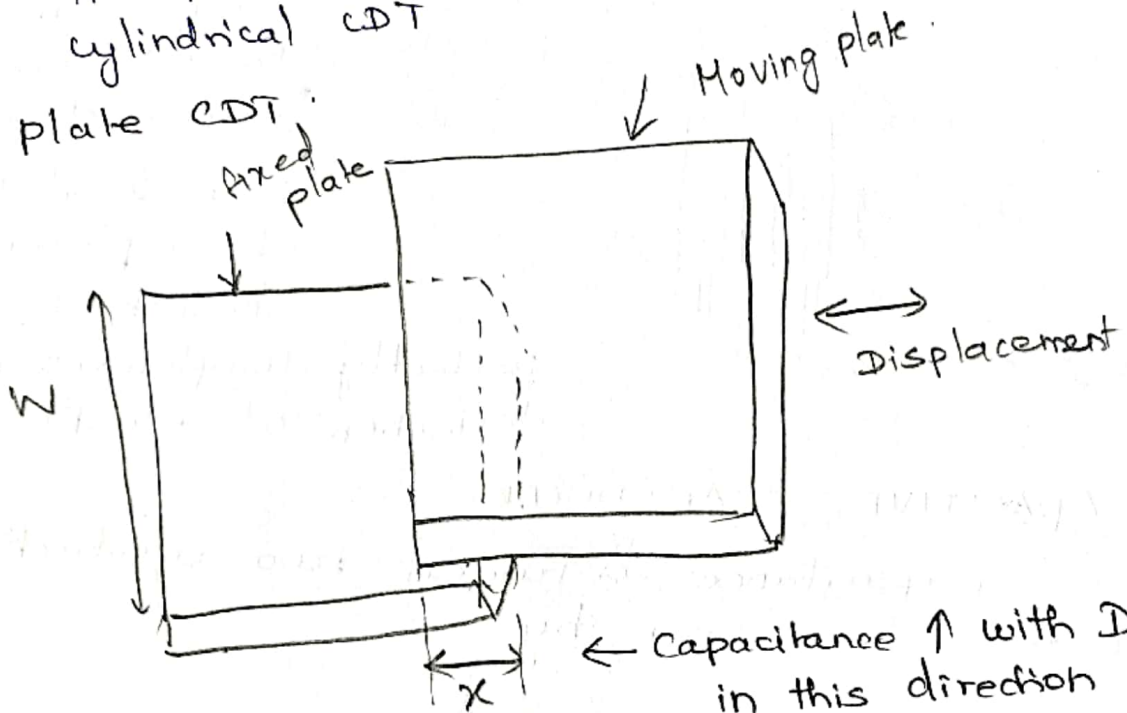
$$C \propto A$$

suitable for measurement of displacements in range of 1mm to several cm.

2 configurations:

- || plate CDT
- cylindrical CDT

1) || plate CDT:



← Capacitance ↑ with D in this direction
 → capacitance ↓ with D in this direction

$$C = \frac{\epsilon A}{d}$$

$$C = \frac{\epsilon x w}{d}$$

$x \rightarrow$ length of overlapping part.
 $w \rightarrow$ width of overlapping part.

1 plate \rightarrow fixed
 another plate \rightarrow movable

D applied to movable
 \downarrow
 Results in overlapping
 \downarrow
 change in capacitance

$$\text{Sensitivity, } S = \frac{\partial c}{\partial x}$$

$$= \frac{\partial}{\partial x} \left(\frac{\epsilon x w}{d} \right)$$

$$S = \frac{\epsilon w}{d} \text{ Farad/m}$$

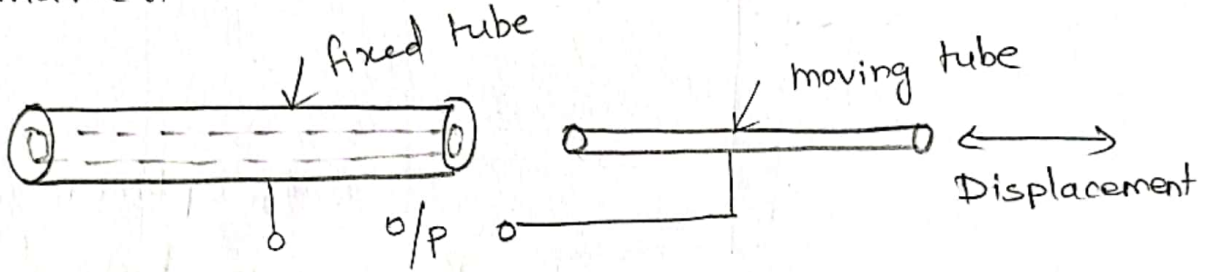
Sensitivity constant.

Relation between C & D is linear.

initial nonlinearity due to edge effects.

Suitable for (1-10mm) displacement.

Cylindrical capacitive displacement transducer



$$C = \frac{2 \pi \epsilon x}{\ln(D_2/D_1)} \text{ Farads}$$

x → length of overlapping parts.

D₂ → inner diameter of outer cylindrical electrode

D₁ → outer diameter of inner cylindrical electrode.

1 electrode → fixed

other electrode → movable

D applied to movable tube

↓
Results in overlapping

↓
cause change in C between cylindrical tube

$$C \propto D$$

$$S = \frac{\partial c}{\partial x}$$

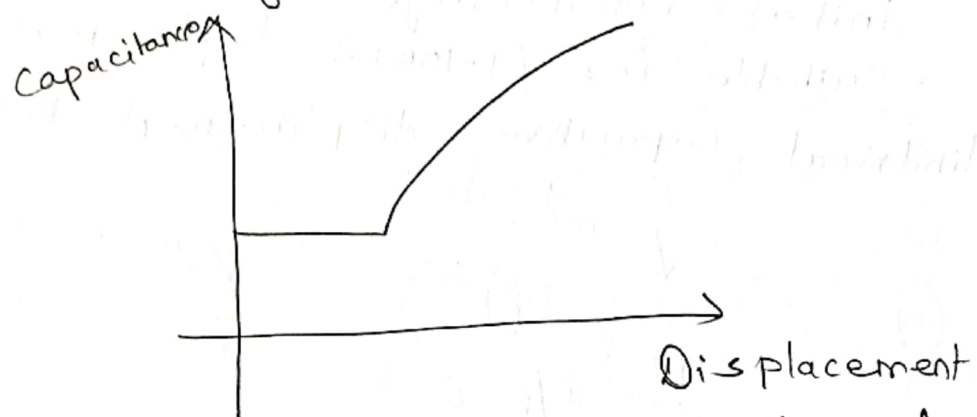
$$= \frac{\partial}{\partial x} \left(\frac{2\pi \epsilon x}{\ln(D_2/D_1)} \right)$$

$$S = \frac{2\pi \epsilon}{\ln(D_2/D_1)} \quad \text{F/m}$$

Sensitivity constant

Relation between C & D is linear.

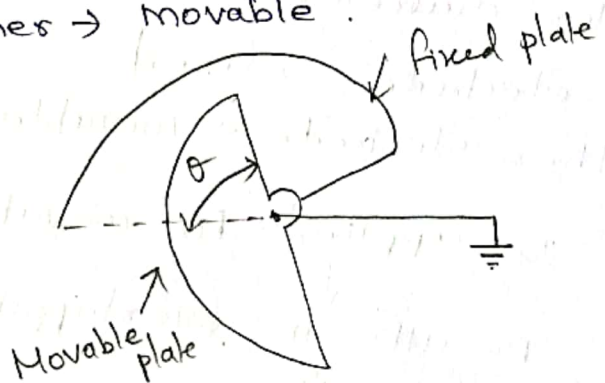
Non linearity due to edge effect.



Capacitive angular displacement based on change in area of plates.

Setup consists of 2 plates forming 2 plates of a capacitor.

- 1 → fixed
- other → movable.



Angular displacement applied to movable plate

↓
change in effective area between plates.

↓
change of capacitance.

C ↑ when 2 plates overlap each other

$$\theta = 180^\circ$$

Capacitance at any angle

$$C = \frac{\sum \theta r^2}{2d}$$

Maximum value,

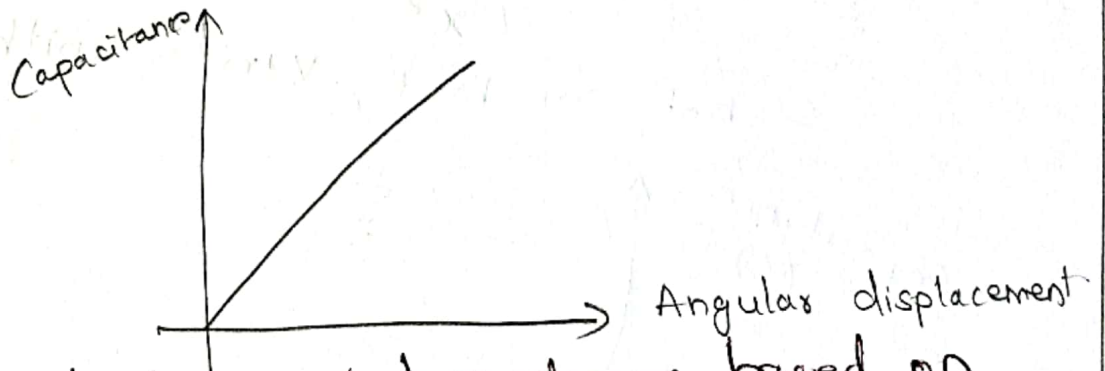
$$C_{max} = \frac{\sum \pi r^2}{2d}$$

$$S = \frac{\partial C}{\partial \theta}$$

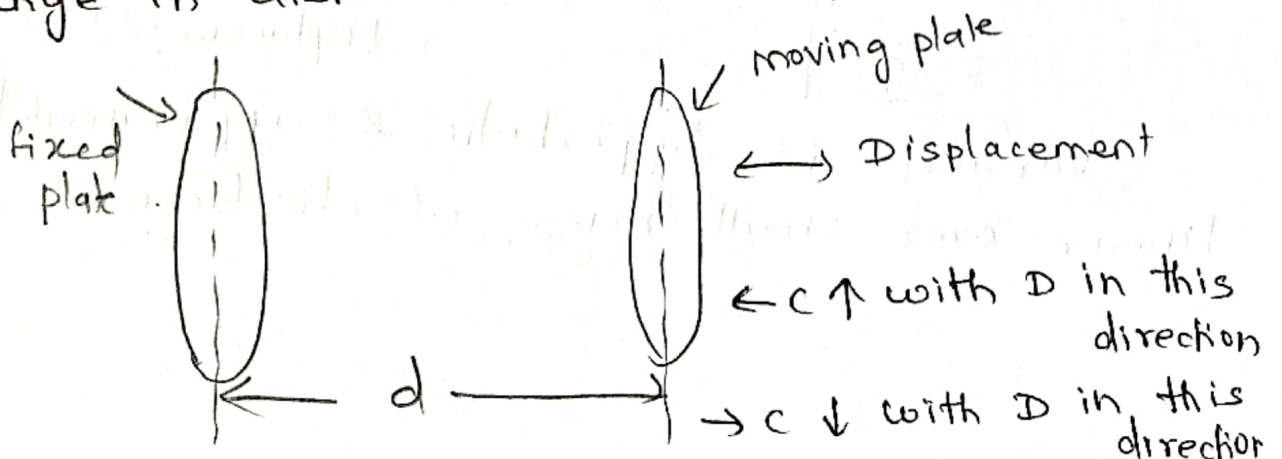
$$= \frac{\partial}{\partial \theta} \left(\frac{\sum \theta r^2}{2d} \right)$$

$$S = \frac{\sum r^2}{2d} \quad \text{F / radian}$$

C is linear with angular displacement
Used to measure maximum of 180°
angular displacement.



Capacitive displacement transducer based on change in distance between plates.



2 plates
1 movable
other fixed

D to be measured applied to movable plate

when D is to left, distance between plates ↓ses & C ↑ses

when D is to right, distance between plates ↑ses & C ↓ses.

∴ C ∝ 1/d

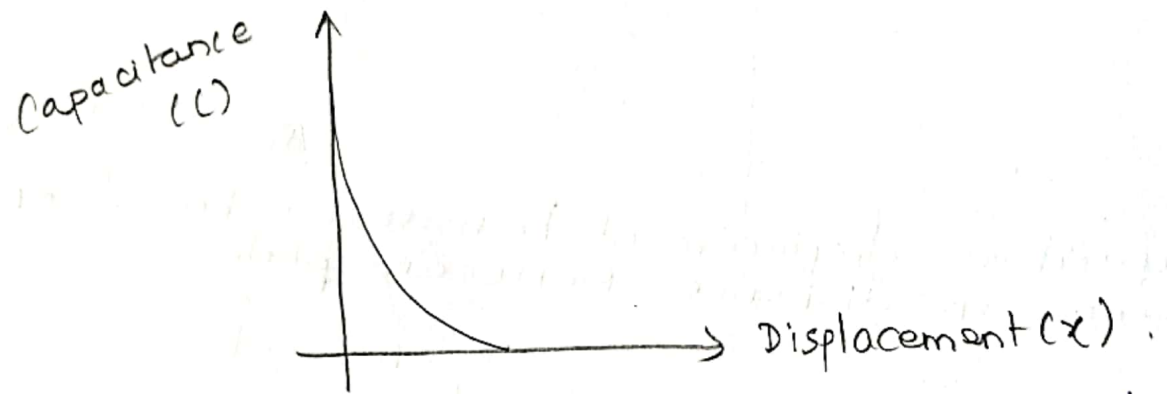
Response of transducer is nonlinear.

Used only for measurement of extremely small displacement.

$$S = \frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left(\frac{\epsilon A}{x} \right)$$

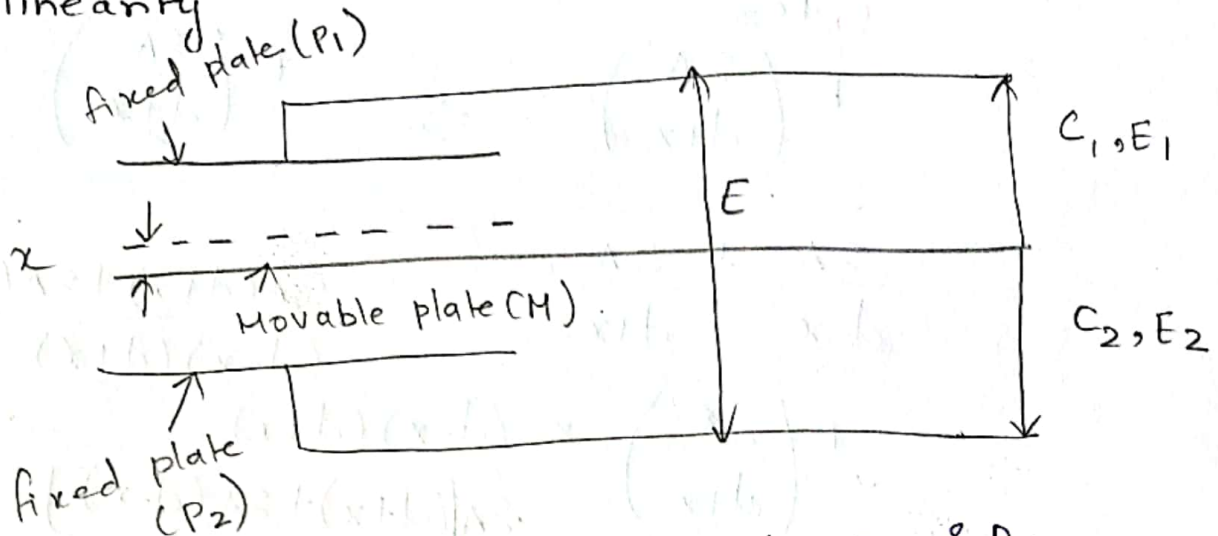
$$S = -\frac{\epsilon A}{x^2}$$

S not constant, varies with D.



Variation is hyperbolic & approximately linear over small range of displacement.

Differential arrangement for improvement of linearity



3 plates \rightarrow 2 fixed P_1 & P_2
 1 Movable plate (M)

D applied to M
 when M is midway between two fixed plates,

C_1 & C_2 = distance between P_1 & M & distance between P_2 & M are same

$$E_1 = \frac{EC_2}{C_1 + C_2} = \frac{E}{2}$$

$$E_2 = \frac{EC_1}{C_1 + C_2} = \frac{E}{2}$$

Differential voltage

$$\Delta E = E_1 - E_2$$

$$= E/2 - E/2$$

$$\Delta E = 0$$

when movable plate moves by distance 'x'.

$$C_1 = \frac{\epsilon A}{d-x} \quad \times \quad C_2 = \frac{\epsilon A}{d+x}$$

$$\begin{aligned}
 E_1 &= \frac{E c_2}{c_1 + c_2} \\
 &= E \left(\frac{\Sigma A}{d+x} \right) \Rightarrow \frac{E \left(\frac{\Sigma A}{d+x} \right)}{\frac{\Sigma A}{d-x} + \frac{\Sigma A}{d+x}} \\
 &= \frac{E \left(\frac{\Sigma A}{d+x} \right) \times \frac{\Sigma A (d+x) + \Sigma A (d-x)}{(d-x)(d+x)}}{\Sigma A (d+x) + \Sigma A (d-x)} \\
 &= \frac{E (d-x)}{d+x+d-x}
 \end{aligned}$$

$$\boxed{E_1 = \frac{E (d-x)}{2d}}$$

$$\begin{aligned}
 E_2 &= \frac{E c_1}{c_1 + c_2} \\
 &= E \left(\frac{\Sigma A}{d-x} \right) \Rightarrow \frac{E \left(\frac{\Sigma A}{d-x} \right)}{\frac{\Sigma A}{d-x} + \frac{\Sigma A}{d+x}} \\
 &= \frac{E \left(\frac{\Sigma A}{d-x} \right) \times \frac{\Sigma A (d+x) + \Sigma A (d-x)}{(d-x)(d+x)}}{\Sigma A (d+x) + \Sigma A (d-x)}
 \end{aligned}$$

$$\boxed{E_2 = \frac{E (d+x)}{2d}}$$

Hence difference voltage

$$\begin{aligned}\Delta E &= E_2 - E_1 \\ &= E \left(\frac{d+x}{2d} \right) - E \left(\frac{d-x}{2d} \right) \\ &= \frac{E}{2d} [d+x - d+x] \Rightarrow \frac{E}{2d} [2x]\end{aligned}$$

$$\Delta E = \frac{x}{d} E$$

output varies linearly with x .

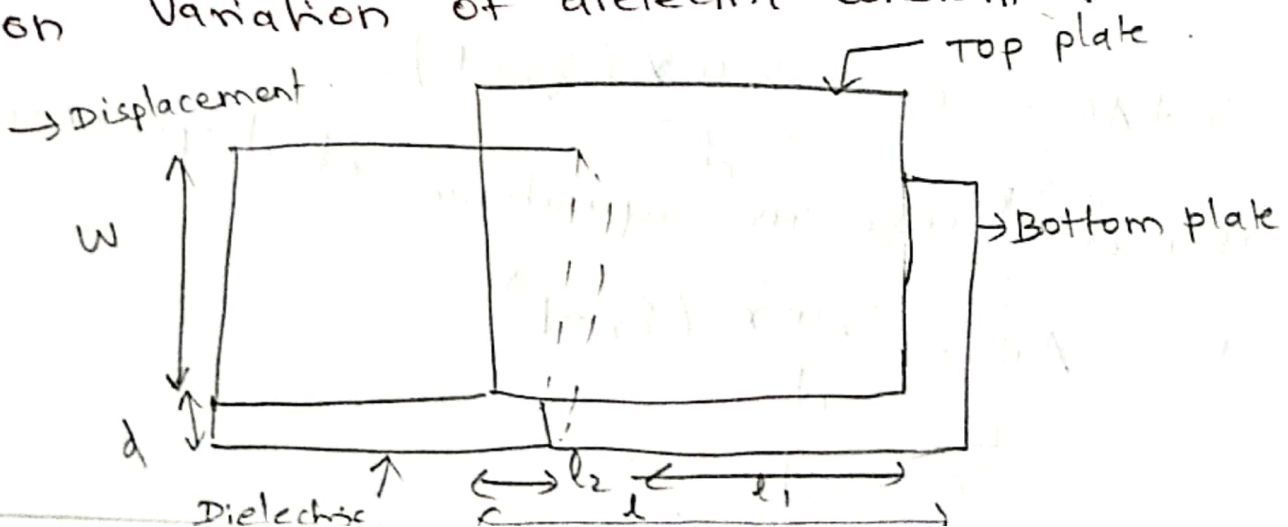
$$\begin{aligned}S &= \frac{\partial c}{\partial x} \\ &= \frac{\partial}{\partial x} \left(\frac{Ex}{d} \right)\end{aligned}$$

$$S = E/d$$

$S \rightarrow$ constant, does not depend on displacement.

suited for displacements ranging from 10^{-8} mm to 10 mm with accuracy of 0.1%.

3) Capacitive displacement transducer based on variation of dielectric constant.



Bottom & top plate → fixed
dielectric Material → Movable

$$C = \frac{\epsilon_0 \epsilon_r \omega l_2}{d} + \frac{\epsilon_0 \omega l_1}{d}$$

$$C = \frac{\epsilon_0 \omega}{d} (l_1 + \epsilon_r l_2)$$

± displacement 'x' applied to dielectric.

$$C + \Delta C = \frac{\epsilon_0 \epsilon_r \omega (l_2 + x)}{d} + \frac{\epsilon_0 \omega (l_1 - x)}{d}$$

$$= \frac{\epsilon_0 \omega}{d} (\epsilon_r (l_2 + x) + l_1 - x)$$

$$= \frac{\epsilon_0 \omega}{d} (\epsilon_r l_2 + \epsilon_r x + l_1 - x)$$

$$C + \Delta C = \frac{\epsilon_0 \omega}{d} (\epsilon_r l_2 + l_1) + \frac{\epsilon_0 \omega}{d} (\epsilon_r x - x)$$

But, $\frac{\epsilon_0 \omega}{d} (l_1 + \epsilon_r l_2) = C$

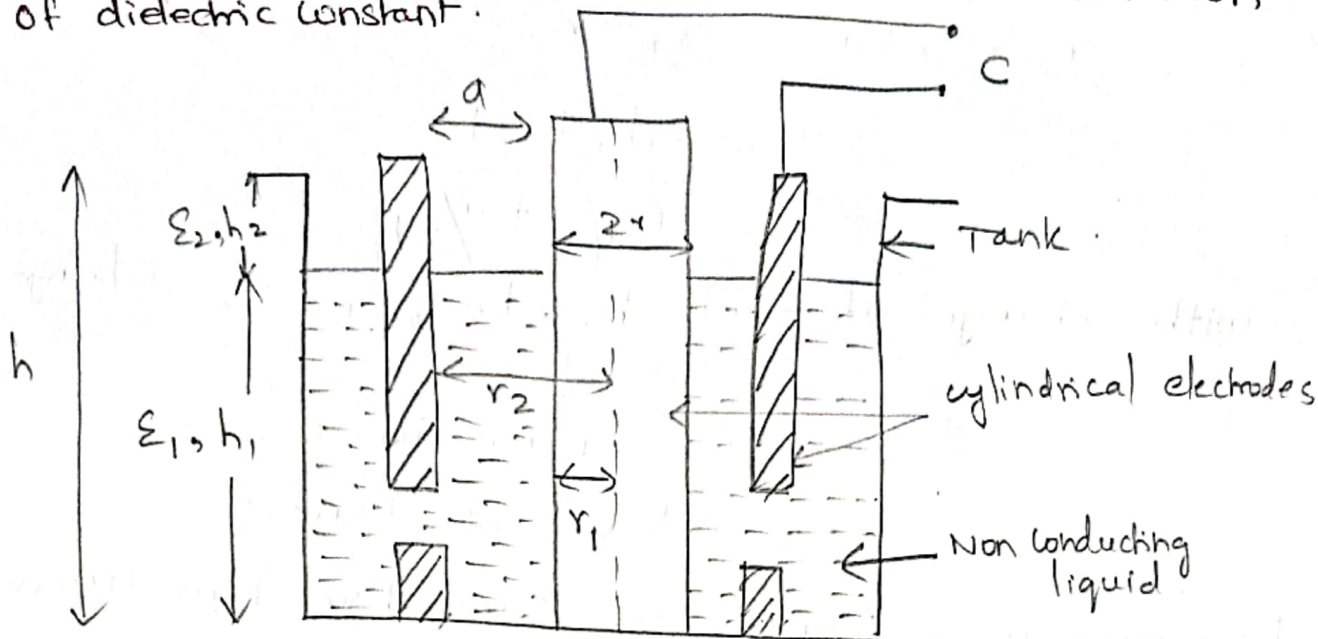
$$C + \Delta C = C + \frac{\epsilon_0 \omega x}{d} (\epsilon_r - 1)$$

Hence change in capacitance of displacement

x is $\Delta C = \frac{\epsilon_0 \omega x}{d} (\epsilon_r - 1)$

$C \propto D$

Capacitive level transducers based on variation of dielectric constant.



2 cylindrical electrode → 2 plates
 non conducting liquid → dielectric
 Lower end of outer cylinder → holes allow passage of liquid.
 Provides mechanical damping of surface variations.

$$C = 2\pi\epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\ln(r_2/r_1)}$$

- h_1 → height of liquid
- h_2 → height of cylinder above liquid
- ϵ_1 → relative permittivity of liquid
- ϵ_2 → relative permittivity of vapor above liquid
- r_2 → inside radius of outer cylinder
- r_1 → outside radius of inner cylinder
- ϵ_0 → permittivity of free space.

Assumption

$$h \gg r_2 \quad \& \quad r_2 \gg r_2 - r_1 \gg a$$

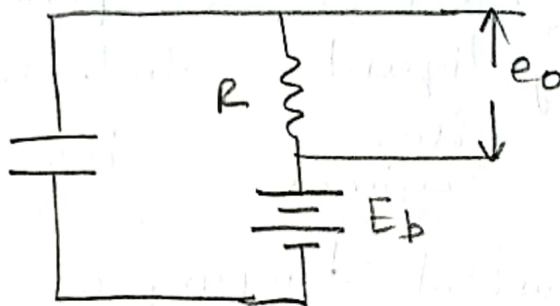
$$r_2 = r + a \quad r_1 = r$$

$$C = 2\pi \epsilon_0 \frac{\epsilon_1 h_1 + \epsilon_2 h_2}{\ln(1 + a/r)}$$

with change of level h_1, h_2, ϵ_1 & ϵ_2 change
 \downarrow
 change in C .

$C \propto$ level of tank.

Frequency response of capacitive transducers.



Distance between plates be x_0 when they are stationary.

Under this condition

no ϵ flows & output voltage $E_o = E_b$.

Transfer function of capacitive transducer is given by

$$\frac{E_o(s)}{X_i(s)} = \frac{kTs}{1+Ts}$$

where $k = \frac{E_b}{x_0}$ V/m

$T =$ Time constant $= RC$

sub $s = j\omega$

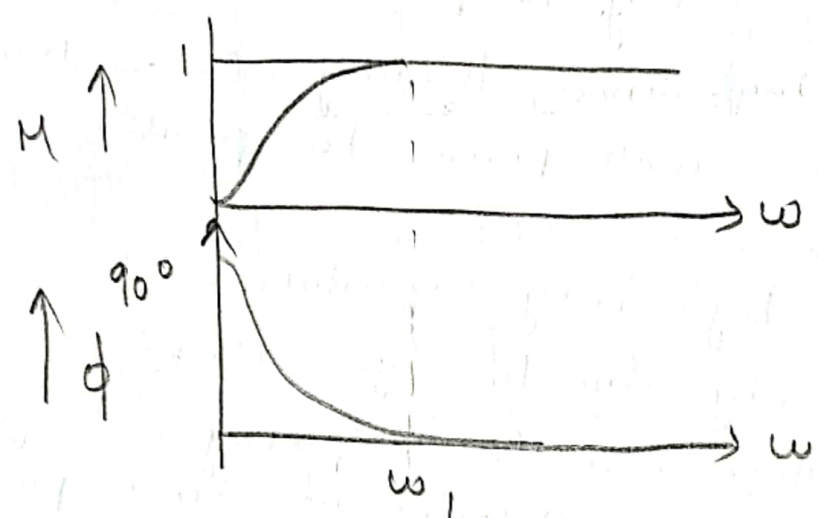
$$\frac{E_o(j\omega)}{X_i(j\omega)} = \frac{j\omega kT}{1+j\omega T}$$

Amplitude ratio can be found

$$M = \left| \frac{E_o(j\omega)}{kx_i(j\omega)} \right|$$

$$M = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$$

phase shift $\phi = \frac{\pi}{2} - \tan^{-1}\omega\tau$



Since output is zero for $\omega=0$ transducers cannot be used for static measurements.

cannot be used for low frequency applications as it results in inaccuracies.

To achieve high accuracy, these transducers should be used for high frequency application only.

Accuracy good for higher frequencies.

- ↑ freq, $\omega\tau \gg 1$, M & ϕ → constant & independent of frequency
- ↓ freq, $\omega\tau \ll 1$, τ ↑, R ↑ ($IM\Omega$).

Problem of loading effect can be avoided by use of measuring devices whose input impedance is atleast $10M\Omega$, like FET.

Advantages:

- Requires small forces & small powers to operate them
- highly sensitive, so small displacements can be easily measured.
- good frequency response for dynamic studies
- \uparrow i/p impedance, loading effects minimal.
- Require small power to operate.

Drawbacks:

- Affected by stray capacitances.
- Nonlinearity due to edge effects.
- output impedance \propto frequency of signal.
- C change on account of dust particles

& moisture.

Too sensitive & affects the performance.

Instrumentation circuitry required with transducer is very complex.

Applications:

Used for measurement of both linear & angular displacement.

Used to measure force & pressure.

Measurement of humidity.

Can also be used to measure volume, density, liquid level, weight etc.,

CAPACITOR MICROPHONE (OR) CONDENSOR

MICROPHONE.
Used for measurement of sound level in air.

For sound level in water, hydrophone is used.

Special type of pressure transducer that converts acoustic energy into electrical energy.

Types:

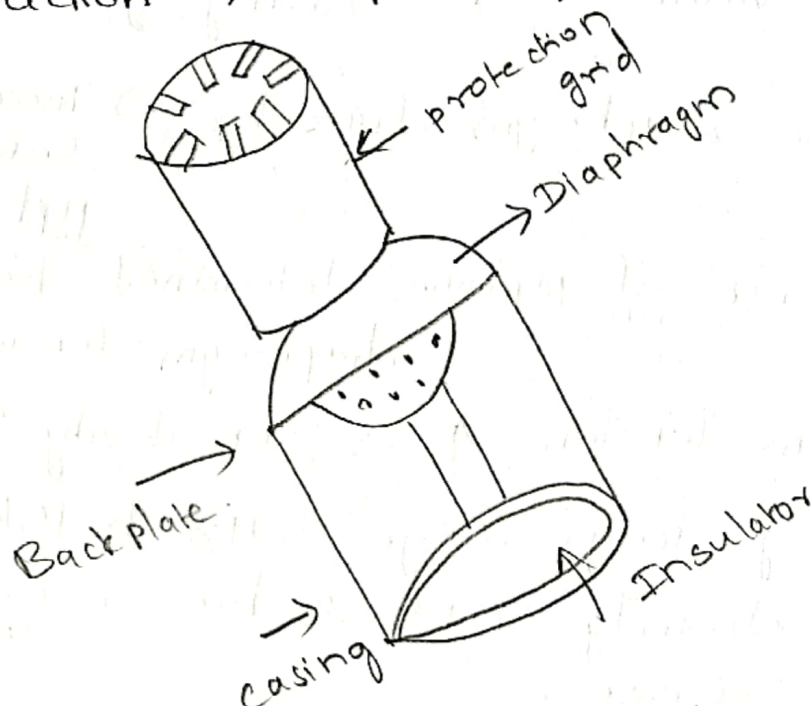
- carbon Microphone
- capacitive Microphone
- Dynamic Microphone
- Inductive Microphone
- Piezoelectric Microphone.

Capacitive Microphone

Principle:

When some amount of electrical charge is held within a capacitor, electrical potential between condenser plates will change if capacitance is changed.

Construction & operation



Diaphragm & back plate form the parallel plates of an air capacitor.

Capacitor is polarized with external voltage supply (or) by an electric charge injected directly into an insulating material on back plate.

when sound pressure in sound field fluctuates

↓
Distances between diaphragm & back plate will change.

↓
Change C of diaphragm / back plate capacitor.

Charge in $C = \text{constant}$.

Change in C will generate output voltage on o/p terminal of microphone.

Diaphragm area,	} determines freq. range sensitivity dynamic range
distance between diaphragm & back plate.	
Stiffness & Mass of diaphragm	
internal volume of μ phone casing	

Larger the diaphragm diameter \rightarrow more sensitive μ phone will be

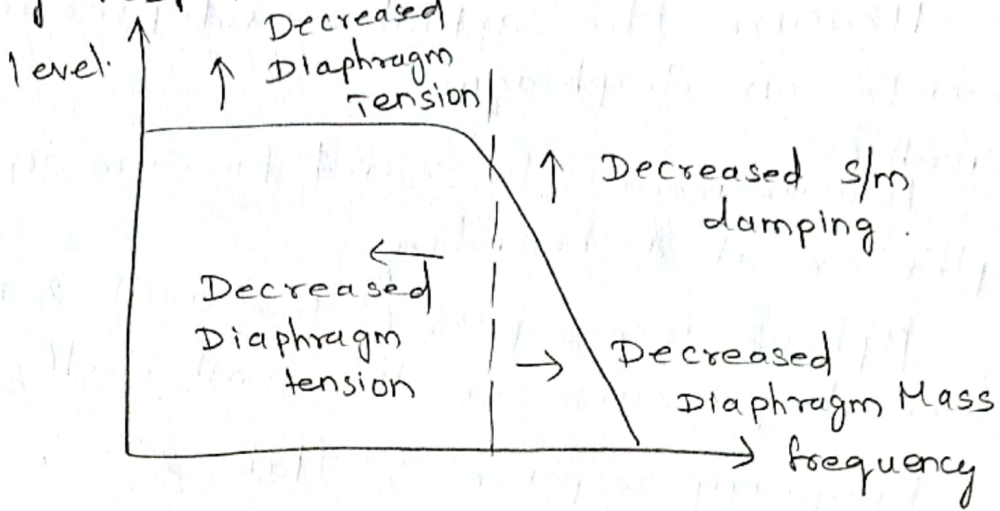
Freq. response of μ phone determined by diaphragm tension.

Diaphragm tension $\uparrow \rightarrow$ sensitivity reduced

operating freq. range 12Hz to 15kHz.

~~is~~ extremely stable & has a ~~wide~~ wide flat freq. response.

Frequency response:



Resonant frequency determined by \rightarrow diaphragm tension + diaphragm mass

Diaphragm tension \uparrow , Resonant freq \uparrow
 Diaphragm mass \uparrow , Resonant freq \downarrow .

Types of capacitor microphone.

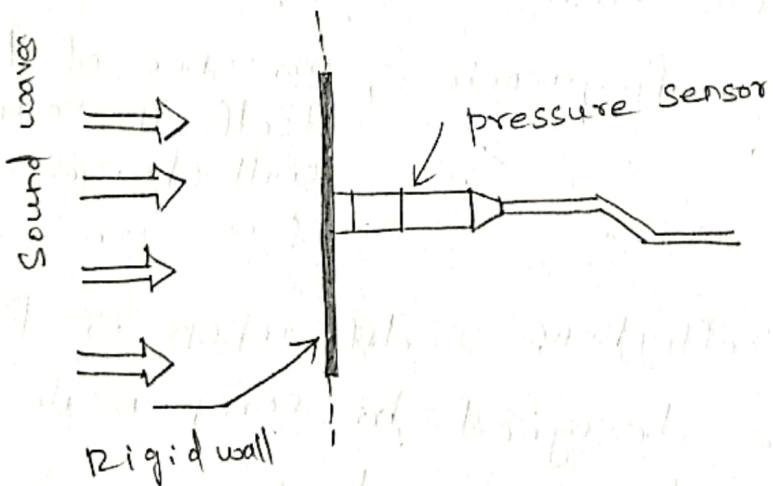
Based on freq. characteristics.

pressure microphone

Free-field microphone

Random incidence microphone.

pressure microphone.



Measure the actual sound pressure as it exists on diaphragm.

Applications:

Measurement of sound pressure in closed coupler or at a boundary.

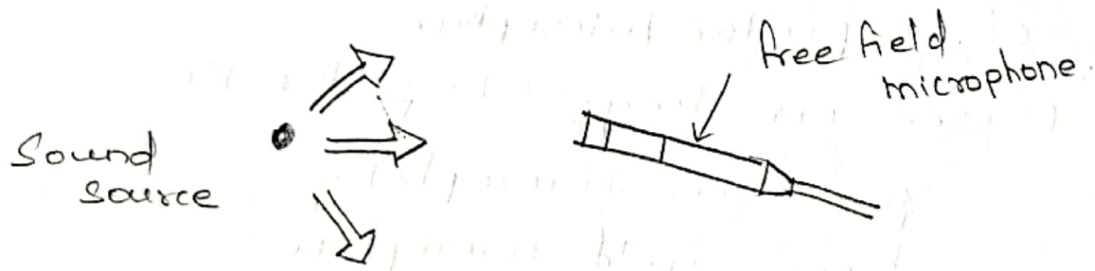
μ phone forms part of the wall & measures the sound pressure on the wall itself.

Frequency response \rightarrow flat & wide as possible.

Frequency range \uparrow sed, sensitivity \downarrow se.

Acoustical damping in air gap between diaphragm & back plate adjusted, so that frequency response is flat upto beyond resonant frequency

Free field microphone.



Designed to essentially measure the sound pressure as it existed before microphone was introduced into the sound field.

At higher frequencies, presence of μ phone itself in sound field will change sound pressure.

Due to reflections & diffraction $\rightarrow P \uparrow$

This type designed to compensate for $P \uparrow$ signal \propto sound pressure

Free-field μ phone should always be pointed toward sound source.

Random incidence microphone.

In some cases, sound waves will not have a well defined propagation direction, but will arrive at μ phone from all directions simultaneously.

Random incidence μ phone is preferred.

Sound waves arriving at μ phone from the front will cause pressure increase while waves arriving from back of μ phone will cause pressure drop to ^{certainly} extent.

Combined influence of waves coming from different directions & distribution of sound waves from different directions.